14.03 Midterm Spring 1996 Solutions

Part I

1. True. First, recall that a concave function satisfies \( \frac{d^2}{dx^2} f(x) \leq 0 \) (you only lost a point for getting this backward).

The standard picture in the text shows that when average costs are declining, we have decreasing returns to scale. This is because declining average costs imply that the cost per unit goes down with the number of units. This can only happen if increasing inputs leads to a more than proportional increase in output. (Many of you tried to interpret the total cost curve in this way).

Second, note that saying that the total cost curve is concave is the same as saying that the marginal cost curve is declining, since \( \frac{d^2}{dq^2} TC(q) = \frac{d}{dq} MC(q) \).

Finally, note that declining marginal cost implies that the average cost is always greater than the marginal cost, since the average cost of the first unit must be equal to the marginal cost of the first unit. Thereafter, a falling marginal cost implies that the last unit produced is cheaper than average. Another way to look at it is that marginal cost must cut through the minimum of the average cost curve from below; if the marginal cost curve never turns up, it can never cut through from below.

But, average costs are declining whenever marginal cost is below average cost. To see this algebraically, the following algebraic expression gives the derivative of average cost with respect to quantity:

\[
\frac{\partial}{\partial q} AC(q) = \frac{\partial}{\partial q} \frac{TC(q)}{q} = -\frac{1}{q^2} TC(q) + \frac{1}{q} MC(q) = \frac{1}{q} [MC(q) - AC(q)]
\]

Note: Obviously, your answer did not need to be this detailed. The assertion of problem 1 was stated in class, though it wasn’t proved, and full credit was received for a sentence which captured the basic logic of the argument.

2. False. It would be true if it were restated substituting “short run average variable costs” for “short run average costs of capital and labor.” Capital is fixed in the short run and thus does not affect the firm’s short-run decision-making at all. The cost of capital must be paid in any event. If SAVC > P (or equivalently, SVC > PQ), the firm could do better by choosing L=0 and Q=0, which yields SVC = 0 and PQ=0. On the other hand, if SAVC < P < SAC, then the firm will produce an interior amount, and the first order conditions for an interior maximum will be valid, so P=SMC. In the long run, the firm will adjust capital, but if LRAC > P the firm will exit (i.e. choose zero capital, zero labor, and zero output).

Note: TC(q) is drawn here increasing, but imagine decreasing TC!!
3. False. It is possible that the change in medicaid funding would affect the pregnancy rate, as discussed in class. In fact, the Kane and Staiger article discussed in class provided evidence in support of the claim that for small decreases in the availability of abortions, the teen birth rate actually declined. This is consistent with a decrease in # abortions / # pregnancies.

4. True. By definition, when the price of the Giffen good rises, demand for that good must rise. But, the consumer is exhausting his income, and the total expenditure on the Giffen good has gone up (both price and quantity increased). Thus, the expenditure on the other good must fall; since its price is unchanged, the demand for that good must fall too.

5. False. The total expenditure in both periods is 10. At the first period prices, the second period bundle (4,3) costs 11, and was as such unaffordable. At the second period prices, the first period bundle (2,6) costs 14, and is unaffordable. Thus, no revealed preference information is obtained. See the attached diagram.

6. False. The market demand before cloning is \( Q^D_0(P) = \sum_{i=1}^{N} q_i(P) \), while after cloning it is \( Q^D_C(P) = 2 \sum_{i=1}^{N} q_i(P) = 2Q^D_0(P) \). Thus, slope doubles and the size of the market doubles, so that the new elasticity is given by \( \varepsilon^C_{Q,P} = \frac{\partial Q^D_C}{\partial P} \frac{P}{Q^D_C} = \frac{\partial 2Q^D_0}{\partial P} \frac{P}{2Q^D_0} = \frac{\partial Q^D_0}{\partial P} \frac{P}{Q^D_0} = \varepsilon^D_{Q,P} \). This illustrates one of the nice features about elasticities -- doubling the size of the market doesn’t change anything, and so the “units” of counting each consumer as “1” doesn’t affect anything.

7. False. Modern manufacturing involves (a), (b), and the opposite of (c): vertical communication and decentralized decision-making. Clearly, the returns to skill and incentives are lower if all of the decisions are made centrally anyway.

Part II

1. False. We cannot conclude that Jenny’s new bundle must cost less than her old bundle at the old prices. Since she is worse off, it must be that she prefers the old bundle to the new bundle. Then there is nothing inconsistent in the first period about choosing the old bundle, even if the new bundle is affordable in that period. However, after the price change, the old bundle might not be affordable anymore. Then Jenny is forced to pick the new bundle, and is thus worse off. In fact, we cannot conclude anything about whether the new bundle was affordable at the old prices, since she wouldn’t have picked it anyway. What we can conclude is that the old bundle is not affordable at the new prices. See the attached diagram.
2. False. This is not a natural experiment because (a) the choices of the firm are endogenous, and the firm might have picked the best methods for each city, and (b) the cities might be very different, and so the "all else constant" hypothesis does not hold.

3. False. While Kmart's prices dropped, the number of consumers stayed the same. This is consistent with the hypothesis that service dropped at the same time. Thus, we cannot conclude that consumers are worse off in 1995 after the price drop. However, the index could be biased, since Kmart customers are not accounted for at all in the index.

4. False. The argument is exactly backwards, just like "steak v. chicken." The Laspeyres index uses the quantities purchased in 1992 as weights. If consumers chose to switch away from those weights, it must have made them better off than continuing to consume the 1992 bundle. So, since some consumers did in fact switch, the Laspeyres index actually overstates, not understates, the increase in the true cost of living.

Part III

1. The ratio of prices is given to be 1. The slope of the indifference curve is -2, and thus the MRS is 2. Since the MRS is diminishing, higher levels of X are required to get Jane's MRS down to 1. Thus, only more X and less Y will get her to the optimizing point, where MRS = 1.

2. Since both supply and demand are shifting together, the economist is not tracing out a single demand curve. The following figure illustrates the problem. The demand curve she estimates is too flat; she thinks that consumers are more price sensitive than they actually are.

3. See attached.

4. See attached.

( sorry - I drew too sleep but you get the idea ...)
Graph illustrates indifference curves for consumer who makes choices in problem.
If choices had been *'s in picture, this would have been violation.

II. 1.

The graph above contradicts the statement.
The consumer is worse off in period 2 at (4,3).
But, (4,3) is outside the period 1 budget set, since 24 + 3 = 11 > 10; however, (2,6) would have been chosen in pd. 1 even if (4,3) was affordable.
Month 0: No violence
Month 1: Some violence
Month 2: Lots of violence

If demand & supply both shift in each month, as described in the question, we will observe $(P_0, Q_0)$, $(P_1, Q_1)$, and $(P_2, Q_2)$.

Fitting a line through the points yields the dashed line in the picture.

The dashed line is not the true demand curve; the contraction in demand leads to an additional decrease in quantity. If demand fixed, going from $S_0 \rightarrow S_i$, causes $\Delta Q = B$; demand shifts causes additional $\Delta Q = A$. Thus, economist thinks demand more responsive to price changes (flatter) than true demand.
3. How does quantity of $X$ demanded change w/ price?

Slutsky equation

$$\frac{\partial d_x}{\partial P_x} = \frac{\partial d_x}{\partial P_x} - X \frac{\partial x}{\partial I}$$

Substitution effect
Income effect

If $\frac{\partial x}{\partial I} > 0$, we can guarantee that $\frac{\partial d_x}{\partial P_x} < 0$, since the substitution effect is always negative.

$\frac{\partial x}{\partial I} > 0 \iff$ normal good.

When can this fail?

$\cdot \frac{\partial x}{\partial I} < 0$ (inferior good)

\[
\text{AND}
\]

$\cdot$ Income effect dominates substitution effect.

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Example where $P_x \downarrow \Rightarrow X \downarrow$

Starting from $P_x^h$ to $P_x^l$:

(A) $1 \rightarrow 2$ is substitution effect

(b) $2 \rightarrow 3$ is income effect

(c) $3 \rightarrow 4$ is substitution effect

(d) $4 \rightarrow 1$ is income effect

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See Nicholson for details & better picture.
a) Crossing pt: \(X+15Y = 30\)
\[
\frac{(X+Y = 25)}{.5Y = 5}
\]
\[Y = 10\]
\[X = 15\]

b) Consumer chooses \(Y=10\) under sales tax
\(\Rightarrow\) indifference curve must be tangent to sales tax b.c. at \((15,10)\), which is crossing pt. of two b.c.'s.

Revealed
Pref argument:
\((15,0)\) is affordable under lump sum tax. Therefore anything chosen is better.

c) Since the indifference curve in part b is tangent to the sales tax b.c., the lump sum tax b.c. must lie above this indifference curve in some region. In my picture, this is the region \([X,15], Y=25-X^3\). All points in this set are affordable under lump sum and preferred to \((15,10)\). I illustrate this with the dashed indiff. curve.