Recitation #1

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Nominal (Engineering) $\sigma$ & $\varepsilon$

- $\sigma = \frac{P}{A_o}$ Force/Original x-sectional area
  - Tensile stress (+)
  - Compressive stress (-)
  - Only valid if uniformly distributed force over entire area, $A$.

- $\varepsilon = \frac{\delta}{L_o}$, elongation/original length
  - Ratio of two lengths (unitless)
  - Also, $\varepsilon = \frac{L-L_o}{L_o} = \frac{L}{L_o}-1 = \lambda-1$
Normal (True) $\sigma$ & $\varepsilon$

- $\sigma = \frac{P}{A}$  Force/Area $\leftarrow$ A changes with time
  - Tensile stress (+)
  - Compressive stress (-)
  - Only valid if uniformly distributed force over entire area, A.

- $\varepsilon = \frac{\delta}{L'}$  elongation/length $\leftarrow$ This changes with time
  - Ratio of two lengths (unitless)
  - $\varepsilon = \int \left( \frac{dL}{L} \right)$ from $L_0$ to $L = \ln(L/L_0)$
Small $\varepsilon$ (strain) condition

- $\varepsilon \sim \varepsilon_{\text{nominal}}$
  - Because, $\varepsilon = \ln(1 + \varepsilon_{\text{nominal}})$

- $\sigma \sim \sigma_{\text{nominal}}$
  - Because, $\sigma = \sigma_{\text{nominal}} (1 + \varepsilon_{\text{nominal}})$

This shows that when dealing with elastic strains (not rubbers), it doesn’t matter whether true or normal or nominal stresses/strains are used in calculation.
Hooke’s Law

- Linear Elasticity – allows conversion from raw data to stress vs. strain curves
- \[ \sigma = E \varepsilon \]
- \( E = \text{Young’s Modulus (Elastic Modulus)} \)
- Similarly,
  - \( F = kd \) (force = constant*distance)
  - \( P = k \delta \) (force = spring constant*displacement)
Elasticity & Poisson’s ratio

- Linear (metals, ceramics) \( \rightarrow \) ALL mat’ls exhibit elastic behavior at \( \varepsilon < 0.001\% \)
- Nonlinear (polymers, rubbers) \( \rightarrow \) Some mat’ls exhibit large strain elasticity
  
  \[ v = - \frac{\varepsilon'}{\varepsilon} \] (if bar stretched in x-direction)
  
  - \( \varepsilon \) is axial strain (the x-related strain)
  - \( \varepsilon' \) is lateral (either y or z strain)
Example Problem 1

A circular aluminum tube of length $L=500\text{mm}$ is loaded in compression by forces $P$. The outside & inside diameters are $60\text{mm}$ & $50\text{mm}$, respectively. A strain gage is placed on the outside to measure normal strains in the longitudinal direction.

Strain gauge $\varepsilon = 540 \times 10^{-6}$
 Problem 1 Continued...

(a) If the measured strain is \(\varepsilon = 540 \times 10^{-6}\), what is the shortening \(\delta\) of the bar?
\[
\delta = \varepsilon L = 0.270\text{mm}
\]

(b) If the compressive stress in the bar is intended to be 40 MPa, what should be the load, \(P\) ?
\[
\sigma = 40\text{MPa}, \quad A = \frac{\pi}{4}[d_2^2 - d_1^2] = 863.9\text{mm}^2,
\]
\[
P = \sigma A = 40\text{MPa} \times 863.9\text{mm}^2 = 34.6\text{kN}
\]
Example Problem 2

Imagine a long copper wire hangs virtually from a high-altitude balloon.

(a) What is the greatest length (feet) it can have without yielding if the copper yields at 25 ksi?

(b) If the same wire hangs from a ship at sea, what is the greatest length?

\[ \gamma_c = \text{wt.} \] Density of copper = 556 lb/ft^3 \[ \gamma_w = \text{wt} \] density of sea water = 63.8 lb/ft^3
(a) \( W = \) total weight of copper wire = \( \gamma_c AL, \sigma_{\text{max}} = \frac{W}{A} = \gamma_c L, \ L_{\text{max}} = \frac{\sigma_{\text{max}}}{\gamma_c} = 25,000 \text{psi}/556 \text{lb/ft}^3 \) (b/c 1,000 psi in 1ksi)

(b) \( F = \) tensile force at top of wire, \( F = (\gamma_c - \gamma_w)AL, \sigma_{\text{max}} = \frac{F}{A} = (\gamma_c - \gamma_w)L, \ L_{\text{max}} = \frac{\sigma_{\text{max}}}{(\gamma_c - \gamma_w)} = 7310 \text{ ft.} \)
Example Problem 3

A prismatic bar of circular cross section is loaded by tensile forces, $P$. The bar has length, $L=3.0\text{ m}$ and diameter $d=30\text{ mm}$. It is made of an aluminum alloy with modulus of elasticity $E=73\text{ GPa}$. And Poisson’s ratio $\nu = 1/3$. If the bar elongates by $7.0\text{ mm}$, what is the decrease in diameter $\Delta d$? What is the magnitude of the load $P$?

![Diagram of a prismatic bar with forces and dimensions labeled.]
Problem 3 Continued...

Axial strain: \( \varepsilon = \frac{\delta}{L} = \frac{7\text{mm}}{3\text{m}} = 0.002333 \)

Lateral strain: \( \varepsilon' = -\nu \varepsilon = -\frac{1}{3} (0.002333) = -0.000778 \)

(Minus sign means shortening)

Decrease in diameter: \( \Delta d = |\varepsilon'|d = (0.000778)(30\text{mm}) \)

Tensile loads: Axial stress \( \sigma = E \varepsilon = 73\text{GPa} \times 0.002333 \)
\[ = 170.3\text{MPa} \]

(This stress is less than the yield stress, so Hooke’s law is applicable)

\( P = \sigma A = (170.3\text{MPa})(\pi/4)(30\text{mm})^2 = 120\text{kN} \)