Back to the equation solved earlier

$$C = 1 - \frac{\phi^2}{4} (1 - \tilde{r}^2)$$

Remember, for our soln to make sense

$$\phi^2 \leq 4$$

Otherwise, reaction is too much faster than diffusion, there will be a core inside the device where $C_{O_2} \approx 0$, and the boundary conditions we had applied would be wrong.

$$\phi^2 = \frac{R^2 \frac{\partial \text{cell}}{V}}{D \cdot C_0}$$

$$R = 0.5 \text{ cm}, \hspace{1cm} \frac{\partial \text{cell}}{V} \approx 1 \times 10^{-17} \text{ mol cm}^{-3} \text{ s}^{-1}$$

$$\frac{\text{Neu}}{V} = 4 \times 10^7 \text{ cell cm}^{-3}, \hspace{1cm} D = 2 \times 10^{-5} \text{ cm}^2 \text{ s}^{-1}, \hspace{1cm} C_0 = 90 \text{ mm Hg}$$

\[ \downarrow \]

123 \text{ \mu M}

\[ \therefore \phi^2 = 41 \gg 4 \]

From calculation it shows that by pure diffusion, at such cell density, with our estimated $O_2$ uptake rate, most of the cells in the center of device will \& die from zero oxygen, which is not consistent with the graph.

(Note that even if we take into account of original $O_2$ content of plasma/blood, the $O_2$ would be depleted by 1 day anyway. Therefore our estimate with the steady state profile should be valid.)
If \( g \) is lower, \( \Phi^2 \) will change, and it will take longer for cells to deplete \( O_2 \). So depending on the \( g \) cell chosen, the comparison may be dramatically different.

\[
\frac{dN}{dt} = kN \quad \Rightarrow \quad \int \frac{dN}{N} = \int k dt
\]

\[
\ln N = kt + A_1
\]

\[
N = N_0 e^{kt}
\]

If \( t=0 \), \( N = N_0 \)

Doubling time \( t_D \)

\[
2N_0 = N_0 e^{kt_D}
\]

\[
t_D = \frac{\ln 2}{k}
\]

From the figure of DNA vs. time

Assuming each cell had the same amount of DNA then \([\text{DNA}] \propto [N]\)

\[
\therefore \quad [\text{DNA}] = [\text{DNA}]_0 e^{kt}
\]

\[
\ln [\text{DNA}] = \ln [\text{DNA}]_0 + kt
\]

If we graph \( \ln [\text{DNA}] \) vs. \( t \), the slope is \( k \)

(see next page)

Using data from L-C series

\( k \approx 0.0775 \text{ (day}^{-1} \text{)} \)
<table>
<thead>
<tr>
<th>Day</th>
<th>DNA</th>
<th>ln (DNA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3.25</td>
<td>1.178655</td>
</tr>
<tr>
<td>6</td>
<td>3.75</td>
<td>1.321756</td>
</tr>
<tr>
<td>9</td>
<td>4.5</td>
<td>1.504077</td>
</tr>
<tr>
<td>12</td>
<td>5</td>
<td>1.609438</td>
</tr>
<tr>
<td>16</td>
<td>7.5</td>
<td>2.014903</td>
</tr>
<tr>
<td>21</td>
<td>13.5</td>
<td>2.60269</td>
</tr>
</tbody>
</table>

\[ y = 0.0775x + 0.8398 \]
\[ R^2 = 0.9557 \]

\[ T_D \approx \frac{\ln 2}{k} = 9 \text{ days} \]

This is too long compared to the oxygen drop time frame ( \( \sim 1-2 \text{ days} \)). So the hypothesis that \([O_2]\) drops sharply due to cell doubling is unlikely.
(f) \( P_{O_2} = 5\% \) or \( 20\% \)

\[
\begin{align*}
\text{For this system:} \\
\frac{\partial C}{\partial t} &= D \frac{\partial^2 C}{\partial z^2} + \text{rxn} \\
\text{There is no rxn in the liquid} \\
\therefore \text{rxn} &= 0 \\
\text{We are looking for steady state soln} \\
\therefore \frac{\partial C}{\partial t} &= 0 \\
\implies \quad \frac{\partial^2 C}{\partial z^2} &= 0 \quad \text{(no } x, y \text{ dependence)} \\
C &= A_1 z + A_2 \\
\text{Boundary Conditions:} \\
\text{at } z=0, \quad C = C_0 \\
\text{at } z=5\text{mm}, \quad \text{flux = oxygen depletion rate} \\
-D \frac{\partial C}{\partial z} &= \frac{\text{flux}}{\text{area}} \\
\text{at } 5\% \text{ } O_2 \quad P_{O_2} \approx 0.05 \text{ atm} \\
\text{at } 20\% \text{ } O_2 \quad P_{O_2} \approx 0.2 \text{ atm} \\
\text{Using Henry's Law again} \\
P_{O_2} \text{ at } 0.05 \text{ atm, } C_{O_2} = 52 \mu M \\
P_{O_2} = 0.2 \text{ atm, } C_{O_2} = 207.5 \mu M
\[ D = 2 \times 10^{-5} \text{ cm}^2 \text{ s}^{-1} \]
\[ \delta \text{cm} = \frac{10^{-17} \text{ mol}}{\text{ cm} \text{ s}} \]
\[ N \text{cm} = 10^5 \]
\[ \text{dish area} = \pi \left(\frac{35 \text{ mm}}{2}\right)^2 = 9.62 \text{ cm}^2 \]

For both cases of \( O_2 \):

\[ C = A_1 z + A_2 \]

at \( z = 0 \), \( C = C_0 \) \( \therefore \) \( A_2 = C_0 \)

at \( z = 0.5 \text{ cm} \), \( \frac{\delta C}{\delta z} = - \frac{\delta \text{cm} \cdot N \text{cm}}{D \cdot \text{area}} \]

\[ = - \frac{10^{-17} \text{ mol}}{\text{ cm} \text{ s}} \cdot 10^5 \]
\[ \frac{1}{2 \times 10^{-5} \text{ cm}^2 \cdot 9.62 \text{ cm}^2} \]

\[ = - 5.2 \times 10^{-9} \frac{\text{mol}}{\text{cm}^2} \]

\[ = A_1 \]

\[ \therefore \] for \( P_{O_2} = 0.05 \text{ atm} \)

\[ C = 52 \mu \text{M} - 5.2 \times 10^{-9} \left( \frac{\text{mol}}{\text{cm}^2} \right) \times z \]

at \( z = 0.5 \text{ cm} \), \( C = 4.94 \times 10^{-5} \frac{\text{mol}}{\text{cm}^2} = 49.4 \mu \text{M} \)

for \( P_{O_2} = 0.2 \text{ atm} \)

\[ C = 207.5 \mu \text{M} - 5.2 \times 10^{-9} \left( \frac{\text{mol}}{\text{cm}^2} \right) \times 2 \]

\[ = 205 \mu \text{M} \]

| \(|5\% \text{ O}_2| \) | \(|20\% \text{ O}_2| \) |
|---|---|
| \( C \) at cell surface | 49.4 \mu \text{M} | 205 \mu \text{M} |