Math Methods: Problem Set # 8

Due: 29 Oct 2012

Suggested reading: 11.5-11.7 in Edition 7, and if you want 11.8. I am back to xeroxes, because I just make too many typos TeXxing this stuff up at 2 am. I have also attached a few pages from chapter 5 of my book on power-series, in case it’s useful.

Problem 1
Do problem 6.5.1 in the attached pages of the book.

Problem 2
Do problem 6.5.4 in the attached pages of the book.

Problem 3
Do problem 6.5.9 in the attached pages of the book.

Problem 4
Do problem 6.5.11 in the attached pages of the book.

Problem 5
Do problem 7.1.1 in the attached pages of the book.
Problem 6
Do problem 7.1.3 in the attached pages of the book. The example 7.1.1 is included in the zeroxes on page 412.

Problem 7
Do problem 7.2.1 in the attached pages of the book.

Problem 8
Do problem 7.2.3 in the attached pages of the book.

Problem 9
Do problem 7.2.17 in the attached pages of the book.

Problem 10
Do problem 7.2.19 in the attached pages of the book.
1. $\frac{1}{2} - \frac{1}{3}x + \frac{1}{4}y = \frac{1}{5}$

2. $12x - 3y = 4$  

3. $2x + 3y = 5$

4. $5x - 2y = 1$  

5. $x + 2y = 3$

6. $3x - y = 2$  

7. $4x + 3y = 7$

8. $2x - 5y = 0$

9. $x + y = 1$

10. $3x - 2y = 4$

11. $5x + 3y = 8$

12. $2x - 3y = -1$

13. $4x + 5y = 9$

14. $3x - 2y = 1$

15. $5x + 4y = 7$

16. $2x + 3y = 5$

17. $4x - 3y = 2$

18. $3x + 2y = 4$

19. $5x - 4y = 3$

20. $2x + 5y = 7$

Exercises

1. $x^2 + y^2 = 1$

2. $x^2 - y^2 = 0$

3. $x^2 + 2xy + y^2 = 1$

4. $x^2 - 2xy + y^2 = 0$

5. $x^2 + 4xy + 4y^2 = 1$

6. $x^2 - 4xy + 4y^2 = 0$

7. $x^2 + 6xy + 9y^2 = 1$

8. $x^2 - 6xy + 9y^2 = 0$

9. $x^2 + 8xy + 16y^2 = 1$

10. $x^2 - 8xy + 16y^2 = 0$
The equation of the curve of the form

$$\frac{1}{4}(x - 2)^2 + \frac{1}{8}(y - 2)^2 = 2$$

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7.2 Calculus of Residues

The residue of a function at a point is the coefficient of the term in its Laurent expansion.

For a function $f(z)$, the residue at a point $z_0$ is given by:

$$ \text{Res}(f(z), z_0) = \lim_{z \to z_0} (z - z_0) f(z) $$

This limit exists if $f(z)$ has a pole of order $n$ at $z_0$, and the residue is then equal to $\frac{1}{(n-1)!} \text{d}^{n-1} \left[ (z - z_0)^n f(z) \right] / \text{d} z^{n-1} \bigg|_{z = z_0}$.

The residue theorem states that the integral of a function around a closed contour is equal to $2\pi i$ times the sum of the residues inside the contour.

$$ \oint_C f(z) \, dz = 2\pi i \sum \text{Res}(f(z), z_k) $$

where $z_k$ are the singular points inside the contour $C$.

Example: For the function $f(z) = \frac{1}{z^2 - 1}$, the residues are $\frac{1}{2}$ at $z = 1$ and $\frac{1}{2}$ at $z = -1$.

$$ \oint_C \frac{1}{z^2 - 1} \, dz = 2\pi i \left( \frac{1}{2} + \frac{1}{2} \right) = 2\pi i $$

This result is useful in evaluating definite integrals of complex functions.

7.3 Functions of a Complex Variable
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PS # 5

\[
\frac{d^2x}{dt^2} + \omega^2 x = 0
\]

where \(x\) is the displacement from equilibrium, \(\omega\) is the angular frequency, and \(t\) is time.

This differential equation describes simple harmonic motion, where the restoring force is directly proportional to the displacement from equilibrium.

\[
\text{Exercises}
\]

(1) \(x(t) = A \cos(\omega t + \phi)\)

(2) \(x(t) = A \sin(\omega t + \phi)\)

(3) \(x(t) = A e^{-\alpha t} \cos(\omega t + \phi)\)

where \(A\) is the amplitude, \(\omega\) is the angular frequency, \(\phi\) is the phase constant, \(\alpha\) is the decay constant, and \(t\) is time.

These equations represent various forms of simple harmonic motion, with different characteristics.

The exercises are designed to help students understand and apply the concepts of simple harmonic motion.
Here are the equations and formulas:

\[
\frac{1 + \cos \theta}{\sin \theta} = \frac{2}{\sin \theta} \cos \left(\frac{\theta}{2}\right)
\]

These equations are used to solve various problems in trigonometry. The diagram illustrates the geometric representation of the equations, showing the relationship between the angles and sides of a right-angled triangle.
\[ f \] is a function of \( x \) if \( \Delta f = f(x + \Delta x) - f(x) \). The derivative of a function at a point is the limit of the difference quotient as \( \Delta x \) approaches zero.

If \( f \) is differentiable at a point, then the function is continuous at that point.

\[ f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \]

If \( f \) is differentiable on an interval, then the function is continuous on that interval.

\[ f'(x) = \frac{df}{dx} \]

**CONTINUITY**

\[ f(x) = \begin{cases} \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} \]

\[ f(0) = \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{1}{x} = \infty \]

The function is not continuous at \( x = 0 \) because the limit does not exist.

**S'**

\[ f'(x) = \frac{d}{dx} e^x = e^x \]

**POWER SERIES**

The function is a power series of the form:

\[ f(x) = \sum_{n=0}^{\infty} a_n x^n \]

Where the coefficients \( a_n \) are the derivatives of \( f \) evaluated at zero.
Math Methods

PS # 5

Example

Write the expression for the coefficient of expansion of a new medium.

\[ a(x) = \frac{x}{1 + \frac{x}{n}} \]

**Exercise**

\[ \sum_{n=1}^{\infty} (\frac{x^n}{n!}) \]

**Example**

The expression for the coefficient of expansion of a new medium is given by:

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**Exercise**

\[ \sum_{n=1}^{\infty} (\frac{x^n}{n!}) \]

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