Prob. 2.7

(a) Relation for derivatives:

\[
\left( \frac{\partial T}{\partial L} \right)_S = \frac{\left( \frac{\partial S}{\partial L} \right)_T}{\left( \frac{\partial S}{\partial T} \right)_L}
\]  

(1)

Second law along with heat content as mass \( M \) specific heat \( c \) temperature change \( dT \):

\[
dS = \frac{dQ}{T}, \quad dQ = McdT =TdS = \left( \frac{\partial S}{\partial L} \right)_T = \frac{Mc}{T}
\]  

(2)

Substituting (2) into (1):

\[
\left( \frac{\partial T}{\partial L} \right)_S = -\frac{T}{Mc} \left( \frac{\partial S}{\partial T} \right)_L
\]  

(3)

(b) First and second law, with \( dU = 0 \) for an ideal rubber:

\[
dU = 0 = dQ + dW = TdS + FdL \Rightarrow \frac{\partial S}{\partial L} = \frac{-F}{T}
\]  

Using this in Eq. (3) of the previous problem:

\[
\frac{\partial T}{\partial \lambda} = \frac{F}{Mc}
\]  

(4)

Extension ratio: \( \lambda = L / L_0 \Rightarrow dL = L_0 d\lambda \). Using this in Eq. (4):

\[
\frac{\partial T}{\partial \lambda} = \frac{FL_0}{Mc} = \frac{FAL}{A_0Mc}
\]  

(5)

Now substituting

\[
\frac{AL}{M} = \frac{V}{M} = \frac{1}{\rho}
\]

where \( V \) is volume and \( \rho \) is density, along with the engineering stress \( \sigma = F / A_0 \) into (5):

\[
\frac{\partial T}{\partial \lambda} = \frac{\sigma}{\rho c} \Rightarrow dT = \frac{\sigma}{\rho c} d\lambda
\]