Relativistic Dynamics
Calibration of a Multi-Channel Analyzer using Barium-133

Erin R. Rhode

MIT Department of Physics
November 25, 2002
Calibration using Ba-133
Barium-133 Emissions

- Barium-133 decays by electron capture

\[ {}_{56}^{133}Ba \rightarrow {}_{55}^{133}Cs + \gamma \]  

(1)

- We see the following energies:
  
  - K X-ray from \( {}_{55}^{133}Cs \)
  
  - \( \gamma \) ray energies
    
    * Photoelectric peaks
    
    * Compton edges

- We measured the energy spectrum using a Silicon PIN diode
\( \gamma \) Ray Energies

\[
\begin{array}{c}
\begin{array}{c}
\text{Energies given in keV}
\end{array}
\end{array}
\]

\[
\begin{array}{c}
\text{Ba}^{133}_{56}
\end{array}
\]

1/2+:
- 356 keV: 86%
- 276 keV: 14%

3/2+:
- 384 keV: 2.8%
- 302 keV: 0.2%
- 223 keV: 81.0%

5/2+:
- 8.5%

5/2+:
- 75%

7/2+:
- 4.5%
- 1.8%
- 91%
K X-ray from $^{133}_{55}$Cs

- Barium-133 decays by electron capture

- An inner shell, usually the K shell, of the Cesium product is left empty

- When this shell is filled, an X-ray is emitted

- The energy of this X-ray for Cesium-133 is 30.97 keV (CSC Handbook, Vol 58)
Photoelectric Peaks

• A gamma ray passing through the detector is absorbed entirely by an electron, giving the electron kinetic energy.

• The amount of kinetic energy is measured in the Silicon diode, which converts the excited electrons into a pulse of charge.

• These are detected as photoelectric peaks in the energy spectrum.

• For a silicon PIN diode, photoelectric peaks are easily detected in the 20 to 100 keV range.
Compton Scattering

- Compton Scattering occurs when only a portion of the $\gamma$ ray energy is absorbed by an electron.

- The energy of a $\gamma$ ray is related to its wavelength by the equation:

$$E = \frac{hc}{\lambda}$$

where $h$ is Planck's constant, $6.626 \times 10^{-34} \text{Js}$, $c$ is the speed of light, $3 \times 10^8 \frac{\text{m}}{\text{s}}$, and $\lambda$ is the wavelength of the gamma ray.

- The amount of energy absorbed is related to the angle of the impact.
Compton Scattering

- Relativistic energy and momentum are conserved in the collision.
Determining the Compton effect

- Conservation of Energy (relativistic)
  \[ \frac{hc}{\lambda} + mc^2 = \frac{hc}{\lambda'} + \sqrt{p^2c^2 + m^2c^4} \]  \hspace{1cm} (3)

- Conservation of Momentum (relativistic)
  \[ \frac{hc}{\lambda} = \frac{hc}{\lambda'} \cos \theta + cp \cos \phi \]  \hspace{1cm} (4)

  \[ 0 = \frac{hc}{\lambda'} \sin \theta + cp \sin \phi \]  \hspace{1cm} (5)
Determining the Compton Effect

- Combine Momentum equations

\[ c^2 p^2 = \frac{h^2 c^2}{\lambda^2} - \frac{2h^2 c^2}{\lambda \lambda'} \cos \theta + \frac{h^2 c^2}{\lambda'^2} \]  \hspace{1cm} (6)

- Square Energy equation

\[ c^2 p^2 = \frac{h^2 c^2}{\lambda^2} + \frac{h^2 c^2}{\lambda'^2} - \frac{2h^2 c^2}{\lambda \lambda'} + 2hmc^2\left(\frac{hc}{\lambda} - \frac{hc}{\lambda'}\right) \]  \hspace{1cm} (7)

- Combine equations to remove \( p \) term

\[ \Delta \lambda = \lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta) \]  \hspace{1cm} (8)

where \( m \) is the rest mass of the electron, \( 9.11 \times 10^{-31} \text{kg} \), book value
The Compton Effect

- The greatest change in wavelength occurs when $\theta = 180^\circ$
  \[
  \Delta \lambda = \frac{h}{mc} (1 - \cos 180^\circ) = \frac{h}{mc} (1 - (-1)) = \frac{2h}{mc} \tag{9}
  \]

- Convert to Energy
  \[
  E' = \frac{E}{1 + \frac{2E}{mc^2}} \tag{10}
  \]

- For a given $\gamma$-ray energy, the Compton shift appears as a plateau, with the right edge corresponding to the greatest $\Delta E$.

- In a silicon diode, Compton scattering is easily detectable for values greater than 200 keV.
What We Expect to See

- Photoelectric peaks for the K X-ray and two of the $\gamma$ rays
  - $E = 30.97\,keV$ (K X-ray)
  - $E = 53\,keV$ ($\sim$ 3% occurrence → smaller peak)
  - $E = 81.0\,keV$ ($\sim$ 91% occurrence → larger peak)

- Compton edges for high occurring $\gamma$ ray energies
  - $E = 356\,keV; \Delta E = 207\,keV$ ($\sim$ 75% occurrence)
  - $E = 302\,keV; \Delta E = 163\,keV$ ($\sim$ 9% occurrence)
  - $E = 384\,keV; \Delta E = 230\,keV$ ($\sim$ 4% occurrence)
Our Calibration Curve
Our Calibration Curve
Conclusions

- Our calibration curve linearly lines up at all expected points

- A possible extra Compton edge was found
  - Unlikely to be a miscalibration since all our expected edges were seen
  - Too high to be from a Barium gamma ray
  - Because of the small number of counts, it’s possible that it’s noise
    * Other calibrations didn’t show this edge because of shorter time

- More occurrence of a γ ray energy is needed to show a clear Compton edge than is needed for a Photoelectric peak