Our experiment used a beam of thermal neutrons from the MIT Research Reactor to determine the velocity spectrum of neutrons and the temperature of the reactor based on the Maxwell-Boltzmann distribution. In the second part of the experiment, we used a copper crystal to demonstrate Bragg diffraction of neutrons and confirm the DeBroglie relation. In the first half of the experiment, we found the most probable neutron speed to be $2295 \pm 40$ m/s and the temperature of the reactor to be $319 \pm 11$ K. In the second half of the experiment, we found the value of $h/m$, where $m$ is the mass of a neutron, to be $3.90 \pm 0.18 \times 10^{-7}$ m$^2$.  

\section*{INTRODUCTION}

The MIT Research Reactor uses a chain-reacting system to create thermal neutrons. Neutrons react with Uranium-235, the “nuclear fuel”, to create a decay reaction in which more neutrons are created. Some of these neutrons can then react with another Uranium-235 atom, thereby forming a chain reaction. We use the resulting neutron beams to study two fundamental properties of diffraction and the DeBroglie relation.

The Maxwell-Boltzmann distribution was determined by Boltzmann in 1871 using Maxwell's kinetic theory of gases. Maxwell had shown that temperature and heat were determined only by molecular movement. The Maxwell-Boltzmann distribution described the probability, $p$, of a system having energy, $E$, based on the temperature, $T$ and the Boltzmann constant, $k_B$. From this, the Maxwell-Boltzmann distribution law was derived. Our experiment shows that thermal neutrons also obey this distribution law.

In 1923, Compton’s scattering experiment showed that photons in light beams act as both waves and particles. He determined the momentum, $p$, of the photons to be related to the wavelength, $\lambda$, by $p = h/\lambda$ where $h$ is Planck’s constant. In his 1924 PhD thesis, DeBroglie suggested that all particles of matter should have this wave-particle duality. This meant that electrons, protons, and any composite form such as atoms, a baseball, or the earth should have a characteristic wavelength, $\lambda$ related to its momentum.

Initially, many, including Albert Einstein, dismissed DeBroglie’s theory until 1927 when G.P. Thomson and Davison and Germer independently discovered electron diffraction. Using techniques developed by Laue and Bragg in 1912 for X-rays, they were able to reflect electron beams off of metal crystals and observe interference patterns. Their results confirmed DeBroglie’s hypothesis. Our experiment will attempt to recreate this observation using neutrons from the research reactor.

\section*{THEORY}

In order to determine the temperature of the reactor, we use the Maxwell-Boltzmann distribution law, which says the neutrons should obey the following:

$$n(v)dv = \frac{4N}{\sqrt{\pi}} \frac{v^2}{v_0^5} \exp\left(-\frac{v^2}{v_0^2}\right)dv$$

where $v$ is the neutron speed, $v_0$ is the most probable neutron speed, $N$ is the volume density of all neutrons, and $n(v)dv$ is the neutron density for those speeds falling within $v$ and $v + dv$. $v_0$ is related to the temperature, $T$ of the reactor by the equation:

$$v_0 = \sqrt{\frac{2k_BT}{m}}$$

where $k_B$ is the Boltzmann constant and $m$ is the neutron mass.

For the second half of the experiment, we attempt to prove the DeBroglie hypothesis that matter has a characteristic wavelength, $\lambda$, related to its momentum, $p = mv$, by:

$$\lambda = \frac{h}{mv}$$

In order to determine the wavelength of the neutrons, we used Bragg diffraction by a crystal lattice. The incident beam is reflected off a crystal, which can be thought of as a series of identical planes. Not all the beam is reflected off the surface plane, as illustrated in Figure (1). The path spacing between the two reflections is $2d \sin \theta_B$ where $d$ is the interplanar spacing and $\theta_B$ is the Bragg angle of incidence in reflection. A property of Bragg diffraction is that the incidence angle and reflected angles are the same. Constructive interference then occurs when:

$$n\lambda = 2d \sin \theta$$

$d$ is determined by a property of the crystal called the Miller indices. Dividing the crystal lattice into identical unit cubes, the Miller indices, $h$, $k$, and $l$, are defined.
by the intersection of the crystal planes with the unit cell axes as illustrated in Figure (2). Using elementary geometry, the interplanar spacing for a given set of Miller indices is:

\[ d_{hkl} = \frac{a_0}{\sqrt{h^2 + k^2 + l^2}} \]  

(5)

where \( a_0 \) is the unit cell size.

**EXPERIMENT**

For the Maxwell-Boltzmann distribution, the neutron beam was collimated through a slit and sent through a chopper wheel before reaching a Boron Trifluoride detector, as shown in Figure (3). The slit was in a neutron absorbing boron-plastic composite. The slotted chopper wheel was made of cadmium, which absorbs thermal neutrons, and rotated at 240 Hz in order to produce pulses of neutrons. The BF\(_3\) detector sent a pulse to the Multi-Channel Scaling card. The MCS outputted a count of neutrons detected for a given time, corresponding to the origin time of the neutron burst, \( C_0 \) was found by comparing the peak times, \( C_1 \) and \( C_2 \), of the two runs.

\[ C_0 = C_1 - \frac{L_1}{L_2 - L_1}(C_2 - C_1) \]  

(6)

For the Bragg diffraction, the setup was similar, but before the beam reached the detector, it was reflected off a copper crystal aligned to have Miller indices of (002). Taking the unit cell size of copper to be 3.6147 Å, this gives an interplanar spacing of \( d = 1.8077 \times 10^{-10} \) meters. A He\(_3\) detector was used and placed at 30°, 40°, 50°, and 60° from the line of the beam. This corresponded to Bragg angles of \( \theta_B = 15°, 20°, 25° \) and 30°. The detector was measured to be a distance 190.0 ± 0.3 cm from the chopper. The MCA was again set to 20 µs per channel. \( C_0 \) from the first half of the experiment was used.

**DATA AND ANALYSIS**

The time distribution as taken from the MCA for the Maxwell-Boltzmann analysis is shown in Figure (5). The near position approximates a triangular distribution as the neutrons had not had enough time to spread out. The far position seems to more accurately approximate a Maxwell-Boltzmann distribution. Each data point corresponds to the number \( N_i \) of neutrons detected for a given flight, \( t_i \). Since the distance \( L = vt \), we can convert equation (1) to:

\[ N_i = D \left( \frac{1}{v_i^4} \right) \exp \left( -\frac{L^2}{v_0^2 v_i^2} \right) \Delta t \]  

(7)

where \( D \) is a constant and \( \Delta t \) is the width of the MCA channel. From this, we can derive that:

\[ \ln \left( \frac{N_i}{N_1} \right) = Q - \frac{v_i^2}{v_0^2} \]  

(8)

where \( Q \) is a constant and \( v_i \) is the velocity for a given channel. We plotted \( \ln(N_i/N_1) \) vs. \( v_i^2 \) in Figure 6. Based on the slope of the resulting line, we determined \( v_0 \) to be 2295 ± 40 m/s. Using equation (2), we can compute the temperature of the reactor to be 319 ± 11 K.

For the second half of the experiment, we were not able to get into the research reactor. We used data provided to us by Scott Sewell, presented in Figure 7. For each of the main peaks in the plots, we assumed the order of di\( f \)raction to be \( n = 1 \). Using the values of \( d \) and \( \theta_B \) obtained from the Miller indices of the copper and placement of the He\(_3\) detector, we were able to compute \( \lambda \) for each plot using equation (5). The velocity was determined similar to the first half of the experiment, where \( v = L/t \) where \( t \) was determined by the peak MCA channel on the plot and \( C_0 \). We plotted \( v \) against \( 1/\lambda \).

**CONCLUSIONS**

We determined the most probable velocity of thermal neutrons in the reactor to be 2295 ± 40 m/s. This results in a reactor temperature of 319 ±11 K. The reactor is known to run at under 323 K, or 50 °C, making our result within expected range. We showed the wave-particle duality of matter and found a value for \( h/m \) to be 3.90 ±0.18 × 10\(^{-7}\) m²/s, well within the expected value of 3.956 × 10\(^{-7}\) m²/s.


ERR thanks her lab partner C. Clarke for all her efforts and insights during the course of this experiment and S. Sewell for providing data for the Bragg Diffraction half of the experiment.
FIG. 1: An illustration of Bragg diffraction through a crystal lattice. The incoming wave is reflected off the crystal, but portions of the wave are reflected off different planes of the lattice. These different reflections display interference patterns based on the Bragg angle, $\theta_B$, the interplanar spacing of the crystal, $d$, and the wavelength of the incident beam.

FIG. 2: (Taken from [1]) A graphical description of the Miller indices used to define a crystal plane. $a$, $b$, and $c$ are the unit vectors defining the unit cell axes.
FIG. 3: The apparatus used to determine the Maxwell-Boltzmann spectrum. The neutron beam was sent through a fixed slit to collimate the beam and a cadmium chopper wheel before being detected by the BF$_3$ detector which sent a signal to the Multi-Channel Scaling card. The experiment was run twice, with the detector in the near position, $L_1 = 5.5$ cm, and in the far position, $L_2 = 156.4$ cm.

FIG. 4: The apparatus used to determine the DeBroglie relation. The beam was sent through a collimating slit and chopper wheel, as in the first half of the experiment, but then it was reflected off a copper crystal and detected with a He$^3$ detector which sent a signal to the MCA. The crystal was angled $\theta_B$ from the line of incidence of the neutrons and the detector was placed an angle $2\theta_B$ in order to catch the diffracted neutrons. This experiment was run four times, for $\theta_B = 15^\circ, 20^\circ, 25^\circ, \text{and} 30^\circ$. 
FIG. 5: The time distribution for near and far positions of the BF$_3$ detector. The near position approximates a triangular distribution and the far position approximates the Maxwell-Boltzmann distribution.

FIG. 6: A plot of $\ln\left(\frac{N_i}{v_i^4}\right)$ vs. $v_i^2$. According to equation (9), the slope corresponds to $\frac{1}{v_0^2}$. 

$$ a = (-1.899 \pm 0.064) \times 10^{-7} \text{ s}^2\text{m}^{-2} $$

$$ \chi^2_{v-1} = 3.4675 $$

$$ P_{\chi^2} = < 0.01 $$
FIG. 7: The Bragg diffraction data provided to us by Scott Sewell for four different Bragg angles, $\theta_B$. 
FIG. 8: A plot of neutron velocity vs. $1/\lambda$ where $\lambda$ was the wavelength determined by Bragg diffraction. The slope of the line corresponds to $h/m$ based on the DeBroglie relation.