Math Methods: Problem Set # 1

Due: 27 Aug 2012

Suggested reading: §1.7, 3.1, 3.2, 3.5, 3.6

Problem 1

Draw a picture that demonstrates (geometrically) the associativity of vector addition.

Problem 2

Prove the law of cosines starting from $A^2 = (B - C)^2$. Hint: draw a triangle with vectors $A$, $B$ and $C$, and choose theta wisely.

Problem 3

Prove that $(A \times B) \cdot (A \times B) = (AB)^2 - (A \cdot B)^2$

Problem 4 Fix your book

Find the error on the first cover page of the textbook. This page will serve you well as a quick reference... once it has fewer typos. Explain what has gone wrong with the equation in question, and fix it with a sharpie or some white out (that was a hint).
Problem 5  Kronecker delta and Levi Cevita Symbol

a. Define the Kronecker delta $\delta_{ij}$

b. Define the Levi-Cevita symbol $\varepsilon_{ijk}$

c. Show that (in 3D space):
   i) $\sum_i \delta_{ij} = 3$
   ii) $\sum_{ij} \delta_{ij} \varepsilon_{ijk} = 0$
   iii) $\sum_{pq} \varepsilon_{ipq} \varepsilon_{jqp} = \delta_{ij}$
   iv) $\sum_{ijk} \varepsilon_{ijk} \varepsilon_{ijk} = 6$

d. Show that (in 3D space): $\sum_k \varepsilon_{ijk} \varepsilon_{pkq} = \delta_{ip} \delta_{jq} - \delta_{iq} \delta_{jp}$

e. Express a cross product using $\varepsilon_{ijk}$

f. Prove $A \times (B \times C) = B(A \cdot C) - C(A \cdot B)$

Problem 6  Leibnitz’s rule

Show that $\nabla(uv) = v\nabla u + u\nabla v$ where $u$ and $v$ are differentiable scalar functions of $x$, $y$ and $z$.

Problem 7  Angular momentum operator

Using the following angular momentum operators:

$$L_x = -i \left( y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y} \right) \quad L_y = -i \left( z \frac{\partial}{\partial x} - x \frac{\partial}{\partial z} \right) \quad L_z = -i \left( x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right)$$

Show that they satisfy commutation relations of the form

$$[L_x, L_y] \equiv L_x L_y - L_y L_x = iL_z$$

and hence

$$L \times L = iL$$
Problem 8

a. Show that $\nabla \times \nabla \phi = 0$ for a scalar field $\phi$

b. Show that $\nabla \cdot (\nabla \times \mathbf{A}) = 0$ for a vector field $\mathbf{A}$

Problem 9
Verify $\nabla \times (\nabla \times \mathbf{V}) = \nabla (\nabla \cdot \mathbf{V}) - \nabla \cdot \nabla \mathbf{V}$ by direct expansion in Cartesian Coordinates.

Problem 10
Prove that $\nabla \times (\phi \nabla \phi) = 0$