**6.334 Summary: Power Factor**

**Definition:**
1. Root-mean-square of a waveform $X$ is

\[ X_{\text{rms}} = \sqrt{\frac{1}{T} \int_{-T}^{T} X^2(t) \, dt} \]

The RMS voltage or current is the dc voltage or current that will deliver the same average power to a resistor as the periodic (time-varying) voltage or current.

2. Two functions are **orthogonal** over $[a,b]$ iff

\[ \int_{a}^{b} x(t) y(t) \, dt = 0 \]

**Identities**

\[ \int_{0}^{2\pi} \sin(m \omega t) \sin(n \omega t + \phi) \, d(\omega t) = 0 \quad \text{if} \quad n \neq m \]

\[ \Rightarrow \text{Sinusoids of different frequencies are orthogonal} \]

\[ \int_{0}^{2\pi} \sin(\omega t) \cos(\omega t) \, d(\omega t) = 0 \]

\[ \Rightarrow \text{Sine and Cosine are orthogonal} \]

In general

\[ \frac{1}{2\pi} \int_{0}^{2\pi} \sin(\omega t) \sin(\omega t + \phi) \, d\omega = \frac{1}{2} \cos \phi \]

These identities are handy for calculating power, etc.

**Power Factor** provides a measure of the utilization of a source (e.g. fraction of possible power drawn from a source with an RMS current limit.)

\[ \text{P.F.} = \frac{<P>}{V_{\text{rms}} I_{\text{rms}}} \quad <\text{real power}, P> \]

\[ = \quad <\text{apparent power}, S> \]
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**Example**

For a resistor

\[ V_s \sin(\omega t) \]

\[ \text{For an inductor} \]

\[ V_s \]

\[ <P> = V_{\text{rms}} I_{\text{rms}} \]

\[ \Rightarrow \text{P.F.} = 1 \]

\[ <P> = 0 \text{ (no average power)} \]

\[ \Rightarrow \text{P.F.} = 0 \]

**For a load that draws nonsinusoidal current:**

\[ V_s(\sin(\omega t)) \]

\[ I(t) = \sum_{n=0}^{\infty} I_n \sin(n\omega t + \phi_n) \]

\[ V_{\text{rms}} = \frac{V_s}{\sqrt{2}} \]

\[ I_{\text{rms}} = \sqrt{\frac{I_1^2}{2} + \frac{I_2^2}{2} + \cdots + \frac{I_n^2}{2} + \cdots} \]

\[ <P> = <V^2> = \frac{1}{2} V_s I_1 \cos(\phi_1) = V_{\text{rms}} I_{\text{rms}} \cos(\phi) \]

\[ \text{P.F.} = \frac{<P>}{V_{\text{rms}} I_{\text{rms}}} = \frac{(I_{\text{rms}}/I_{\text{rms}})}{K_d \cdot K_a} \]

Power Factor can be broken into two factors:

- **K_d** (distortion factor) (K_d < 1) tells how much utilization of source is reduced because of harmonic currents (that cannot contribute to average power by orthogonality)

- **K_a** (displacement factor) (K_a < 1) tells how much utilization is reduced because of phase shift between voltage and fundamental current.