Lecture 16 - The pn Junction Diode (II)

Equivalent Circuit Model

April 10, 2001

Contents:

1. I-V characteristics (cont.)
2. Small-signal equivalent circuit model
3. Carrier charge storage: diffusion capacitance

Reading assignment:

Howe and Sodini, Ch. 6, §§6.4, 6.5, 6.9

Announcements:

Quiz 2: 4/18, 7:30-9:30 PM, Walker (lectures #10-17) open book, must bring calculator
Key questions

- How does a pn diode look like from a small-signal point of view?
- What are the leading dependences of the small-signal elements?
- In addition to the junction capacitance, are there any other capacitive effects in a pn diode?
1. I-V characteristics (cont.)

Diode current equation:

\[ I = I_o \left( \exp \frac{qV}{kT} - 1 \right) \]

Physics of forward bias:

- potential difference across SCR reduced by \( V \) \( \Rightarrow \) minority carrier injection in QNR’s
- minority carrier diffusion through QNR’s
- minority carrier recombination at surface of QNR’s
- large supply of carriers available for injection
  \( \Rightarrow I \propto e^{qV/kT} \)
Physics of reverse bias:

- potential difference across SCR increased by $V$ 
  $\Rightarrow$ minority carrier extraction from QNR’s 
- minority carrier diffusion through QNR’s 
- minority carrier generation at surface of QNR’s 
- very small supply of carriers available for extraction 
  $\Rightarrow$ $I$ saturates to small value
I-V characteristics: $I = I_o (\exp \frac{qV}{kT} - 1)$

- Linear scale
- Semilogarithmic scale

$0.43 \cdot \frac{q}{kT} = 60 \text{ mV/dec @ 300K}$
Source-body pn diode of NMOSFET (linear scale):
Source-body pn diode of NMOSFET (logarithmic scale):
Temperature extraction from forward bias characteristics:

\[ T \approx \frac{q}{k} \frac{1}{d(ln I)/dV} \]
Key dependences of diode current:

\[ I = qA n_i^2 \left( \frac{1}{N_a w_p - x_p} D_n + \frac{1}{N_d w_n - x_n} D_p \right) \left( \exp \frac{qV}{kT} - 1 \right) \]

- \( I \propto \frac{n_i^2}{N} \left( \exp \frac{qV}{kT} - 1 \right) \equiv \text{excess minority carrier concentration at edges of SCR} \)
  
  - in forward bias: \( I \propto \frac{n_i^2}{N} \exp \frac{qV}{kT} \): the more carrier are injected, the more current flows
  
  - in reverse bias: \( I \propto -\frac{n_i^2}{N} \): when minority carrier concentration drops to zero, the current saturates

- \( I \propto D \): faster diffusion \( \Rightarrow \) more current

- \( I \propto \frac{1}{wQR} \): shorter region to diffuse through \( \Rightarrow \) more current

- \( I \propto A \): bigger diode \( \Rightarrow \) more current
2. Small-signal equivalent circuit model

Examine effect of small signal overlapping bias:

\[ I + i = I_o \left[ \exp \frac{q(V + v)}{kT} - 1 \right] \]

If \( v \) small enough, linearize exponential characteristics:

\[ I + i \simeq I_o \left( \exp \frac{qV}{kT} \exp \frac{qv}{kT} - 1 \right) \simeq I_o \left[ \exp \frac{qV}{kT} (1 + \frac{qv}{kT}) - 1 \right] \]

\[ = I_o \left( \exp \frac{qV}{kT} - 1 \right) + I_o \left( \exp \frac{qV}{kT} \right) \frac{qv}{kT} \]

Then:

\[ i = \frac{q(I + I_o)}{kT} v \]

From small signal point of view, diode behaves as conductance of value:

\[ g_d = \frac{q(I + I_o)}{kT} \]
Small-signal equivalent circuit model, so far:

\[ g_d \]

\( g_d \) depends on bias. In forward bias:

\[ g_d \approx \frac{qI}{kT} \]

\( g_d \) is linear in diode current.
Must add capacitance associated with depletion region:

\[ C_j = \frac{q\varepsilon_s N_a N_d}{\sqrt{2(N_a + N_d)(\phi_B - V)}} \]

Depletion or junction capacitance:

\[ C_j = A \frac{q\varepsilon_s N_a N_d}{\sqrt{2(N_a + N_d)(\phi_B - V)}} \]

Can rewrite as:

\[ C_j = A \frac{q\varepsilon_s N_a N_d}{\sqrt{2(N_a + N_d)\phi_B}} \frac{\phi_B}{\phi_B - V} \]

Or,

\[ C_j = \frac{C_{jo}}{\sqrt{1 - \frac{V}{\phi_B}}} \]

where

\[ C_{jo} \equiv \text{zero-voltage junction capacitance} \]
3. Carrier charge storage: diffusion capacitance

What happens to the majority carriers?

Carrier picture so far:

If in QNR minority carrier concentration ↑ but majority carrier concentration unchanged
⇒ quasi-neutrality is violated.
Quasi-neutrality demands that at every point in QNR:

\[ \text{excess minority carrier concentration} = \text{excess majority carrier concentration} \]

Mathematically:

\[ p'(x) = p(x) - p_o \approx n'(x) = n(x) - n_o \]

Define integrated carrier charge:

\[
q_{Pn} = qA \frac{1}{2} p'(x_n)(w_n - x_n) = qA \frac{w_n - x_n}{2} \frac{n_i^2}{N_d} (\exp \frac{qV}{kT} - 1) = -q_{Nn}
\]
Now examine small increase in $V$:

Small increase in $V \Rightarrow$ small increase in $q_{Pn} \Rightarrow$ small increase in $|q_{Nn}|$

Behaves as capacitor of capacitance:

$$C_{dn} = \frac{dq_{Pn}}{dV} = q A \frac{w_n - x_n}{2} \frac{n_i^2}{N_d kT} \frac{q}{kT} \exp\left(\frac{qV}{kT}\right)$$
Can write in terms of $I_p$ (portion of diode current due to holes in n-QNR):

$$C_{dn} = \frac{q}{kT} \frac{(w_n - x_n)^2}{2D_p} qA \frac{n_i^2}{N_d} \frac{D_p}{w_n - x_n} \exp\left(\frac{qV}{kT}\right)$$

$$\approx \frac{q}{kT} \frac{(w_n - x_n)^2}{2D_p} I_p$$

Define *transit time* of holes through n-QNR:

$$\tau_{Tp} = \frac{(w_n - x_n)^2}{2D_p}$$

Transit time is *average time for a hole to diffuse through n-QNR* [will discuss in more detail in BJT]

Then:

$$C_{dn} \approx \frac{q}{kT} \tau_{Tp} I_p$$
Similarly for p-QNR:

\[ C_{dp} \approx \frac{q}{kT} \tau_{Tn} I_n \]

where \( \tau_{Tn} \) is transit time of electrons through p-QNR:

\[ \tau_{Tn} = \frac{(w_p - x_p)^2}{2D_n} \]

Both capacitors sit in parallel \( \Rightarrow \) total diffusion capacitance:

\[ C_d = C_{dn} + C_{dp} = \frac{q}{kT} (\tau_{Tn} I_n + \tau_{Tp} I_p) \]

Complete small-signal equivalent circuit model for diode:

![Diode Circuit Diagram]
Bias dependence of $C_j$ and $C_d$:

- $C_j$ dominates in reverse bias and small forward bias ($\sim 1/\sqrt{\phi_B - V}$)

- $C_d$ dominates in strong forward bias ($\sim e^{qV/kT}$)
Key conclusions

Small-signal behavior of diode:

- **conductance**: associated with current-voltage characteristics
  
  \[ g_d \sim I \quad \text{in forward bias, negligible in reverse bias} \]

- **junction capacitance**: associated with charge modulation in depletion region
  
  \[ C_j \sim \frac{1}{\sqrt{\phi_B - V}} \]

- **diffusion capacitance**: associated with charge storage in QNR’s to keep quasi-neutrality
  
  \[ C_d \sim e^{qV/kT} \]