Prob. 15.5

A 60°/0°/60° layup gives an example of what are called "quasi-isotropic" laminates, having equal stiffnesses in the \(x\) and \(y\) directions, regardless of the laminate orientation. Verify that this is so for two laminate orientations, one having the middle ply fibers oriented along the \(x\) axis and the other with the middle ply fibers oriented at 30° from the \(x\) axis. Examine the \(B\) submatrices (Eqn. 15.22) for these cases, and explain how the laminates do not act as isotropic materials.

First laminate:

1> plate
assign properties for lamina type 1...
enter modulus in fiber direction...
(enter -1 to stop): 230e9
enter modulus in transverse direction: 6.6e9
enter principal Poisson ratio: .25
enter shear modulus: 4.8e9
enter ply thickness: .13e-3
assign properties for lamina type 2...
enter modulus in fiber direction...
(enter -1 to stop): -1
define layup sequence, starting at bottom...
(use negative material set number to stop)
enter material set number for ply number 1: 1
enter ply angle: 60 \(\Rightarrow 60/0/-60\) layup
enter material set number for ply number 2: 1
enter ply angle: 0
enter material set number for ply number 3: 1
enter ply angle: -60
enter material set number for ply number 4: -1

laminate stiffness matrix:

\[
\begin{bmatrix}
0.3576E+08 & 0.1110E+08 & 0.0000E+00 & 0.0000D+00 & 0.0000D+00 & -0.8528D+03 \\
0.1110E+08 & 0.3576E+08 & 0.0000E+00 & 0.0000D+00 & 0.0000D+00 & -0.2423D+04 \\
0.0000E+00 & 0.0000E+00 & 0.1233E+08 & -0.8528D+03 & -0.2423D+04 & 0.0000D+00 \\
0.0000E+00 & 0.0000E+00 & -0.8528E+03 & 0.1485D+00 & 0.1996D+00 & 0.0000D+00 \\
0.0000E+00 & 0.0000E+00 & -0.2423E+04 & 0.1996E+00 & 0.6402D+00 & 0.0000D+00 \\
-0.8528E+03 & -0.2423E+04 & 0.0000E+00 & 0.0000D+00 & 0.0000D+00 & 0.2152D+00 \\
\end{bmatrix}
\]

laminate compliance matrix:

\[
\begin{bmatrix}
0.3114E-07 & -0.5507E-08 & 0.0000E+00 & 0.0000D+00 & 0.0000D+00 & 0.6140D-04 \\
-0.5507E-08 & 0.1188E-06 & 0.0000E+00 & 0.0000D+00 & 0.0000D+00 & 0.1316D-02 \\
0.0000E+00 & 0.0000E+00 & 0.3276E-06 & 0.3699D-03 & 0.1125D-02 & 0.0000D+00 \\
0.0000E+00 & 0.0000E+00 & 0.3699E-03 & 0.1201D+02 & -0.2345D+01 & 0.0000D+00 \\
0.0000E+00 & 0.0000E+00 & 0.1125E-02 & -0.2345D+01 & 0.6548D+01 & 0.0000D+00 \\
0.6140E-04 & 0.1316E-02 & 0.0000E+00 & 0.0000D+00 & 0.0000D+00 & 0.1970D+02 \\
\end{bmatrix}
\]

Second laminate:

2> plate
assign properties for lamina type 1...
enter modulus in fiber direction...
(enter -1 to stop): 230e9
enter modulus in transverse direction: 6.6e9
Note that the 1,1 and 2,2 elements of the stiffness matrix are the same, and also the same for the second laminate oriented 30° from the first. This is an apparent isotropy: stiffness the same in all directions (though only verified here for four directions). Neither laminate is symmetric, so there are nonzero coupling terms in the $B$ submatrices, which is not expected in isotropic materials.

The appearance of isotropy is not evident in the compliance matrix, where the 1,1 and 2,2 elements are different; this is a consequence of the extension-curvature coupling that occurs in the nonsymmetric laminate. If the laminate is made quasi-isotropic and symmetric, as in the 60/0/-60/-60/0/60 layup below, the appearance of isotropy occurs in both the stiffness and compliance matrices.
enter material set number for ply number 2: 1
enter ply angle: 0

enter material set number for ply number 3: 1
enter ply angle: -60

enter material set number for ply number 4: 1
enter ply angle: -60

enter material set number for ply number 5: 1
enter ply angle: 0

enter material set number for ply number 6: 1
enter ply angle: 60

enter material set number for ply number 7: -1

laminate stiffness matrix:

<table>
<thead>
<tr>
<th></th>
<th>0.7152E+08</th>
<th>0.2221E+08</th>
<th>0.0000E+00</th>
<th>0.2441D-03</th>
<th>-0.1221D-03</th>
<th>0.0000D+00</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2221E+08</td>
<td>0.7152E+08</td>
<td>0.0000E+00</td>
<td>0.0000D+00</td>
<td>0.0000D+00</td>
<td>0.2441D-03</td>
<td>0.1221D-03</td>
</tr>
<tr>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.2466E+08</td>
<td>0.0000D+00</td>
<td>0.0000D+00</td>
<td>0.2441D-03</td>
<td>0.0000D+00</td>
</tr>
</tbody>
</table>

laminate compliance matrix:

<table>
<thead>
<tr>
<th></th>
<th>0.1547E-07</th>
<th>-0.4804E-08</th>
<th>-0.1010E-22</th>
<th>-0.1685D-11</th>
<th>0.5872D-12</th>
<th>0.8656D-12</th>
</tr>
</thead>
<tbody>
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<td>-0.4804E-08</td>
<td>0.1547E-07</td>
<td>0.5762E-23</td>
<td>0.7364D-12</td>
<td>0.3160D-11</td>
<td>-0.7484D-11</td>
<td>-0.1046D-10</td>
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<td>-0.1349E-23</td>
<td>0.9467E-24</td>
<td>0.4055E-07</td>
<td>0.3092D-12</td>
<td>0.4847D-11</td>
<td>-0.1046D-10</td>
<td>-0.1046D-10</td>
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<tr>
<td>-0.1667E-11</td>
<td>0.1170E-11</td>
<td>0.1193E-11</td>
<td>0.3821D+00</td>
<td>-0.8927D-01</td>
<td>-0.6247D-01</td>
<td>-0.1046D-10</td>
</tr>
<tr>
<td>0.3895E-12</td>
<td>-0.2733E-12</td>
<td>-0.2482E-11</td>
<td>-0.8927D-01</td>
<td>0.7403D+00</td>
<td>-0.9792D+00</td>
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<tr>
<td>0.2726E-12</td>
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<td>-0.6247D-01</td>
<td>-0.9792D+00</td>
<td>0.2114D+01</td>
<td>-0.1046D-10</td>
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</tbody>
</table>