I. Finding the Open Circuit Time Constants $\tau_j$’s

A. Example: CE Amplifier

- Small signal model

(a)
B. Procedure

- Eliminate all independent sources (e.g., $V_s \rightarrow 0$)
- Open-circuit all capacitors
- Find Thevenin resistance by applying $i_t$ and measuring $v_t$.

C. Time Constant for $C_\pi$

• Result: (by inspection)

\[
R_{T\pi} = R_s \parallel r_\pi \\
\tau_{C_{\pi O}} = R_{T\pi} C_\pi
\]
D. Time Constant for $C_{\mu}$

- Same procedure:

$$i_t = \frac{v_t + v_\pi}{R'_{out}} + g_m v_\pi$$

$$\frac{v_\pi}{R'_{in}} = -i_t \quad \text{eliminate } v_\pi$$

$$\frac{v_t}{i_t} = R_T \mu = R'_{out} + R'_{in} \left( 1 + g_m R'_{out} \right)$$

$$\tau_{C_{\mu o}} = R_T \mu C_{\mu} = \left[ R'_{out} + R'_{in} \left( 1 + g_m R'_{out} \right) \right] C_{\mu}$$
E. Dominant Pole for CE Amplifier

- Sum Individual time constants

\[
b_1 = (R_T r C_p + R_T \mu C_p)
\]

\[
b_1 = R'_in C_p + R'_in \left( 1 + g_m R'_out \right) C_p + R'_out C_p
\]

- Assume \( \tau_1 \gg \tau_2 \)

\[
b_1 = \tau_1 + \tau_2 \approx \tau_1
\]

\[
\omega_{3dB} \approx \frac{1}{b_1} = \frac{1}{R'_in C_p + R'_in \left( 1 + g_m R'_out \right) C_p + R'_out C_p}
\]

- Very similar result to the Miller effect calculation
- Additional term \( R'_out C_p \) is taken into account
II. Common Collector Frequency Response

A. Small Signal Model

- Add $C_\mu$ and $C_\pi$ to the two-port model from Chapter 8
B. Low Frequency Voltage Gain

\[ \frac{v_{out}}{v_s} = \left( \frac{R_{in}}{R_S + R_{in}} \right) (1) \left( \frac{R_L}{R_L + R_{out}} \right) \]

- Substituting values for input and output resistance

\[ \frac{v_{out}}{v_s} = \left( \frac{r_\pi + \beta_o R_L}{R_S + r_\pi + B_o R_L} \right) (1) \left( \frac{R_L}{R_L + (1/g_m) + (R_S/\beta_o)} \right) \]

C. Use Miller Approximation to Find Dominant Pole

- Voltage gain from B to E across \( C_\pi \)

\[ A_vC_\pi = \frac{R_L}{R_{out} + R_L} = \frac{R_L}{1/g_m + R_L} \]

- Total Capacitance seen at the input \( C_T = (1 - A_vC_\pi)C_\pi + C_\mu \)

\[ C_T = C_\pi \left( \frac{1/g_m}{1/g_m + R_L} \right) + C_\mu \]

- Thevenin resistance seen by \( C_T \)

\[ R_T = R_S \parallel R_{in} \]
D. Common Collector Dominant Pole

- The dominant time constant for a CC amplifier is

\[ \tau = \left( R_s \parallel R_{in} \right) \left[ \frac{C}{1 + g_m R_L} + C \mu \right] \]

- Substitute for \( R_{in} \) and look at \( \omega_{3dB} \)

\[
\omega_{3dB} = \frac{1}{\left( R_s \parallel \left( r_o + \beta_o R_L \right) \right) \left( \frac{C}{1 + g_m R_L} + C \mu \right)}
\]

- Effect of \( C_\pi \) is small

- In general \( R_s < R_{in} \)----> frequency response is dominated by \( R_s C \mu \)
III. Common Base Amplifier Frequency Response

A. Small Signal Model

- DC Gain

\[
\frac{i_{\text{out}}}{i_s} = \left( \frac{R_S}{R_{\text{in}} + R_S} \right) \left( \frac{R_{\text{out}}}{R_L + R_{\text{out}}} \right)
\]
B. Use Method of Open Circuit Time Constants

- Thevenin resistance across $C_\pi$

$$R_{T\pi} = R_S \parallel R_{in} = R_S \parallel \left(\frac{1}{g_m}\right) = \frac{1}{g_m}$$

- Thevenin resistance across $C_\mu$

$$R_{T\mu} = R_{out} \parallel R_L \approx \beta_o r_o \parallel R_L \approx R_L$$

- Summing the open circuit time constants and taking reciprocal

$$\omega_{3dB} = \frac{1}{\left(C_\pi/g_m\right) + R_L C_\mu}$$

IV. Summary of Frequency Response of Single-Stages

CE/CS: with voltage output - suffers from Miller effect

CE/CS: with current output - “wideband”

CB/CG: “wideband”

CC/CD: “wideband”

- “Wideband” means that the stage can operate near the frequency limit of the device ... $f_T$

- Frequency limitation is set by external circuit $R_S$ and $R_L$