Directions:  This exercise should be completed independently.  It is due in class on Monday, October 19.  Please answer all of the parts (a)-(d).  Good luck!

You have been hired as the economic advisor to a small island economy that is trying to configure its tax and expenditure system optimally.  The island’s leaders have agreed that social policies should be designed to maximize a utilitarian social welfare function.  The island’s government provides a pure public good, shark control services, for the benefit of island residents. Island residents supply labor and consume a private good in addition to the public good.

(a) Centuries of isolation from the rest of humanity has produced a homogeneous population with identical preferences given by:

\[ u = \log c + \log (1-L) + \log (G) \]

where \( c \) denotes private good spending, \( L \) denotes labor supply, and \( G \) denotes aggregate outlays on shark control services.  Assume that the government has access to a lump sum tax, levied at an amount \( \delta \) per person, that everyone supplies labor at a wage of \( w \), and that there are \( N \) islanders in this economy.  Find the optimal level of the lump sum tax and the associated level of public goods that maximizes the sum of utilities in the population.  [Hint:  First find the indirect utility for each individual as a function of \( w \) and \( \delta \), then find the value of \( \delta \) that maximizes social welfare.]

(b) One of the island’s political leaders travels to the United States and discovers that income taxes, rather than lump sum taxes, are used to finance government expenditures in most developed nations.  She directs you to design an income tax that will finance the optimal level of public spending on shark control.  You assume that the labor income tax will be a simple proportional tax, and that there will be no lump sum taxes once the income tax is introduced.  (Taxes paid by each person will be \( t \times w \times L \) in this case.)

(b1) Find labor supply for the individuals who live on this island, as a function of their wage rate and the income tax rate.  Note that individuals assume that the level of public spending, \( G \), is fixed when they make their labor supply decisions.  What does your answer suggest about the deadweight loss of taxing labor income in this economy?

(b2) Now find the optimal income tax rate in this economy.  [Hint: Write each individual’s utility as a function of the marginal tax rate, then maximize the sum of utilities in the economy, recognizing (as the social planner does) that the sum of income tax revenue equals expenditures on shark control.]  Compare the level of spending in this case with that in part (a), when the government was financed using lump sum taxes.  Can you explain any similarities or differences?

(c) The same political leader who suggested the income tax now suggests that the island nation annex a small adjacent island.  The new island acquiesces in this plan.  It has the same number of inhabitants as the original island that you were advising, but each individual on the island has a wage that is twice that of the inhabitants on the original island.  The political leadership has decided to apply the same income tax rate to residents on the two islands, and to use the proceeds to finance shark control.

How does the addition of the second island change the optimal level of the income tax rate, relative to that for the single original island?  What happens to the level of shark control spending?
Would the inhabitants of either island find it in their best interest to secede from the two-island federation and provide their own shark control, assuming that they were restricted to using an income tax on their own island to finance the public good spending? (Explain.)

(d) Now reconsider the two-island economy that you analyzed in part (c), but assume that the government decides to return to lump-sum taxes to finance spending on public goods. Assume that it levies the same lump-sum tax on residents of the two islands. What is the optimal lump sum tax in this case? Can you provide any intuition for the relationship between this tax and that in (a) above?