Lamina Constitutive Relations

- **Isotropic**

\[
\begin{bmatrix}
E_x \\
E_y \\
\nu_{xy}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{E_x} & -\nu_x & 0 \\
-\nu_x & \frac{1}{E_y} & 0 \\
0 & 0 & \frac{1}{G_{xy}}
\end{bmatrix} \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}
\]

- **Transversely isotropic**

\[
\begin{bmatrix}
E_1 \\
E_2 \\
\nu_{12}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{E_1} & -\nu_{21} & 0 \\
-\nu_{21} & \frac{1}{E_2} & 0 \\
0 & 0 & \frac{1}{G_{12}}
\end{bmatrix} \begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix}
\]

\[
\frac{\nu_{21}}{E_2} = \frac{\nu_{12}}{E_1} \rightarrow 4 \text{ constants}
\]
Axis Transformations

\[ \sigma' = \begin{bmatrix} \sigma_1' \\ \sigma_2' \\ \sigma_{12}' \end{bmatrix} \quad \sigma = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \]

\[ \{ \sigma' \} = [T] \{ \sigma \} \quad T = \begin{bmatrix} c^2 & s^2 & zsc \\ s^2 & -c^2 & -zsc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \]

\[ c = \cos \theta \quad s = \sin \theta \]

Strains: almost the same

\[ \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{bmatrix} = [T] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix} \]

\[ \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix} = R \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix} = RT^{-1} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{bmatrix} = RT \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{bmatrix} \]

\[ = RT^{-1} S \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = RT \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \]

\[ \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix} = \begin{bmatrix} \bar{S} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \]

\[ \bar{S} = RT^{-1} S \]

\[ \bar{S} = R^{-1} S \]
Example 19.1 - Kevlar/epoxy ply at 30 degrees

Read linear algebra package

\[ \text{with(linalg):} \]

Warning, new definition for \texttt{norm}
Warning, new definition for \texttt{trace}

Define compliance matrix (Eq. 19.4):

\[ S := \text{matrix}(3,3, \begin{bmatrix} \frac{1}{E_1} & \frac{-\nu_{21}}{E_2} & 0 \\ \frac{-\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}); \]

Numerical parameters for Kevlar/epoxy:

\[ \text{Digits} := 4; \text{unprotect(E); } E_1 := 82 \times 10^{11}; \]
\[ E_2 := 4 \times 10^{10}; \]
\[ G_{12} := 2.8 \times 10^{10}; \]
\[ \nu_{12} := .25; \]
\[ \nu_{21} := \nu_{12} \times \frac{E_2}{E_1}; \]

Compliance matrix evaluated:

\[ S_2 := \text{map(eval,S)}; \]

Transformation matrix (Eq. 19.6)

\[ A := \text{matrix}(3,3, \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2-s^2 \end{bmatrix}); \]

Evaluate transformation matrix at 30 degrees (convert to radians):

\[ s := \sin(\theta); \]
\[ c := \cos(\theta); \]
\[ \theta := 30 \times \pi/180; \]
\[ s := \sin(\theta) \]
\[ c := \cos(\theta) \]
\[ \theta := -\frac{\pi}{6} \]

\[
A_2 := \text{evalf(map(eval,A))};
\]

\[
A_2 := \begin{bmatrix}
0.7500 & 0.2500 & 0.8660 \\
0.2500 & 0.7500 & -0.8660 \\
-0.4330 & 0.4330 & 0.5000
\end{bmatrix}
\]

Reuter's matrix (Eq. 19.9):

\[
R := \text{matrix}(3,3,[[1,0,0],[0,1,0],[0,0,2]]);
\]

\[
R := \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 2
\end{bmatrix}
\]

Transformed compliance matrix (Eq. 19.11):

\[
S_{\text{bar}} := \text{evalf(evalm(R &* inverse(A2) &* inverse(R) &* S_2 &* A2 ))};
\]

\[
S_{\text{bar}} := \begin{bmatrix}
0.8828 \times 10^{-10} & -0.1968 \times 10^{-10} & -0.1222 \times 10^{-9} \\
-0.1969 \times 10^{-10} & 0.2071 \times 10^{-9} & -0.8370 \times 10^{-10} \\
-0.1222 \times 10^{-9} & -0.8377 \times 10^{-10} & 0.2905 \times 10^{-9}
\end{bmatrix}
\]

Stiffness in x-direction:

\[
E_x = \frac{1}{S_{\text{bar}}[1,1]};
\]

\[ E_x = 0.1133 \times 10^{11} \]