6.334 Summary: Magnetics

**Reluctance Model:** Analogy of electric circuit model for magnetics (Magnetic Circuits)

<table>
<thead>
<tr>
<th>Electric</th>
<th>Magnetic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage (EMF)</td>
<td>Magnetomotive Force (MMF), Ampere turns (NA)</td>
</tr>
<tr>
<td>Current</td>
<td>flux</td>
</tr>
<tr>
<td>Resistance $\frac{V}{I}$ (Ω)</td>
<td>Reluctance $\frac{2}{(\mu A)}$</td>
</tr>
</tbody>
</table>

\[ R_c = \frac{\ell_c}{\mu_c A_c}, \quad R_g = \frac{\ell_g}{\mu_g A_g} \]

\[ \Phi = \frac{NI}{R_c + R_g} = \frac{NI}{\ell_c/\mu_c A_c + \ell_g/\mu_g A_g} \]

Flux direction determined by right-hand rule.

To find terminal voltage $V$: Voltage of each turn is $V_\text{turn} = \frac{d\Phi}{dt}$. Voltage of whole coil is $N$ times this.

Hence, the definition of flux linkage:

\[ \lambda = N\Phi \quad \text{(flux linkage)} \]

\[ V = \frac{d\lambda}{dt} = \left[ \frac{N^2}{\ell_c/\mu_c A_c + \ell_g/\mu_g A_g} \right] \frac{d\Phi}{dt} \]
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Notes:
1. \( L \propto N^2 \)
2. For accuracy, we need \( M_c \gg \mu_0 \), \( L_c \gg \sqrt{A_c} \), \( h_c \ll \sqrt{A_c} \)
3. We rely on material property \( B \propto H \). This is an approximation to real behavior, satisfied for a limited range \( B \ll B_{sat} \)

Transformers (2 Winding)

\[
\begin{align*}
\begin{bmatrix}
\lambda_1 \\
\lambda_2
\end{bmatrix} &= \begin{bmatrix}
\frac{N_1^2}{R_c} + \frac{N_2^2}{R_{21}} & \frac{N_1 N_2}{R_c} \\
\frac{N_1 N_2}{R_c} & \frac{N_2^2}{R_c} + \frac{N_2^2}{R_{22}}
\end{bmatrix}
\begin{bmatrix}
\lambda_1 \\
\lambda_2
\end{bmatrix}
\end{align*}
\]

This is equivalent to:

Practical Transformer Model:

\[
\begin{align*}
L_{m1} &= \frac{Z_1^2}{R_c} \\
L_{r1} &= \frac{Z_1^2}{R_{21}} \\
L_{r2} &= \frac{Z_2^2}{R_{22}}
\end{align*}
\]

If \( R_c \to 0 \Rightarrow L_{m1} \to \infty \)
\( R_{21}, R_{22} \to \infty \Rightarrow L_{r1}, L_{r2} \to 0 \)

We then get an "ideal transformer with characteristics:"

\[
N_1 V_2 = N_2 V_1 \quad \text{and} \quad N_1 \lambda_1 = -N_2 \lambda_2
\]
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Transformers with more than 2 windings (ideal case: \( R_e \to \infty, R_c \to 0 \))

**Series-Wound Structure**

\[ N_1 \lambda_1 + N_2 \lambda_2 + N_3 \lambda_3 = 0 \]

\[ \frac{N_1}{N_1} = \frac{N_2}{N_2} = \frac{N_3}{N_3} \]

If \( R_c \to 0 \), no leakage

**Parallel-Wound Structure**

\[ N_1 \lambda_1 = N_2 \lambda_2 = N_3 \lambda_3 \]

\[ \frac{N_1}{N_1} + \frac{N_2}{N_2} + \frac{N_3}{N_3} = 0 \]

- We get different relations in series, parallel, and other cases.
- In the general case, the "dot" convention is no longer a sufficient description.
- If we include nonlinearities in an M-winding system, we get an \( M \times M \) inductance matrix with \( (\frac{m}{d}) \) independent parameters.