14.02 Principles of Macroeconomics

Problem Set #3, Questions and Answers

Posted: Thursday, February 28, 2002

Due Date: Thursday, March 7, 2002

Total points: 100 + 2 bonus points

Please remember to write your TA’s name and section time on the front page or your problem set.

Part I: True, False Questions. Decide whether each statement is true or false and justify your answer with a short argument. (2 points each, 12 points total)

1.) Investment does not count in current GDP, because it increases the capital stock that will be used in the future.
   FALSE: Investment counts in final sales, and is part of GDP.

2.) Over the past century, technological convergence allowed some poor countries to grow faster than rich countries.
   TRUE: For some countries, in particular a number of Asian and European countries, convergence in per capita GDP occurred. However, many countries that were poor 100 years ago remain poor today.

3.) Bond prices are negatively related to the interest rate.
   TRUE: Take a bond that pays off $100 in one year. Denote the bond price by P. The rate of return on the bond is then (100-P)/P, which is simply the interest rate i. Rewriting, P=100/(1+i), the price is the discounted future payment of $100.

4.) The estimate from an OLS regression of dC on dY gives the marginal propensity to consume.
   TRUE: Consider the specification dC=α+βdY+ε. Then the estimate of β is the marginal propensity to consume, dC/dY.

5.) The Central Bank fully controls inflation.
   FALSE: The central bank controls the supply of money. Changing the supply of money influences inflation, but many other influences such as productivity, fiscal policy etc. also influence inflation.

6.) Inflation is good, as it increases wages.
   FALSE: What matters to employees are real wages. If inflation is anticipated, nominal wages will indeed be higher, to keep the purchasing power of wages constant. This can be seen in the wage setting equation.
Part II: (50 points total)
You take a summer job at the central bank of New Zealand. The governor of the central bank leaves for a holiday right when you start working. You are in charge of monetary policy over the next three months....

1. You want to learn something about money demand. You remember from class $M^d = SYL(i)$
   a) **(2 points)** Explain why money demand is proportional to nominal income $Y$.

   Answers:
   (a) As nominal income increases, more monetary transactions are made, and more money is demanded. Both an increase in real income and an increase in the level of prices make more transactions necessary.

   b) **(2 points)** How does money demand depend on $i$? Give an economic explanation for this dependency.

   Answers:
   (b) Money demand depends negatively on $i$, as the interest rate is the opportunity cost of holding money instead of investing in bonds.

2. Now you want to obtain an econometric model from this specification.
   a) **(2 points)** You specify the following functional form
      \[ L(i) = e^{-\alpha - \beta i} \]
      Replace in the money demand equation from question 1, take logarithms of your equation, and impose equilibrium by setting $M = M^d$. Call it equation 2.a.

   b) **(2 points)** By using the following change of variable, $v = \ln Y - \ln M$, in equation 2.a., explain in words why the following econometric model makes sense:
      \[ v = \alpha + \beta i + \varepsilon \]

   c) **(2 points)** Give the formulas for the estimates of $\alpha$ and $\beta$ and interpret.

   Answers:
   a) Replacing in the money demand equation gives $M = SY\exp(-\alpha - \beta i)$. Taking logarithms gives $\ln(M) = \ln(SY) - \alpha - \beta i$.

   b) Note that $Y/M$ is velocity, so that $v$ is defined as the log velocity. Replacing in equation from 2.a gives $v = \alpha + \beta i$. As relationships in economics only hold approximately, an error term is added to the econometric specification.

   c) The log-velocity of money is regressed on the interest rate. $\beta$ denotes the sensitivity of log-velocity with respect to the interest rate, and $\alpha$ is the constant part of log-velocity. Both coefficients are expected to be positive. The estimates of the coefficients are:
\[ \tilde{v} = (1/T) \sum v_i \]
\[ \tilde{i} = (1/T) \sum i_i \]
\[ \hat{\alpha} = \overline{v} - \hat{\beta} \overline{i} \]
\[ \hat{\beta} = \frac{\sum (V_t - \overline{V})(i_t - \overline{i})}{\sum (i_t - \overline{i})^2} \]

The constant adjusts for the means of \( i \) and \( V \). The slope depends on the correlation between \( i \) and \( V \).

3. Now you go to the data. You want to estimate your money demand function. You find quarterly data from Mar 1994 to Sep 2001. You have a quarterly series of nominal GDP, that you denoted \( Y \), money supply in circulation denoted \( M \), and 90 day nominal interest rates \( i \). You can download the data directly from the website of the central bank of New Zealand at [www.rbnz.govt.nz](http://www.rbnz.govt.nz) or you simply download the spreadsheet from web.mit.edu/14.02/www.

a) (5 points) Plot \( Y \) versus \( M \) and interpret.

b) (5 points) Construct \( v \), and plot \( v \) versus \( i \). Interpret.

Answers:

(a) This graph plots \( Y \) against \( M \). There is a very strong positive relationship between the two. The slope looks close to 1.

(b) This Graph plots \( V \) against \( i \), and shows a positive relationship, as expected from the theory.
4. Run your regression of \( v \) on \( i \). (Hint: you can run the regression in Excel, under Tools, select Data Analysis. If it is not there, go to Tools, and then Add-Ins, and select to install the Analysis ToolPak. After selecting the Data Analysis, choose to run a regression. Then all you need to is to specify your dependent variable and explanatory variable.
   
   a) \((8\text{ points})\) What is your estimate of \( \alpha \) and \( \beta \)?
   
   b) \((8\text{ points})\) Interpret your t-statistics and the R2 that you find.

   **Answers:**

   *The regression gives the following estimates:*

   \[
   V = 0.37 + 0.05 i
   \]

   \[(5)\quad (4.9)\quad \text{R-squared} = 0.46\]

   *The t-statistics are in parenthesis.*

   (a) The sensitivity of \( V \) with respect to \( i \) is 0.05, the estimate of the intercept is 0.37.

   (b) Both t-statistics are greater than 2, so the coefficient estimates are significant. The \( R^2 \) indicates that 46% of the variation in log velocity is explained by the interest rate.

5. Now you want to check for misspecification. Run the following regression:

   \[
   \ln M = \alpha + \beta i + \gamma \ln Y + \varepsilon
   \]

   a) \((1\text{ Points})\) What coefficient do you expect for \( \gamma \)?

   b) \((8\text{ Points})\) Interpret your regression results. Is the coefficient you find in your regression significantly different from the one you expected?

   **Answers:**

   a) You expect a coefficient of 1.

   b) Running the regression gives:
\[
\ln(M) = -6.7 - 0.36i + 1.6 \ln(Y)
\]

\[(-4.3) \quad (-4) \quad (10.5) \quad \text{R-squared} = .87\]

The estimate of the slope of the exchange rate is practically the same as before, except that it is negative, which we would expect, as the equation was inverted. However, the coefficient on log nominal income is significantly different from 1, which is unexpected.

6. **(5 points)** The next day you go to work, you look at the regressions that you estimated in parts 4 and 5. Did you really estimate a money demand function? **Explain.** (Hint: think about IS-LM)

**Answers:**

The money demand function is \(M^d = \$YL(i)\), or, taking logs, \(\ln(M^d) = \ln(\$Y) + \ln(L(i))\). If it is assumed that \(L(i) = \exp(\alpha + \beta i)\), then the specification looks like the one of either part 4 or 5. However, the coefficient on \(\ln(\$Y)\) is not 1. The reason is that money demand cannot be observed, only the equilibrium when money supply equals money demand can be observed. In order to get correct estimates, instrumental variables must be employed.

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**Part III: A simple AS-AD model (Total points: 40)**

(For simplicity, we assume a closed economy: \(X=IM=0\))

**Important note:** All questions of this problem build one onto the next, so answer questions and solve equations sequentially.

1. **IS: The goods market**
   1. **(2 points)** State the definition of private savings and government saving, and use the following accounting identity to relate \(I\) and \(S\):

   \[Y = C + I + G\]

   2. **(2 points)** From now on and for the rest of the question, consider the following consumption and investment functions: \(C = 50 + .6(Y-T)\) and \(I = 50 - i\). Solve for \(i\) and draw aggregate investment in the \((i,Y)\) space.

3. **(2 points)** What does this curve represent?

4. **(2 points)** Explain what happens when \(G\) increases, holding \(T\) fixed.

**Answers:**

a) The definition of private savings is \(Sp = Y^d - C\), or using \(Y^d = Y - T\), we get: \(Sp = (Y-T) - C\), where the definition of government savings is: \(Sg = T - G\). Therefore: \(Y = C + G + I \Rightarrow Y - C - G = I \Rightarrow [(Y - T) - C] + [T - G] = I \Rightarrow Sp + Sg = I \Rightarrow S = I\) This identity states that aggregate savings of the economy equals aggregate investments of the economy which is the source of the name IS.

b) Solving for \(i\) gives \(i = 100 + G - .6T - .4Y\). This is a downwardsloping function in \((i,Y)\) space, with slope -.4, and intercept 100 + G - .6T.
c) It represents the pairs \((i,Y)\) that support equilibrium in the goods market.

d) When \(G\) increases, holding \(T\) fixed, the IS curve shifts North East.

2. LM: The money market

a) **(2 points)** Money demand is given by

\[
M_d^i = PYe^{-i} \Rightarrow \ln M_d = \ln P + \ln Y - i
\]

Explain in words why money demand depends positively on the price level and aggregate output, and negatively on the interest rate.

b) **(2 points)** The central bank controls money supply, so that the equilibrium condition on the money market is money supply=money demand \([M^s=M^d=M]\). This is the LM curve. Solve the equilibrium condition for \(i\), and draw it in the \((i,Y)\) space.

c) **(2 points)** What does this curve represent?

d) **(2 points)** What happens when \(P\) increases? Explain.

e) **(2 points)** Explain the economics of expansionary monetary policy via an open market operation, and draw the appropriate graph.

f) **(3 points)** Suppose that the White House decides to increase spending, and the fed decides to accommodate the fiscal policy by changing it's monetary policy. For a given change in \(G\), and assuming that \(T=0\), by how much needs money supply change to keep the interest rate fix?

**Answers:***

a) Money demand depends on aggregate output for transaction reasons: higher GDP leads to more transactions in the economy, which increases money demand. Furthermore, what matters is nominal GDP, if prices are high, more money needs to be in circulation. Money demand depends negatively on the interest rate, as the opportunity cost of holding money (and not investing in bonds) increases with the interest rate.

b) Solving for \(i\) gives \(i=\ln P + \ln Y - \ln M\). This is a upward sloping in \((i,Y)\).

c) It represents the pairs of \((i,Y)\) that support equilibrium in the money market.

d) When \(P\) increases, the LM curve shifts up: to accept the same amount of money in circulation, and keeping the interest rate fix, output must decline.

e) In an open market operation, the central bank buys bonds, and thus puts more money into circulation. Buying bonds puts pressure on bond prices to go down, and interest rates therefore go up. In \((i,Y)\) space, the LM curve shifts to the right.

f) To accommodate fiscal policy such that the interest rate is fixed, take the differential of IS and LM, setting \(di=dP=dT=0\) and replace for \(dY\) which gives:

\[
\frac{dM}{dG} = 2.5M/Y
\]
3. **AD: The aggregate demand curve**
   a. **(2 points)** The IS-LM curves give you the equilibrium on the money and goods market. Assume $T=0$. By combining IS and LM from questions 1. and 2., eliminate the interest rate. Solve your equation for $P$. This is the AD curve.
   b. **(2 points)** What does this curve represent?
   c. **(2 points)** Your AD curve from 3 a) depends on $P^e$, $G$ and $M$. Draw the curve in space $(P,Y)$. What happens when $M$ increases? Explain. What happens to the AD curve when $G$ increases? Explain.

   **Answers:**
   
   a.) The AD curve is: $P=(M/Y)\exp(100+G-.6T-.4Y)$
   
   b.) The AD curve represents the pairs of $(P,Y)$ that support equilibrium in both the money and the goods markets.
   
   c.) **Typo:** the AD curve depends on $P$, not on $P^e$.
   The AD curve is downward sloping in space $(P,Y)$. Both expansionary fiscal and expansionary monetary policies increase aggregate demand, and, holding real GDP fix, increase prices. This corresponds to an upward shift of the AD curve in space $(P,Y)$.

4. **AS: Aggregate supply**
   a. **(1 points)** Denote the number of people who would like to work by $L$, and the number of people who are employed by $N$. Write down the definition of the unemployment rate and call it 4.a
   b. **(1 point)** Assume that the wage setting equation is given by: $W/P^e=10-u$ What is the natural rate of unemployment implied by this equation?
   c. **(1 point)** Replace your definition of the unemployment rate from part a. Call your new equation 4.c.
   d. **(1 point)** Assume that the price setting equation is given by $P=2W$. How big is the mark-up implied by this equation?
   e. **(1 point)** Assume $L=100$ and use the price setting equation to eliminate $W$ in equation 4.c. Interpret, and call your new equation 4.e.
   f. **(1 point)** Assume that the production technology is $Y=N$. What happens to the curve when expected prices shift? Explain
   g. **(1 point)** What does this curve represent?
   h. **(1 point)** Draw the curve in $(P,Y)$.

   **Answers:**
   
   (a) Equation 4.a. is $u=(1-N/L)$
   (b) The natural rate of unemployment is 9.5. You get this by using the equation in part 4.d., and setting $P=P^e$
   (c) Substitute Equation 4.a in 4.b. gives equation 4.c $W/P^e=9+N/L$.
   (d) The equation for the mark-up is $P=(1+\mu)W$, so the mark-up is 100%
(e) Equation 4e is: \( P = P^e (18 + N/50) \). This is the static version of the Phillips curve, stating that the price level depends negatively on the unemployment rate, holding expected prices fix.

(f) Replacing for the production function gives \( P = P^e (18 + Y/50) \). This is an upward sloping curve in \((P, Y)\) space, with slope \( P^e/100 \), and intercept \( P^e 9 \).

(g) It represents the pairs of \((P, Y)\) that support equilibrium in the production factor market.

(h) When \( P^e \) is rising, the AS curve is shifted north west. The economics of this comes from bargaining: higher expected prices leads employees to demand higher wages, which in turn leads to higher prices by the wage setting equation.

5. Equilibrium
   
a) **(2 points)** Draw your AD curve from part 3. together with your AS curve from part 4. in space \((P, Y)\).

b) **(3 points)** Assume that \( P^e \) increases. What kind of fiscal or monetary policy would you advise the government to do? What is the trade-off? Argue using graphs.

**Answers:**

a.) AS curve slopes up, AD curve slopes down.

b.) The increase in expected price level leads to a north west shift of the AS curve, i.e. holding prices fix, to lower output, or holding \( Y \) fix, to higher prices. Fiscal and monetary policy can only shift the AD curve, which leads either to a further decline in output when the aim is to detain inflation, or else to inflation when the aim is to reduce unemployment. The government is only able to influence the AD curve, so a shift in the AS curve is difficult to accommodate with fiscal and monetary policy.