Unit 3: Motion in a Plane

In this unit we shall continue our investigation of kinematics, extending the concepts introduced in the previous unit to motion of a particle in a plane. Projectile motion and circular motion are two common examples of this motion, and we shall investigate both of them.

Motion in a plane displays characteristics which are not present in linear motion and which are essential to our study. For example, in linear motion the path of the particle is always a straight line by definition, whereas in a plane there is an infinite variety of curves which the particle may follow in its motion. Furthermore, in linear motion the vectors describing that motion—displacement, velocity, acceleration—could possess but two directions, to the right or to the left. In a plane these vectors may now point in any direction and we must learn how to manipulate these quantities when there are two degrees of freedom. Once we master the kinematics of planar motion, the extension to motion in space is straight-forward.

Objectives: After completing this unit you should be able to:

1. Predict the motion of a body given its acceleration as well as the initial values of its position and velocity in a plane, and

2. Calculate the velocity and acceleration of a particle (in polar as well as Cartesian coordinates) given its position in a plane as a function of time.

Suggested Procedure:

1. Read H&R, Chapter 4

2. Read Berkeley, Chapter 2, pg. 40 to end. This contains a good treatment of polar vectors and vector derivatives.

3. Do problems in H&R, pg. 68–70, #13, 28, 29, 34 and in the PANIC Problem Booklet, pg. VII–8 to VII–10, #17, 18, 19, 20.
4. If you encounter difficulties with the problems, you might want to consult H&R SSGWPP, Chapter 4.

5. Take a unit test.

**Optional Problem:** Starting from $a_\theta = \dot{v}_\theta + 2\dot{\theta}v_r$, show that

$$ra_\theta = \frac{d}{dt} \left( \dot{\theta}r^2 \right).$$