Determining the mean speed of cosmic-ray muons

Danny Ben-David

MIT - Department of Physics
Cosmic rays bombard the upper atmosphere, causing particle showers.

Among the showers are near-luminal muons.

Muon has a short lifetime, how does it make it to the ground? Answer: Time dilation.
Scintillators

- Incident muons cause the plastic to release photons
- Photodetector collects the photons and emits a signal announcing an event
Experimental Setup

Delay set to 12ft
Add to the measured time of flight, so the data isn't crushed into the first few ns

CFDs clip event rate to 100Hz

50ns range

Source: 8.13 "Cosmic-Ray Muons" Lab Guide.
Timing Data and a Double Gaussian Fit

- Integrated data for $\Delta z =$
  - 300.0 cm
  - 175.0 cm (top)
  - 100.0 cm
  - 75.0 cm
  - 50.0 cm (bottom)
  - 25.0 cm [mis-calibrated]

- Mean time: arithmetic mean of two Gaussians, weighted by their amplitudes

\[
y(x) = A \exp\left(-\frac{(x-\mu_A)^2}{2\sigma_A^2}\right) + B \exp\left(-\frac{(x-\mu_B)^2}{2\sigma_B^2}\right)
\]
Mean Path Length via Monte Carlo

- Muons take different paths with different lengths
- $10^6$-trial Monte Carlo (MC) for each $\Delta z$
- Weighted by angle and position
- Smaller $\Delta z$ allow more angles $\rightarrow$ larger mean path length

Intensity $\propto \cos^2 \theta$

Position along paddle $\left(1 - \frac{x}{\ell}\right)^3$

Photons $\propto \frac{1}{\cos \theta}$
Mean Time of Flight vs. Mean Path Linear Fit

- Linear fit with constant offset from delays (intentional and systematic)
- slope\(^{-1}\) = velocity
- More data on \(\Delta z = 300.0\text{cm}\) → smaller \(\sigma_{300\text{cm}}\)

\[
\frac{\Delta t}{\Delta z} = \frac{1}{\nu} + \tau
\]

\(\nu = 0.99271 \pm 0.07279\) \(c\)

\(\tau = 21.1280 \pm 0.6878\) \(\text{ns}\)

\(\chi^2 = 0.0946\) for 3 degrees of freedom

99.50% Probability of fit
Uncertainties

- Calculated uncertainty on the mean from error propagation on $\Delta \mu_A$ and $\Delta \mu_B$, which are controlled by the curve’s width.
- Factors in the Gaussian fit’s width (for $\Delta z = 175.0$ cm):

<table>
<thead>
<tr>
<th>Source</th>
<th>Amount</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum</td>
<td>$+0.1160_{-0.0047}$ns</td>
<td></td>
</tr>
<tr>
<td>Path length</td>
<td>$\pm 0.0405$ns</td>
<td>Calculated with MC</td>
</tr>
<tr>
<td>Delay within paddle</td>
<td>$\pm 0.4907$ns</td>
<td>Calculated with MC</td>
</tr>
</tbody>
</table>

- Remaining spread attributed to uncertainty in timing circuitry.
- Systematic uncertainty contributes constant offset to all measurements, which affect the value of the delay, but not the slope (or velocity), of the linear fit.
$v_{\text{measured}} = 0.9927 \pm 0.0728 c$

From the momentum distribution, we find $v_{\text{accepted}} = 0.9988^{+0.1140}_{-0.0047} c$, which is $0.084\sigma$ away from $v_{\text{measured}}$.

Lorentz factor: $\gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}} = 8.291$

Longer integration would have reduced uncertainties further.
Cosmic-ray muons go *very fast*

At near-luminal speeds, relativistic effects are in play, especially time dilation

This allows a non-negligible fraction of the muons produced to reach sea level; for more see Dakota’s presentation