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Professional Portfolio Selection Techniques: From Markowitz to Innovative Engineering

Introduction to Portfolio Management Techniques and Introduction to the PID Model

Antonella Sabatini
in collaboration with Gino Gandolfi and Monica Rossolini

MIT – Jan 06, 2014
12:00-2:00pm, 32-124
The process of portfolio construction
Asset allocation:
- strategic asset allocation
- tactical asset allocation

PID Model:
a new tactical asset allocation technique
PID feedback controller theory
Applications and future research
The Process of Portfolio Construction: 3 Macrophases

1st
To know the Clients

2nd
To manage Clients’ Expectations

3rd
To Manage Clients’ Portfolios
Phase 1: To know the Clients

*Portfolio Manager’s Skills:*

- To know the Client
- Consulting and Constructing a plan
- Seeking Solutions Identifying those suitable Financial Services and Tools which are Available
- Managing Client’s Portfolio
Phase 1: To know the Clients and Analysis of Clients’ Needs

Building up a Personalized and Individual Relation with the Client, Outlined in 5 Steps

Step 1
Analysis of Financial Needs and Priorities
Phase 1: Analysis of Clients’ Needs

Step 2
Identifying and Verifying Constraints

Objective Constraints:

\[ \downarrow \]
Analysis of Client’s Assets, Current Income and Future Expectations

Subjective Constraints:

\[ \downarrow \]
Client’s Risk and Time Horizon grid
Phase 1: Analysis of Clients’ Needs

**Step 3:** Presentation of Possible Solutions, given First Priorities and Needs; Illustration of Possible Scenarios, given the Macroeconomic Situation and the Bank/Institution Policies.

**Step 4:** Actuating and managing the chosen Financial Activities

**Step 5:** Continuous Monitoring of Client’s Needs, of the chosen Financial Instruments and Investments, and Market Conditions.
The Process of Portfolio Construction: 3 Macrophases

1st

To know the Clients

2nd

To manage Clients’ Expectations

3rd

To Manage Clients’ Portfolios
Phase 2: To manage Clients’ Expectations

Requires:

Adopting and conveying to a framework in order to understand performance (Time-Series of Interest, Holding Period-Performance Relationship, etc.)

Aiming to:

• Have the Client Intuitively Understand the Trade off between Risk and Reward
• Have the Client Acknowledge the range of the possible (attainable) results and performance
Phase 2: To manage Clients’ Expectations

In Addition:

Sharing the Investment Philosophy And Strategic approach

With the Purpose of:

• Defining the Risk Level implicit to all the possible managing techniques
• Improving Communication and Understanding between Client and Manager, by furthering on Client’s side, the Dynamics and Risk embedded in the specified investment management technique
The Process of Portfolio Construction: 3 Macrophases

1st

To know the Clients

2nd

To manage Clients’ Expectations

3rd

To Manage Clients’ Portfolios
Phase 3: To Manage Clients’ Portfolios

PORTFOLIO OPTIMIZATION: Defining the Asset Allocation And the Portfolio Design

Implementing the Investment Strategy and the Management Techniques

Performance Measurement and Monitoring and revising the Portfolio
Asset Allocation (AA): Definition

Asset Allocation is defined as follows:
• Investment Analysis tool leading to the desired Portfolio:
• Portfolio construction is obtained through the identification of the optimal asset mix
  – given a desired time horizon (Holding Period) and
  – given investor’s risk averse level
AA: Introduction

Asset Allocation focuses on and supplies various elements:

- Risk Reduction through Diversification.
- Portfolio comprising those Assets exhibiting the best opportunity to achieve positive returns.
- Impulsive and Emotional factors Reduction.
Asset Allocation

Strategic Asset Allocation

Tactical Asset Allocation

1

2
1. Strategic Asset Allocation

Component of Asset Allocation, implemented by the identification of the optimal long term mix, and by monitoring results and performance on yearly intervals.

In contrast, Tactical Asset Allocation (TAA), aims to periodically take the most interesting Investment opportunities by temporarily and partially deviating from the main strategic portfolio structure.
1. Strategic Asset Allocation

A study on the contributions to portfolio performance by the various determinants (strategic, stock selection, market timing) has estimated that the 91.5% of performance is given by strategic Asset Allocation.

% Contribution to Portfolio Performance

![Graph showing the contribution to portfolio performance over time.](image)
1. Strategic Asset Allocation

- The longer the investment time horizon the more is the performance contribution of the activity provided by Strategic Asset allocation.

Long Term Asset Allocation is less sensitive to short term market fluctuations providing more stable returns than short term techniques.
1. Strategic Asset Allocation: Exemples

Case A:
Investor’s type: **Strongly Conservative**
Level of Risk Aversion: Highest

Case B
Investor’s type: **Conservative**
Level of Risk Aversion: High

Case C
Investor’s type: **Aggressive**
Level of Risk Aversion: Low

<table>
<thead>
<tr>
<th>Case</th>
<th>Investor’s type</th>
<th>Level of Risk Aversion</th>
<th>Bonds</th>
<th>Stocks</th>
<th>Cash</th>
<th>Treasury Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Strongly</td>
<td>Highest</td>
<td>15%</td>
<td>0%</td>
<td>25%</td>
<td>60%</td>
</tr>
<tr>
<td>B</td>
<td>Conservative</td>
<td>High</td>
<td>20%</td>
<td>40%</td>
<td>15%</td>
<td>25%</td>
</tr>
<tr>
<td>C</td>
<td>Aggressive</td>
<td>Low</td>
<td>35%</td>
<td>40%</td>
<td>5%</td>
<td>15%</td>
</tr>
</tbody>
</table>
The Modern Portfolio Theory

Modern Portfolio Theory (MPT) is the traditional approach to the identification of the Optimal Portfolio for investors, in terms of Risk and Expected Return.

Main studies:
Markowitz (1952) Mean-Variance principle, Efficient Frontier
Sharpe (1964)
Lintner (1965)
Mossin (1966)
Ross (1976)  Arbitrage Pricing Theory

Capital Asset Pricing Model
The Relationship Return/Risk

- Fundamental Hypothesis of the MPT: investors are Risk Averse.
- MPT states that expected return on an investment and investment risk are directly proportional.
Investor’s Risk aversion levels

• The problem is that all investors would like to achieve high returns, but not all of them are able to bear high risk. Risk can be represented by a loss in invested capital (downside risk) or by excessive capital fluctuation (standard deviation).

• In other words, not all investors have the same risk aversion level.
Risk Aversion or Volatility Aversion?

- The term “risk” tends to have a negative meaning; in reality, since risk embeds uncertainty of results, a higher risk implies:
  - a higher probability of losses;
  - a higher probability of achieving higher returns.

- Risk is, therefore, a negative element and a positive element.
The Risk-Return Relationship

Based on these assumptions it is possible to choose different asset classes:

- **Given the same Return, the activity with lower risk is preferable**
  - *C is better than B*
- **Given the same Risk, the activity with higher return is preferable**
  - *A is better than B*

Choosing is not always easy!

*Between A and C, which is better?*
Portfolio Optimization

Assuming to know:

- The investor’s risk tolerance
- Determining of the asset classes to be included in the portfolio

AIM To determine the optimal composition of a portfolio
Markowitz’s Model

Aim

- To define the optimal portfolio able to provide the investor with the highest expected return given a risk level, or, viceversa, the lowest possible risk given a value of expected return
Markowitz’s Hypotheses

- Investors choose their portfolios according to 2 parameters: average expected return and expected risk; the latter is measured as variance of returns (mean variance principle).
- Investors are risk adverse and they maximize expected utility
- Uniperiodicity
From 1 asset to a portfolio

• Portfolio return is given by the weighted mean of all the asset returns
• Portfolio risk is less than or equal to weighted risk of all assets
• Thus, investing in a Portfolio is better than in a single asset since a portfolio has a lower risk due to diversification
From 1 asset to a portfolio

• The best Portfolio is not the one formed by the less risky assets taken individually.

• Correlations among the assets are important.
From 1 asset to a portfolio: example

- 3 assets: A, B, C
- 3 years: 1st, 2nd, 3rd
- The assets have the following returns:

<table>
<thead>
<tr>
<th>ASSET</th>
<th>YEAR 1</th>
<th>YEAR 2</th>
<th>YEAR 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>C</td>
<td>30</td>
<td>5</td>
<td>-5</td>
</tr>
</tbody>
</table>

- Which is the best Asset?
From 1 asset to a portfolio: example

• Calculate the risk and average return of the three assets:

<table>
<thead>
<tr>
<th>ASSET</th>
<th>Avrg RET</th>
<th>RISK</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10.00</td>
<td>between 5.0 and 15.0</td>
</tr>
<tr>
<td>B</td>
<td>10.00</td>
<td>between 0.0 and 20.0</td>
</tr>
<tr>
<td>C</td>
<td>10.00</td>
<td>between 30.0 and -5.0</td>
</tr>
</tbody>
</table>

• The risk is intuitively represented by the range of the possible returns in the 3 year period:
  – Asset A dominates asset B
  – Asset B dominates asset C
From 1 asset to a portfolio: example

Which is the best portfolio?
(Note: 2 assets of equal weight)

<table>
<thead>
<tr>
<th>ASSET</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Avrg Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>2.50</td>
<td>10.00</td>
<td>17.50</td>
<td>10.00</td>
</tr>
<tr>
<td>BC</td>
<td>15.00</td>
<td>7.50</td>
<td>7.50</td>
<td>10.00</td>
</tr>
<tr>
<td>AC</td>
<td>17.50</td>
<td>7.50</td>
<td>5.00</td>
<td>10.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ASSET</th>
<th>Min Ret</th>
<th>Max Ret</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>2.50</td>
<td>17.50</td>
</tr>
<tr>
<td>BC</td>
<td>7.50</td>
<td>15.00</td>
</tr>
<tr>
<td>AC</td>
<td>5.00</td>
<td>17.50</td>
</tr>
</tbody>
</table>
From 1 asset to a portfolio: example

- Considering risk and return of the 3 portfolios:

<table>
<thead>
<tr>
<th>ASSET</th>
<th>Avrg Ret</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>10.00</td>
<td>15.00</td>
</tr>
<tr>
<td>BC</td>
<td>10.00</td>
<td>7.50</td>
</tr>
<tr>
<td>AC</td>
<td>10.00</td>
<td>12.50</td>
</tr>
</tbody>
</table>

1) Portfolio BC dominates all the others, despite asset A, the best of the 3 assets was not picked.
2) Investors prefer to select portfolio BC in spite of the single asset A; even though A, individually, dominates the other assets.
Correlations among Assets

• The previous example shows that the portfolio return equals the weighted average of the individual asset returns; whereas, portfolio risk decreases as correlations among the assets decrease.

• Correlation is a statistical measure of how much the movement of two securities or asset classes are related. The range of possible correlations is between -1 and +1.
Correlations among Assets

- Positive Correlation equal to 1: Assets move in the same direction and with the same intensity;
- Positive correlation (> 0): Assets move, in general, in the same direction;
- Zero correlation (= 0): assets move independently one from the other;
- Negative correlation (< 0): assets move, in general, in opposite directions;
- Negative correlation equal to -1: assets move in opposite directions with the same intensity.
Portfolio Diversification

Aim

• To reduce portfolio risk. The risk is calculated as the variance of the returns, $\sigma_p^2$.

\[
\sigma_p^2 = \sum_i X_i^2 \cdot \sigma^2(R_i) + \sum_i \sum_j X_i \cdot X_j \cdot \sigma(R_i) \cdot \sigma(R_j) \cdot \rho_{i,j}
\]

where $X_i$ and $X_j$: portfolio i-th and j-th asset weights
The benefits of diversification

<table>
<thead>
<tr>
<th>Asset</th>
<th>Return</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset A</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Asset B</td>
<td>20%</td>
<td>20%</td>
</tr>
</tbody>
</table>

Correlation = +1

- 50% Asset A, 50% Asset B
The benefits of diversification

- 50% Asset A, 50% Asset B
- Correlation = +1
- Correlation = 0
- 50% Asset A, 50% Asset B

<table>
<thead>
<tr>
<th>Asset</th>
<th>Expected Return</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset A</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Asset B</td>
<td>20%</td>
<td>20%</td>
</tr>
</tbody>
</table>

10% Return

15% Return

20% Return
The benefits of diversification

Asset A
Asset B

Expected Return
10% 15% 20%

Return 10% 15% 20%
Risk 10% 20%

Correlation = +1
Correlation = 0
Correlation -1

50% Asset A, 50% Asset B

10% 15% 20%

Asset A
Asset B

Risk
Portfolio Diversification

How

• Picking and including in the portfolio those assets with a correlation different than 1

Effect

• Portfolio Risk is different from the simple average of the individual asset risks included in the portfolio
Portfolio Diversification

Scenario 1:

• Portfolio composed by N assets. Every Asset has risk equal to $\sigma$ and is zero correlated ($\rho=0$).

• Portfolio Risk $\sigma_p$ is

  that is $\sigma_p<\sigma$

  $$\sigma_p = \frac{\sigma}{\sqrt{N}}$$
Portfolio Diversification

Scenario 2

• Portfolio composed by N assets. Every Asset has risk equal to $\sigma$, weights $1/N$, and correlations $\rho < 1$

$$\sigma_p = \sigma \left( \sqrt{\frac{1 + \rho(N-1)}{N}} \right)$$

• That is $\sigma_p < \sigma$
Markowitz’s Model

**inputs**

- Asset expected returns
- Linear correlations
- Asset risk ($\sigma^2$)
2-Asset Model

Given...

\[ X_i \rightarrow \text{weight of the i-th asset} \]

Constraint \[ \sum_{i} X_i = 1 \quad \text{with} \quad i=1, 2, \ldots, n \]

Given...

a portfolio P formed by 2 assets, A and B with expected returns \( E(R_A) \) and \( E(R_B) \) and with weights \( (X) \) and \( (1- X) \), respectively,
2-Asset Model

- The portfolio expected return $\mu_p$ is:
  
  $$\mu_p = E(R_p) = X \cdot E(R_A) + (1-X) \cdot E(R_B)$$

  where $X + (1-X)=1$

- The portfolio risk $\sigma^2_p$ is:
  
  $$\sigma^2_p = X^2 \cdot \sigma^2_A + (1-X)^2 \cdot \sigma^2_B + 2 \cdot X \cdot (1-X) \cdot \sigma_A \sigma_B \rho_{AB}$$

  where
  
  $\sigma^2_A$: Asset A variance
  $\sigma^2_B$: asset B variance
  $\sigma_A$: asset A standard deviation
  $\sigma_B$: Asset B standard deviation
  $\rho_{AB}$: correlation between A and B.
2-Asset Model

- By varying the weight of A, (X), we get a series of points P(µ, σ²) on the plane [µ, σ²] (mean-variance), which define the region of market opportunities.
- The upper edge of such region is called the Efficient Frontier.
Portfolio Optimization

Expected Returns

Standard Deviation (Risk)

0.00 3.00 6.00 9.00 12.00 15.00 18.00 21.00 24.00 27.00 30.00 33.00 36.00 40.00

0.00 1.00 2.00 3.00 4.00 5.00 6.00 7.00 8.00 9.00 10.00 11.00 12.00 13.00 14.00 15.00 16.00 17.00 18.00 19.00

A

B

C

Ing. Antonella Sabatini, P.E.
Efficient Portfolio given N assets

**Given**: 3 assets A, B, C
AB: Efficient Frontier Assets A and B.
BC: Efficient Frontier Assets B and C.

If we consider portfolio D on the line AB, it is possible to construct an efficient frontier DC between asset C and D. The curves constructed by all the combinations among assets and portfolios form the efficient frontier AC for the 3 assets.
Efficient Portfolio given N assets

\[
\begin{align*}
\mu & \\
\sigma^2 & 
\end{align*}
\]

- A
- B
- C
- D
Efficient Portfolio given N assets

By iteratively repeating the process N times, we obtain the efficient frontier for N assets; the efficient frontier points have coordinates \((\mu, \sigma^2)\) given by:

\[
E(R_p) = \sum_i E(R_i) \cdot X_i
\]

\[
\sigma^2_p = \sum_i X_i^2 \cdot \sigma^2(R_i) + \sum_i \sum_j X_i \cdot X_j \cdot \sigma(R_i) \cdot \sigma(R_j) \cdot \rho_{i,j}
\]

with \(i, j = 1, 2, \ldots, n\)
Efficient Portfolio given N assets

In case of more assets, the procedure is more complicated because all the correlations are to calculated.
The Efficient Frontier

• Includes the optimal portfolios given the trade-off risk/return.

• It is not possible to calculate, ex-ante, any portfolio which goes above the efficient frontier.

• All the portfolios which are located below the efficient frontier are not efficient; in fact,
  – there always exists a better portfolio with the same risk (and higher return)
  – and there always exists a better portfolio at the same return level (and lower risk)
The Efficient Frontier

Efficient Frontier

- Expected Return
- Standard Deviation (Risk)

Conservative
Balanced
Aggressive

Ing. Antonella Sabatini, P.E.
• After defining the Efficient Frontier, the optimal portfolio for a specific investor needs to be chosen.

• Markowitz’s model uses Indifference Curves based on the Utility Function Squared.

\[ E(u) = E(r) - \frac{1}{2} \lambda \sigma^2 \]

Where:
- \( \sigma \) standard deviation
- \( E \) expected value operator
- \( \lambda \) Risk Aversion Coefficient
- \( u \) utility function
- \( r \) return
Given a set of Indifference Curves, the Optimal Portfolio is determined by the Risk and Return values defined by the tangential point of the Efficient Frontier with the highest Utility Curve of the set [each point on the indifference curve renders the same level of utility (=satisfaction) for the investor]
Some Negative Aspects and Weaknesses of Markowitz’s Model

- Simplifying assumptions: i.e. all assets are risky
- Parameter estimation
- Incompleteness of the picking criteria
- Uniperiodicity
- Extreme optimal portfolio weights enhanced by using asset allocation constraints
- Symmetric definition of risk
The Capital Asset Pricing Model - CAPM

- MPT by Markowitz has proven the existence of the Efficient Frontier, CAPM introduces risk free asset and risk free loan
- Which portfolio is preferable?
- It depends on the investor’s risk aversion level

E(R)

\[ \sigma \]

- CAPITAL MARKET LINE
- EFFICIENT FRONTIER
- RETURN/RISK INDIFFERENCE CURVES
The Capital Asset Pricing Model - CAPM

• If an investor can go short, and can get a loan at the Risk Free Rate (RFR), among the portfolios on the efficient frontier, it is possible to identify a portfolio (M) which is the preferred one.

• Portfolio M, named market portfolio, is the only one in which investors are interested in. The line connecting the RFR and M is the Capital Market Line (CML).
The Capital Asset Pricing Model - CAPM

Market equilibrium requires:
1) that the Risk Free Rate is such that offering risk free rate is equal to asking risk free rate;
2) All investors hold only portfolio M.
Market Model

\[ r_i = \alpha_i + \beta_i r_{mkt} + \varepsilon_i \]

Where:
- \( r_i \): asset return
- \( \alpha_i \): asset return when market return is zero
- \( \beta_i \): systematic risk
- \( r_{mkt} \): market return
- \( \varepsilon_i \): random error term

Courtesy Dobbins R. Witt S.F., "Portfolio Theory and Investment Management"
- \( \beta > 1 \) aggressive securities; larger price variation than market trend.
- \( 0 < \beta < 1 \), defensive securities; smaller price variation than market trend.
- \( \beta < 0 \) anticyclical securities; price variation opposite to market trend (theoretical Hypothesis)
Portfolio Diversification

Risk ($\sigma$) = $R_{\text{systematic}} + R_{\text{specific}}$

• **Systematic Risk (market risk)** ⇒ Risk Component given by the asset sensibility to market oscillations ($\beta$)

$$\beta = \rho_{i,m} \frac{\sigma_i}{\sigma_m}$$

Where:
- $\rho$ correlation
- $\sigma$ standard deviation
• Specific Risk (non-market risk) ⇒ Risk component derived from specific factors (business investment plans, dividends, etc.)

*Diversification reduces non-systematic risk only*
Portfolio Diversification

Number of Assets: chart analysis …

- Specific Risk [Removable]
- Systematic Risk – non removable -
Black and Litterman – B&L: parameters estimation

• One of the main limits of Markowitz’s model concerns expected returns estimation.

• Black and Litterman [1992] model estimates asset class returns, calculated as the weighted average of equilibrium returns [excess return generated by an equilibrium risk premium – strategic returns] and investor views (tactical returns).
• Market Equilibrium:
\[
\lambda = \frac{R_{mkt} - R_f}{\sigma_{mkt}^2}
\]

• Investor views: B&L Methodology allows to indicate two types of views:
  Absolute views (fixed levels of returns for a single asset class)
  Relative views (levels of outperformance/underperformance of an asset class compared to another)
For each view, a confidence level must be specified showing the asset-manager’s confidence about his view.
• B&L returns, obtained by the combination of strategic returns and views, are integrated in a mean-variance optimization process (i.e. Markovitz’s process).

• Constraints can also be considered (i.e. constraints about minimum and maximum exposition for each asset class or for each macro-asset class)
Asset Allocation

1. Strategic Asset Allocation
2. Tactical Asset Allocation
Tactical asset allocation

• Strategic asset allocation aims to define expected return and risk, and then the optimal portfolio in the medium and long period according to investor’s features.

• **Tactical asset allocation**: is composed by all actions to manage portfolio in short period, within strategic lines defined by strategic asset allocation
Tactical asset allocation

- Dynamic management techniques
  - Rigorous trading rules providing weight adjustments in the portfolio when price of the asset class changes

- Pure tactical management techniques
  - Predominance of asset-manager’s role in deciding on rebalancing with the aim to catch better opportunities
Tactical asset allocation strategies

Courtesy Sampagnaro G., “Asset Management: Tecniche e Stile di Gestione del Portafoglio”

- **BUY-AND-HOLD**
- **CONSTANT PROPORTION**
- **CONSTANT PROPORTION and PORTFOLIO INSURANCE**
- **CORE-SATELLITE**
- **ACTIVE STRATEGIES**

<table>
<thead>
<tr>
<th>Asset manager intervention</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure tactical techniques</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dynamic techniques</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
BUY-AND-HOLD

• The necessity to periodically rebalance a portfolio is due to asset classes price fluctuations.

• ➔ change in asset weights ➔ generating a change in risk/return ratio (defined by strategic asset allocation for strategic/ optimal portfolio)
Example:

- **Portfolio composed by:**
  - risky asset (50%)
  - Non-risky asset (50%)
- portfolio value 100

- **1 month later:**
  - risky asset increase its value by 10%
  - What does it happen?
  - Portfolio value increases from 100 to 105

- **Change in asset weights:**
  - risky asset: 55/105 = 52.38%
  - Non-risky asset: 50/105 = 47.62%

- **Is it a good composition for an investor’s risk tolerance?**
• In this scenario:
  – Buy-and-Hold strategies do nothing
  – the weights change depending on the asset price

Advantage:
• Low cost

Disadvantage:
• High correlation between a risky asset and the market
CONSTANT PROPORTION

Constant Mix Strategy: the aim is to conserve the initial weight composition that tends to variate with price fluctuations.
Example:

- **Portfolio composed by:**
  - risky asset (50%)
  - Non-risky asset (50%)
  - value 100

- **After 1 month:**
  - risky asset increases its value by 10% (from 50 to 55)
  - What does it happen?

- **Portfolio value increases from 100 to 105**

- **Change in asset weights**
  - risky asset: $\frac{55}{105} = 52.38\%$
  - Non-risky asset: $\frac{50}{105} = 47.62\%$
• In this scenario:
  – the aim of constant mix strategy is to conserve the initial composition
    • 50% risky asset
    • 50% non-risky asset

• Selling a part of risky asset for a value of 2.38% of portfolio value and with this money buying non-risky asset.
When does the asset-manager rebalance a portfolio?

- Periodic rebalancing (fixed time intervals)
- Threshold rebalancing (rebalancing takes place when asset class weights change more than a specific level (Threshold) as a consequence of asset price fluctuations.
- Range rebalancing (similar to the previous one; rebalancing does not aim to restore the strategic asset allocation weights; but it provides maximum deviation)
- Volatility based rebalancing (rebalancing takes place when the volatility increases above a specific level)
CONSTANT PROPORTION AND PORTFOLIO INSURANCE

• CPPI, asset allocation strategy defined by dynamic rebalancing, offers
  – the possibility to capture market opportunities
  – to protect the portfolio value, with a combination of risky assets and non-risky assets
CORE-SATELLITE

• The portfolio is divided into 2 parts:
  – the core-portfolio
  – the satellite-portfolio
• Core portfolio: passive strategy aiming to obtain benchmarked performance (i.e. ETF)
• Satellite-portfolio: active strategy aiming to obtain an overperformance compared to
  – the benchmark
  – the core portfolio
Advantages:
To obtain differential returns with lower costs

<table>
<thead>
<tr>
<th>Fund</th>
<th>Active risk constraint</th>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5%</td>
<td>40 b.p.</td>
</tr>
<tr>
<td>B</td>
<td>0%</td>
<td>20 b.p.</td>
</tr>
<tr>
<td>C</td>
<td>20%</td>
<td>55 b.p.</td>
</tr>
</tbody>
</table>

Example:
100% fund A, Active risk 5%, Load 40 b.p.
Otherwise
75% (core) fund B, Active risk 0%, Load 15 b.p.
25% (satellite) fund C, Active risk 5%, Load 13.75 b.p.
Portfolio (core+satellite): Active risk 5%, Load 28.75 b.p.
ACTIVE STRATEGIES

- Active strategies aim to obtain an extra performance compared to the benchmark
- Passive strategies aim to obtain the same performance of the benchmark.
Active strategies

MARKET TIMING

STOCK SELECTION
2. Market Timing

‘Forecasting of market dynamics – Portfolio Rebalancing in consequence of Trend Forecasts in order to improve portfolio performance
2. Market Timing

**Market timing requires:**

- Forecasting skills (scenario analysis, Macro trends etc.);

- Timing Skills;

- Managing the interacting factors and market variables (Inflation/Interest Rates; Interest Rate /Returns; Beta/Returns, etc.)

- Technical Analysis indicators and charting skills
2. Market Timing

Typical Activity of Tactical Strategies of short term investing/disinvesting activities

**Timing and economic Cycles**

Portfolio manager tends to take investment opportunities deriving from adapting SAA techniques to the current economic cycles via rebalancing among the different asset classes.

Timing and economic cycles via rebalancing among the different asset classes.
2. Market Timing

During an activity where market timing predominates, the Manager tends to focus on a small number of securities for which he/she has quantitative and qualitative data in support of his/her analysis.

The objective in this case is:

Outperforming relative to the benchmark by increasing the portfolio sensitivity to the expected returns.

The Risk is:

Excessive reduction of portfolio diversification and asymmetry between risk tolerance and market volatility exposure.

...continua
## 2. Market Timing

Market timing and $\beta$-based strategies for the stock portion of the portfolio:

<table>
<thead>
<tr>
<th>Beta Values</th>
<th>Market Phase</th>
<th>Securities Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta \leq -1$</td>
<td>Market Recession</td>
<td>Atypical securities</td>
</tr>
<tr>
<td>$\beta \leq -0.5$</td>
<td>Down-trending market</td>
<td>Anticyclical securities</td>
</tr>
<tr>
<td>$-0.5 \leq \beta \leq 0.5$</td>
<td>Non-trending market</td>
<td>Conservative Securities</td>
</tr>
<tr>
<td>$0.5 \leq \beta \leq 1$</td>
<td>Up-trending market</td>
<td>Aggressive Securities</td>
</tr>
<tr>
<td>$\beta &gt; 1$</td>
<td>Expanding market</td>
<td>High Risk securities</td>
</tr>
</tbody>
</table>

..continued
2. Market Timing

Market timing and $\beta$-based strategies for the stock portion of the portfolio:

- **Strongly Up-trending**
  - **Strategy**: Increase portfolio $\beta$
  - **Focus and Overweighing** aggressive securities ($\beta > 1$)

- **Down-trending**
  - **Strategy**: Reduce portfolio $\beta$
  - **Focus and Overweighing** on anticyclical and negatively correlated securities ($\beta < 0$)
2. Market Timing

Market Timing and fixed income Duration-analysis

Based on the *Modified Duration Formula*

\[ \frac{dP}{P} = -D \frac{di}{1 + i} = -DMdi \]

[A measure of the price sensitivity of a bond to interest rate movements ]
it is possible to follow a continuous duration analysis of the portfolio

<table>
<thead>
<tr>
<th>SCENARIO</th>
<th>DURATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>RISING INTEREST RATES</td>
<td>LOW</td>
</tr>
<tr>
<td>FALLING INTEREST RATES</td>
<td>HIGH</td>
</tr>
</tbody>
</table>
2. Market Timing

Market Timing and fixed income Duration-analysis

Negative Aspects of Duration Analysis
- Numerous and complex data flow analysis (price, rate of return, cash flow of all the bonds);
- High frequency of calculations required (based on market fluctuations);
- Risk covered is Interest Rate risk only. Duration takes into account interest rate risk only and not global risk phenomena (credit risk, exchange rate risk, etc.)
3. Stock Picking

Analysis Activity and Securities Picking for insertion in a portfolio

Possible criteria for stock picking
Analysis of technical characteristics:
- Liquidity
- Risk
- Return
- Expiration date
### 3. Stock Picking

<table>
<thead>
<tr>
<th>Securities</th>
<th>Liquidity</th>
<th>Risk</th>
<th>Return</th>
<th>Expiration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Certificates of Dep.</td>
<td>High</td>
<td>Low</td>
<td>Low</td>
<td>fixed</td>
</tr>
<tr>
<td>Treasury Notes</td>
<td>High</td>
<td>Low/Medium</td>
<td>Medium</td>
<td>Various</td>
</tr>
<tr>
<td>Bonds</td>
<td>Good</td>
<td>medium</td>
<td>Medium/Low</td>
<td>Medium/Long</td>
</tr>
<tr>
<td>LT Bonds</td>
<td>Medium</td>
<td>High</td>
<td>High</td>
<td>Various</td>
</tr>
<tr>
<td>Stocks</td>
<td>Good</td>
<td>Highest</td>
<td>Highest</td>
<td>no expiration</td>
</tr>
</tbody>
</table>

gennaio 5, 2014
Q&A

Thank you for your attention.

Break!!
1. INTRODUCTION

INNOVATION

Use of the Feedback controller, widely applied in most industrial processes, as a technique for financial portfolio management.**

AIM

Tactical Portfolio Asset Allocation Technique.

METHOD

Rebalancing of Assets determined by the controlled value of Risk Adjusted Return subject to the action of the Controller.

The innovative procedure consists in the controlling action over the uncertain behavior of the plurality of assets comprising the portfolio. The controller attempts to regulate the dynamics of the portfolio by rebalancing the weights of the different assets in such a way to force the portfolio risk adjusted return to approach the Set Point.

(*) ** Patent Pending – International - National
1. INTRODUCTION

The Innovation
Comprises
Seeking
STABILITY
CONSISTENCY
of Portfolio Return
over the time Horizon
by “controlling” Return
2. BACKGROUND

- **Strategic Asset Allocation** = Selecting a Long Term Target Asset Allocation
  - most common framework: mean-variance construction of Markowitz (1952)

- **Tactical Asset Allocation** = Short Term Modification of Assets around the Target
  - systematic and methodic processes for evaluating prospective rates of return on various asset classes and establishing an asset allocation response intended to capture higher rewards
2. BACKGROUND

• Tactical Asset Allocation (TAA)
  – asset allocation strategy that allows active departures from the Strategic asset mix based upon rigorous objective measures
  – active management.
  – It often involves forecasting asset returns, volatilities and correlations.
  – The forecasted variables may be functions of fundamental variables, economic variables or even technical variables.
4. SYSTEMS: MANUAL VS AUTOMATIC SYSTEMS

• Manual Control = System involving a Person Controlling a Machine.

• Automatic Control = System involving Machines Only.
4. SYSTEMS: MANUAL VS AUTOMATIC SYSTEMS
(ESAMPLES)

- Manual Control: Driving an Automobile
- Automatic Control: Room Temperature Set by a Thermostat
4. SYSTEMS: REGULATORS VS TRACKING (SERVO) SYSTEMS

- Regulators: Systems designed to Hold a System Steady against Unknown Disturbances
- Servo: Systems designed to Track a Reference Signal
4. OPEN-LOOP SYSTEMS

- The Controller does not use a Measure of the System Output being Controlled in Computing the Control Action to Take.
4. FEEDBACK SYSTEMS

- Feedback Systems (Processes): defined by the Return to the Input of a part of the Output of a Machine, System, or Process.
- Controlled Output Signal is Measured and Fed Back for use in the Control Computation.
4.1 OPEN AND CLOSED LOOPS

OPEN LOOP SYSTEM

CLOSED LOOP SYSTEM

System 1 affects system 2

System 2 affects system 1
4.1 CLOSED LOOP (EXAMPLE)

- Household Furnace Controlled by a Thermostat:

![Diagram of a closed loop system with a thermostat controlling a gas valve, which in turn affects house temperature.](Fig. 01 – BLOCK DIAGRAM)
4.1 CLOSED LOOP (EXAMPLE)

- Household Furnace Controlled by a Thermostat: Plot of Room Temperature and Furnace Action

- Initially Room Temperature << Reference (or SET POINT) Temperature.
- Thermostat ON
- Gas Valve ON
- Heat Qin supplied to House at rate > Qout (Heat loss)
- Room temperature will rise until > Reference Point
- Gas Valve OFF
- Room Temperature will drop until below Reference point
- Gas Valve ON......
4.1 CLOSED LOOP (Components)

- ACTUATOR = Gas Furnace
- PROCESS = House
- OUTPUT = Room Temperature
- Disturbances = Flow of Heat from the house via wall conduction, etc.
- PLANT = Combination of Process and Actuator
- CONTROLLER = components which compute desired controlled signal
- SENSOR = Thermostat
- COMPARATOR = Computes the difference between reference signal and sensor output.
4.2 FEEDBACK SYSTEM PARAMETERS

• **Set-Point** = Target Value that an Automatic Control System will aim to Reach.

• **Output** = Current Output of the System.

• **Error** = Difference between **Set Point** and Current Output of the System.

• **Block Diagram of Plant** = Mathematical Relations in Graph Form
4.3 DYNAMICS

- Dynamic Model = Mathematical Description via equation of motion of the system
- Three domains within which to study dynamic response
  - S-plane
  - Frequency Response
  - State Space
4.3 DYNAMICS

- Feedback allows the Dynamics (Behavior) of a System to be modified:
  - Stability Augmentation.
  - Closed Loop Modifies Natural Behavior.
4.3 DYNAMICS - Superposition

• **PRINCIPLE OF SUPERPOSITION** – if input is a sum of signals ⇒ Response = Sum of Individual Responses to respective Signals
  – It works for Linear Time-Invariant Systems
  – Used to solve Systems by System responses to a set of elementary signals
    • Decomposing given signal into sum of elementary responses
    • Solve subsystems
    • General response = sum of single subsystem solutions
    • Elementary signals
      – **Impulse** = Intense Force for Short Time
      \[
      \int_{-\infty}^{\infty} f(\tau) \delta(t-\tau) d\tau = f(t)
      \]
      – **Exponential** \( e^{st} \)
4.3 DYNAMICS – Transfer Function

- Exponential input \( u(t) = e^{st} \)
- \( \Rightarrow \) Output of the form \( y(t) = H(s)e^{st} \)
- Where:
  - \( S \) can be complex \( S = \sigma + j\omega \)

- Transfer Function = Transfer gain from \( U(S) \) to \( Y(S) = \frac{Y(S)}{U(S)} = H(S) \)
  - Ratio of the Laplace Transform of Output to Laplace Transform of Input
4.3 DYNAMICS – Laplace Transform

Definition

\[ F(s) = \int_{0}^{\infty} f(t) e^{-st} dt \]
4.3 DYNAMICS – Laplace Transform

S-Plane

\[ \delta(t) \iff \frac{1}{s} \quad \text{Impulse} \]

\[ 1(t) \iff \frac{1}{s} \]

\[ e^{-kt} \iff \frac{1}{s+k} \]

\[ t \iff \frac{1}{s} \]

\[ te^{-kt} \iff \frac{1}{(s+k)^2} \]

\[ 1-e^{-kt} \iff \frac{k}{s(s+k)} \]

\[ \sin(kt) \iff \frac{k}{s^2+k^2} \]

\[ \cos(kt) \iff \frac{s}{s^2+k^2} \]

\[ h(t) = e^{-kt} \quad 1(t) \iff H(S) \]

\[ f(t - k) \iff F(S) e^{-ks} \]

\[ f(kt) \iff \frac{1}{k} F\left(\frac{S}{k}\right) \]

\[ e^{-kt} f(t) \iff F(S + k) \]
4.3 DYNAMICS – Frequency Response

**Frequency response** is the measure of any system's spectrum response at the output to a signal of varying frequency (but constant amplitude) at its input.

\[
u(t) = A \cos(\omega t) = \frac{A}{2} \left( e^{j\omega t} + e^{-j\omega t} \right)\]

\[
s = j \omega\]

\[
u(t) = e^{j\omega t}\]

\[
y(t) = H(j \omega) e^{j\omega t}\]

\[
u(t) = e^{-j\omega t}\]

\[
y(t) = H(-j \omega) e^{-j\omega t}\]

\[
y(t) = \frac{A}{2} \left[ H(j \omega) e^{j\omega t} + H(-j \omega) e^{-j\omega t} \right]\]

\[
H(j \omega) = M(\omega) e^{j\varphi(\omega)}\]

\[
y(t) = \frac{A}{2} \left[ M(\omega) e^{j(\omega t + \varphi)} + M(\omega) e^{-j(\omega t + \varphi)} \right]\]

\[
= AM \cos(\omega t + \varphi)\]

\[
M = |H(j \omega)|, \varphi = \angle H(j \omega)\]
4.3 DYNAMICS – Frequency Response

Bode Plot

for $k=1$:

$$H(s) = \frac{1}{s+k}$$

$$H(j\omega) = \frac{1}{j\omega+k}$$

$$M = \frac{1}{\sqrt{\omega^2+k^2}}$$

$$\phi = -\tan^{-1}\left(\frac{\omega}{k}\right)$$

$$y(t) = AM \cos(\omega t + \phi)$$
4.4 BLOCK DIAGRAM

\[ \frac{Y_2(s)}{U_1(s)} = G_1G_2 \]

(a)

\[ \frac{Y(s)}{U(s)} = G_1 + G_2 \]

(b)

\[ \frac{Y(s)}{R(s)} = \frac{G_1}{1 + G_1G_2} \]

(c)
Fig. 01

Transfer Function = Linear Mapping of the Laplace Transform of the Input, $R$, to the Output $Y$

$$\frac{Y}{R} = \frac{G_c G_p}{1 + G_c G_p}$$

Where $Y = \text{Process Output}$; $R = \text{Set-Point}$; $G_p = \text{Process Gain}$; $G_c = \text{Controller Gain}$

$$\frac{Y(S)}{U(S)} = H(S)$$
4.5 STABILITY – Poles & Zeros

\[ H(s) = \frac{b(s)}{a(s)} \]

Such S-values \( \Rightarrow \)

Poles of \( H(s) \)

Transfer Function

Denominator factors

\[ a(s) = 0 \rightarrow H(s) = \infty \]

\[ H(s) = \frac{b(s)}{a(s)} \]

Such S-values \( \Rightarrow \)

Zeros of \( H(s) \)

Transfer Function

Numerator Factors

\[ b(s) = 0 \rightarrow H(s) = 0 \]
4.5 STABILITY – Poles & Zeros

Exponential decay ➔ Stability

\[ H(s) = \frac{1}{s + k} \]
\[ h(t) = e^{-kt} 1(t) \]
\[ k > 0 \rightarrow s < 0 \]

Exponential growth ➔ Instability

\[ H(s) = \frac{1}{s + k} \]
\[ h(t) = e^{-kt} 1(t) \]
\[ k < 0 \rightarrow s > 0 \]

\[ \tau = \frac{1}{k} \]
\[ \tau = \text{Time Constant} \]
4.5 STABILITY – Poles & Zeros

\[ H(s) = \frac{2s + 1}{s^2 + 3s + 2} = \frac{-1}{s+1} + \frac{3}{s+2} \]
4.5 STABILITY – Poles & Zeros

EXPLORING THE S-PLANE.....
4.5 STABILITY – Poles & Zeros

STABLE

UNSTABLE

LHP

RHP

Im(s)

Re(s)
4.5 Complex Poles

\[ H(s) = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \]

\( \zeta \) Damping Ratio

\( \omega_n \) Natural Frequency

\( s = -\sigma \pm j \omega_d \)

\( \sigma = \zeta \omega_n \)

\( \theta = \sin^{-1} \zeta \)

\( \omega_d = \omega_n \sqrt{1 - \zeta^2} \)
4.5 Impulse Response

For Low Damping ➔
Oscillator Response

For High Damping (near 1) ➔
No Oscillations

- $\sigma < 0$ ➔ Unstable
- $\sigma > 0$ ➔ Stable
- $\sigma = 0$ ➔ n.a.

$y(t)$
4.5 Step Response (Unit Step Response) 
Time Domain Specifications

- RISE TIME – Time necessary to Approach Set Point ($t_r$)
- SETTLING TIME – Time necessary for Transient to Decay ($t_s$)
- OVERSHOOT – % of Overshoot value to Steady State Value ($M_\%$)
- PEAK TIME – Time to reach highest point ($t_p$)
4.5 Step Response (Unit Step Response) 
Time Domain Specifications

- **RISE TIME** – Time necessary to Approach Set Point ($t_r$)
- **SETTLING TIME** – Time necessary for Transient to Decay ($t_s$)
- **OVERSHOOT** – % of Overshoot value to Steady State Value ($M_{\%}$)
- **PEAK TIME** – Time to reach highest point ($t_p$)
4.5 Step Response (Unit Step Response) Time Domain Specifications

- RISE TIME – Time necessary to Approach Set Point ($t_r$)

Rise Time \[ t_r \approx \frac{1.8}{\omega_n} \]

For $\zeta = 0.5$
4.5 Step Response (Unit Step Response) 
Time Domain Specifications

- PEAK TIME – Time to reach highest point ($t_p$)

$\omega_d = \omega_n \sqrt{1 - \zeta^2}$

For $\zeta = 0.5$
4.5 Step Response (Unit Step Response) 
Time Domain Specifications

- OVERSHOOT – % of Overshoot value to Steady State Value ($M_\%$)

\[
\text{Overshoot } M_p = e^{-\pi \xi / \sqrt{1-\xi^2}}
\]

For $\xi = 0.5$
4.5 Step Response (Unit Step Response)
Time Domain Specifications

- SETTLING TIME – Time necessary for Transient to Decay ($t_s$)

Settling Time

$$t_s = \frac{4.6}{\zeta \omega_n} = \frac{4.6}{\sigma}$$

For $\zeta = 0.5$

$$\sigma = \zeta \omega_n$$
4.5 Step Response (Unit Step Response)
Time Domain Specifications → Design

- Specify $t_r$, $M_p$ and $t_s$:

\[
\begin{align*}
\omega_n & \geq \frac{1.8}{t_r} \\
\zeta & \geq \zeta(M_p) \\
\sigma & \geq \frac{4.6}{t_s}
\end{align*}
\]
4.5 Step Response (Unit Step Response)
Time Domain Specifications  Design

- Specify $t_r$, $M_p$ and $t_s$:  

\[
\sin^{-1} \xi
\]

\[
t_r \leq 0.6 \text{ sec}
\]
\[
M_p = 10\%
\]
\[
t_s \leq 3 \text{ sec}
\]
\[
\omega_n \geq \frac{1.8}{t_r} \Rightarrow \omega_n \geq 3.0 \text{ rad / sec}
\]
\[
\xi \geq \xi(M_p) \Rightarrow \xi \geq 0.6
\]
\[
\sigma \geq \frac{4.6}{t_s} \Rightarrow \sigma \geq 1.5 \text{ sec}
\]
4.5 Step Response (Unit Step Response)
Time Domain Specifications ➔ Design

• **Adding a Zero ➔ Adding a Derivative Effect ➔**
  - Increase Overshoot
  - Decrease Rise Time

• **Adding a Pole ➔ s-term in the denominator ➔**
  pure Integration ➔ Finite Value ➔ Stability
  - Integral of Impulse ➔ Finite Value
  - Integral of Step Function ➔ Ramp Function ➔ Infinite Value
4.5 Step Response (Unit Step Response)
Time Domain Specifications ➔ Design

• For a 2°-order system with no zeros:

\[ t_r \approx \frac{1.8}{\omega_n} \]

\[ M_p \approx 16\%, \xi = 0.5 \]

\[ t_s \approx \frac{4.6}{\sigma} \]

• Zero in LHP ➔ Increase Overshoot
• Zero in RHP ➔ Decrease Overshoot
• Pole in LHP ➔ Increase Rise Time the denominator ➔ pure integration
4.6 Model From Experimental Data

• Transient Response – input an impulse or a step function to the system
• Frequency Response Data – exciting the system with sinusoidal input at various frequencies
• Random Noise Data
4.6 Model From Experimental Data

- PID Model ➔ Transient Response to a step function representing the SP value = Desired value of the Returns.
5.1 FEEDBACK CONTROLLER

Several parameters characterize the process.

- The difference ("error") signal is used to adjust input to the process in order to bring the process' measured value back to its desired Set-Point.

- In Feedback Control the error is less sensitive to variations in the plant gain than errors in open loop control

- Feedback Controller can adjust process outputs based on
  - History of Error Signal;
  - Rate of Change of Error Signal;
  - More Accurate Control;
  - More Stable Control;
  - Controller can be easily adjusted ("tuned") to the desired application.
5.1 FEEDBACK CONTROLLER

The ideal version of the Feedback Controller is given by the formula:

\[ u(t) = K_p \left( e(t) + \frac{1}{T_i} \int e(\tau)d\tau + T_d \frac{de(t)}{dt} \right) \]

where \( u \) = Control Signal;
\( e \) = Control Error;
\( R \) = Reference Value, or Set-Point.

Control Signal = \( \sum \)

- Proportional Term \( P \)
- Integral Term \( I \)
- Derivative Term \( D \)
5.2 FEEDBACK CONTROLLER

- Adjusts Output in Direct Proportion to Controller Input (Error, \( e \)).
- Parameter gain, \( K_p \).
- Effect: lifts gain with no change in phase.
- Proportional - handles the immediate error, the error is multiplied by a constant \( K_p \) (for "proportional"), and added to the controlled quantity.
5.3 FEEDBACK CONTROLLER

- The Integral action causes the Output to Ramp.
- Used to eliminate Steady State Error.
- Effect: lifts gain at low frequency.
- Gives Zero Steady State Error.
- Infinite Gain + Phase Lag.
- Integral - To learn from the past, the error is integrated (added up) over a period of time, and then multiplied by a constant $K_I$ and added to the controlled quantity. Eventually, a well-tuned Feedback Controller loop's process output will settle down at the Set-Point.
5.4 FEEDBACK CONTROLLER

**Derivative Term, D**

- The derivative action, characterized by parameter $K_d$, anticipates where the process is going by considering the derivative of the controller input (error, $e$).
- Gives High Gain at Low Frequency + Phase Lead at High Frequency
- Derivative - To handle the future. The 1st derivative over time is calculated, and multiplied by constant $K_d$, and added to the controlled quantity. The derivative term controls the response to a change in the system. The larger the derivative term, the more the controller responds to changes in the process's output. A Controller loop is also called a "predictive controller." The $D$ term is reduced when trying to dampen a controller's response to short term changes.
6. METHOD - the *PID model*

- Novel approach to Portfolio Tactical Asset Allocation.
- Recalling TAA Constant Proportion, Core Satellite and Active Strategies….
- Portfolio Assets Rebalancing is dictated by an Asset Selection Technique Consisting in the Optimization of Risk Adjusted Return (or simply, Return) by means of the *PID model*. 
6.1 METHOD - the *PID model*

The analysis is been performed by using the following data:

- **Period:** January 2000 – August 2009
- **Time horizon:** 8 Years and 8 Months
- **Sampling Frequency:** Monthly
- **Source:** Bloomberg
- **Number of Assets Classes:** 50+1 (0-risk 0-return Asset=out of the market)
- **Control Variable:** Monthly **Returns**
- **Set Point Value:** 0.5% (Monthly)

> Constraints for Rebalancing = Constraints of Benchmark (i.e. max 10% exposure for any Security)
6. METHOD - the **PID model**

• Return is not Optimized via Rebalancing of Asset Weights following a Forecasting Methodology of the Expected Return Vector.

• Investors seek Consistent and Stable Portfolio Performance over Time.

• Return is induced towards Stability → Return is *Controlled*. 
6. METHOD - the *PID model*

- For a portfolio to be tactically managed over a time horizon by means of the *PID model*:
  - Given an initial asset allocation mix (Initial Portfolio), the assets are rebalanced at a predetermined frequency (monthly, or bimonthly, or quarterly);
  - the rebalancing process is determined by choosing that particular mix of assets such at, at each iteration (monthly, or bimonthly, or quarterly), the current risk adjusted return approaches the current controlled system output.
6.2.1 METHOD - the *PID model*

1. Choose Return parameter (Set-Point);
2. Set Return value;
3. Set Controller parameters;
4. Choose Initial Portfolio (IP); i.e
   1. All equivalents weights among the plurality of all the assets of the portfolio; or
   2. Initial Portfolio could be dictated by Markovitz Asset Allocation.
6.2.2 METHOD - the *PID model*

**PID [Continuous]**

\[
u(t) = k_p \left( e(t) + \frac{1}{K_i} \int e(\tau) \, d\tau + K_d \frac{de(t)}{dt} \right)
\]

**PID [Discrete]**

\[
 u_n = k_p \left( e_n + \frac{1}{K_i} \sum_{m=0}^{n} e_m + K_d (e_n - e_{n-1}) \right)
\]

**PID [Simple Lag Implementation]**

\[
 u_n = \left( k_p e_n + K_i (e_n - u_{n-1}) + u_{n-1} + K_d (e_n - e_{n-1}) \right)
\]

gennaio 5, 2014
6.2.3 METHOD - the *PID. model*

1. Calculate Return (control parameter) for the initial portfolio.

2. Controller indicates the controlled value ➔ Rebalancing ➔ Minimization of Error.

3. New market Data acquisition.

4. New Return is calculated.

5. Items 2, 3 and 4 are iteratively repeated until END of observation period.

6. Approaching Set Value ➔ Stabilility.
### 6.2.4 METHOD - the PID model

Set Value = Return 0.005 = 0.5%

\[ u_n = (k_p e_n + K_i (e_n - u_{n-1}) + u_{n-1} + K_d (e_n - e_{n-1})) \]

<table>
<thead>
<tr>
<th>Portfolio Return</th>
<th>Benchmark Return</th>
<th>PV</th>
<th>Derivative Error</th>
<th>ABS Error</th>
<th>PID Output</th>
<th>Integral</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6028</td>
<td>0.6578</td>
<td>0.0000</td>
<td>0.0090</td>
<td>0.0090</td>
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Gennaio 5, 2014

Ing. Antonella Sabatini, P.E.
Results: Cumulative Return

PID 6.41%  EUROSTOXX50 index -37.13%

![Graph showing cumulative return comparison between PID and EUROSTOXX50 index]
Results: 1-Year Volatility of the Returns

**Avrg PID** 3.81%  
**Avrg EUROSTOXX50 Index** 4.48%
The G.A.M. Model peculiarity is to provide STABILITY to the controlled variable.

The cumulative returns are improved

Volatility has decreased

Correlations are lower in negative market phases and approach 1 in positive market conditions
Conclusions

PID Model provides:

– Better long term Returns
– Lower Volatility
– Low Correlation to Benchmark in difficult market conditions
– High Correlation to Benchmark in favorable market conditions
– Long Term portfolio Stability
8. CONCLUSIONS AND FUTURE WORK

ENHANCING FINANCIAL MARKET ANALYSIS

= FINANCE

+ ELECTRICAL ENGINEERING
CONTACT INFORMATION

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APPENDIX $\beta$eta

- $Ra = RFR + \beta (Rm - RFR)$
- Where $Ra$ = Return of an asset $A$
- $RFR$ = Risk Free Rate
- $Rm$ = Expected Market Return
- The measure of an asset's risk in relation to the market
Appendix D

Duration ($D$) is a weighted average of the times to payment of a bond's cash flows, where the weights ($\frac{X_t/y^t}{B}$) are the relative present values of the corresponding cash flows.

$$D = \sum_{t=1}^{T} \left[ \frac{X_t/y^t}{B} \right] \times t \quad \text{where} \quad B = \sum_{t=1}^{T} X_t/y^t$$

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$$C_f/(1+\%)^\text{Year}$$
Appendix A Simple Lag Derivation
Appendix A Simple Lag Derivation

\[ \tau \frac{dx}{dt} + x = e \quad \text{continuous time} \]

Integral term of PID implemented as a

\[ \frac{1}{ki} (x_n - x_{n-1}) + x_{n-1} = e_n \]

\[ x_n - x_{n-1} + ki x_{n-1} = ki e_n \]

\[ x_n = ki (e_o - x_{n-1}) + x_{n-1} \]
Appendix Z
Ziegler-Nichols Tuning for PID Controller

\[ k_p \approx 0.6 k_u \]
\[ k_i \approx 0.5 P_u \]
\[ k_d \approx 0.125 P_u \]

\( P_u = \text{Period of oscillation} \)
\( k_u = \text{Proportional gain at the edge of oscillatory behavior} \)
Appendix F
Frequency Response and Bode Plots

• Dynamic compensation can be based on Bode Plots
• Bode Plots can be determined experimentally