Does Reinsurance Need Reinsurers?

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December 3, 2001

Abstract

Since Borch's seminal work, reinsurance has been mainly regarded as an instrument for optimal risk sharing among risk averse direct insurers. However, crucial features of the actual reinsurance market, such as the existence of specialized reinsurers, are not reproduced by risk sharing modelings. In this paper, we adapt an incentive model of financial intermediation to the insurance industry in order to capture such features more satisfactorily. More precisely, we consider reinsurers to be able to alleviate moral hazard between direct insurers and their shareholders, because they are credible monitors of monitoring efforts of direct insurers. By contracting reinsurance, direct insurers are able to raise enough external funds and thus to fulfill capital requirements. The comparative static properties of the equilibrium in our setup are in line with some recent stylized trends of the reinsurance market.

1 Introduction

1.1 Motivation

Since Borch's [1962] pioneering article, which is written in the reinsurance terminology, it is generally admitted that optimal risk sharing is the essential microeconomic rationale for the existence of a reinsurance market.

However, the actual Industrial Organization (IO) of the reinsurance market differs dramatically from the structure that optimal risk sharing among insurers would predict. In pure risk sharing modelings, reinsurance market is not "pyramidal". Insurance companies buy and sell primary underwritten
risks to each other and aim at pooling the aggregate risk at the equilibrium. The actual reinsurance market works in a quite different way. There are two types of "pure players", primary insurers and reinsurers. Primary insurers exclusively transfer underwritten risks to reinsurers and give them cash (the reinsurance premiums or ceded premiums). Conversely, reinsurers are exclusive buyers of primary risks.

This situation could be due to the fact that "reinsurers" are less risk averse than "insurers". This is not a convincing story. First, applying the risk aversion concept to large insurance or reinsurance firms suffers from a methodological criticism. Then, even if one would admit that firms "risk aversion" decreases with respect to their size and/or ability to diversify their portfolio, one could not explain why very large and worldwide diversified insurers buy significant reinsurance covers to equivalently large or even smaller reinsurers. Hence, the "pyramidal" feature of risk trading in reinsurance market is not satisfactorily captured by optimal risk sharing modelings.

1.2 Link with the literature

Other rationales for reinsurance have emerged since Borch. Most of them are listed in Mayers and Smith (1990) and Reissaus and Wambach (forthcoming). This subsection reviews some of them. We focus on tax management, costs of financial distress, capital requirements and on one aspect of informational asymmetry inside insurance corporations.

1.2.1 Tax management

Smith and Stulz (1985) state that when taxes are a convex function of income, it is optimal for any firm aiming at maximizing the net expected profit, either insurance company or not, to reduce income volatility, which insurers may achieve through reinsurance. In a more insurance specific setup, Garven and Loubergé (1996) prove that if the tax system is asymmetric (id est losses are not subject to taxes), a reinsurance market provides insurers with Pareto optimal both risk sharing and tax savings allocation.

1.2.2 Bankruptcy costs

Smith and Stulz (1985) argue that under the existence of exogenous bankruptcy costs, risk neutral and expected profit maximizing corporations have
incentives to hedge so as to reduce their probability of default. In the insurance industry, Doherty and Tinic (1981) endogenize in a sense bankruptcy costs. In their story, sophisticated enough insurance takers internalize insurers' probabilities of default in their insurance demand. Reinsurance (as well as appropriate equity ratios) are tools to reduce default risk and thus raise collected premiums.

Note that tax management and bankruptcy costs arguments are straightforward applications of corporate hedging theory to insurance industry. Although Mayers and Smith state that reinsurance is insurance bought by insurers, such a parallel between reinsurance and corporate hedging is to be taken cautiously. While industrial firms hedge risks which are mechanical consequences of their core business (fire, product liability...), insurers reinsurance risks that they have deliberately underwritten, and thus that they could avoid to bear (except maybe for pure commercial reasons). This is not to be compared to the fact that they also cover, say, their premises or computers against fire, or the death of a key executive like any corporation.

Note also that optimal tax management and reduction of default probability are rationales for reinsurance, not for reinsurers. Such goals can be reached through a secondhand risk market between insurers and do not require the empirical pyramidal organization of reinsurance markets.

1.2.3 Capital requirements

Both Mayers and Smith and Reissaus and Wambach stress that reinsurance is a specialized financing which might help direct insurers to meet regulatory capital requirements. This is one of the two crucial points of our setup which is detailed in subsection 1.3.

1.2.4 Informational concerns

A vast literature emphasizes that corporate hedging, very broadly speaking, may alleviate inefficiency due to informational asymmetry among parties of a corporate structure (see e.g. Breeden and Viswanathan 1990, DeMarzo and Duffie 1995, Sung 1997). Dewatripont and Tirole (1994) also stress that financial institutions face a particularly strong moral hazard between insiders and outsiders of the firm. Thus, moral hazard induced by the well-known inversion of the production cycle in insurance business is the second key feature, detailed in next subsection, of our setting.
1.3 The basic intuition

In this paper, we aim at providing a new rationale for reinsurance market pyramidal IO by considering reinsurers to be essentially the venture capitalists of the insurance industry. More precisely, the following stylized facts suggest that reinsurance is a specialized source of financing for direct insurers.

First, reinsurance treaties and equity have essentially the same purpose: reducing insurers probability of ruin. Either reinsurers or stockholders finance the states of the world where losses exceed premiums (of course within the limits of their commitments). Nevertheless, reinsurers do not give ex ante financing, like stockholders, but ex post, and conditioned by losses occurrence, financing to primary insurers. In other words, reinsurance covers are contingent capital\(^1\). Thus, reinsurers financing is a kind of "shadow equity", in the sense that it doesn't appear in insurers balance sheets.

The second key element which suggests to model reinsurance as a source of financing is the fact that in most countries, the calculation of the minimum legal capital requirement depends on insurers net (after reinsurance) liabilities\(^2\). Thus, reinsurance reduces minimum capital requirements. This means that reinsurance and equity are explicitly considered as substitute resources to finance legal capital requirements and allows to consider reinsurers as financiers.

It is well known that financial intermediaries and specialized financiers have no role to play when financial markets are complete and efficient, in the sense that they make zero profit and that the size of their balance sheets are neutral for the economy. An extensive literature has produced rationales for their existence. Most of them are summarized in Freixas-Rochet (1997), chapters 1 & 2. One of them, the asymmetry of information between lenders and borrowers, seems particularly relevant regarding reinsurers.

More precisely, we consider in this paper that the relationship between inside and outside insurance companies shareholders is characterized by moral hazard. The performance of insurance managers depends broadly indeed on their ability and efforts to monitor their risks portfolio. These ability

\(^{1}\)Note that this has not always been the case through history. The so-called contracts of bottomry, which was the prevailing form of reinsurance in Italy in XIV\(^{th}\) century, consisted in an ex ante financing whose repayment (with a risk premium) was conditioned by the absence of loss. Note also that most "Cat-bonds" or other reinsurance securitizations work this way.

\(^{2}\)see e.g. the American "Risk based capital" or the European "Solvency margin" computations.
and efforts are not verifiable by outside shareholders before a long time has elapsed after risks are underwritten, which causes moral hazard. This is due to the well known inversion of the production cycle in the insurance industry. Production costs, \textit{id est} losses, are unveiled several years after premiums are paid in many insurance branches, either because claims are so rare and volatile that the law of large numbers works in the long run only (see e.g. natural events or industrial damages), or because long assessments and trials may occur between claims and settlements (see e.g. risks linked with medical liability or asbestos).

Reinsurers are yet capable of checking insurers ability and monitoring their monitoring efforts for three reasons:

1. They have more auditing capacities than outside shareholders.
2. They have the same technical skills as insurers for claims management.
3. They are often involved in longer term relationships with direct insurers than outside shareholders.

Thus, reinsurers are able to alleviate moral hazard.

We study a model where insurers may get either financing from outside investors or specialized financing from reinsurers. Our setup is very close to Holmstrom-Tirole (1997) (HT). The purpose of their paper is to build an incentive model of financial intermediation in which firms and banks are capital constrained in order to reproduce some stylized facts of financial markets in the last ten years. We adapt this model to the reinsurance market: Firms are insurers and reinsurers play the bankers role.

The main innovation of our modeling consists in the fact that an insurer has an initial choice between insurance and reinsurance business. The more reinsurers, the less insurers and thus the smaller insurance market capacity.

In our story, all agents are risk neutral and protected by limited liability:

- \textbf{Risk neutrality.} We aim to explain some empirical features of the reinsurance market which are not captured by a classical, à la Borch, risk sharing modeling. This is the reason why risk neutrality assumption is made. Such an assumption enables to isolate the pure role played by moral hazard in the existence and in the organization of such a market.
• **Limited liability.** Since capital and reinsurance are tools to improve insurers solvency, limited liability assumption is necessary. Solvency is indeed fully relevant in a limited liability world.

Note that risk neutrality is only assumed for insurers, reinsurers and investors but not needed for policyholders.

The next section describes the basic model. Sections 3 and 4 characterize possible equilibria. In section 5 a static analysis of the equilibrium is carried out. Section 6 concludes with some caveats and remarks, in particular about recent attempts of reinsurance securitization such as "catastrophe bonds".

## 2 The Model

Our model is very closely inspired from HT.

There are two dates, \( t = 0 \) and \( t = 1 \). At \( t = 0 \), insurance policies and reinsurance treaties are underwritten, investment decisions are made. At \( t = 1 \), insurance losses occur, reinsurance and financial contracts are settled.

There are two types of agents, insurers (more precisely, inside investors of insurance companies) and outside investors. As mentioned in introduction, all parties are risk neutral and protected by limited liability.

### 2.1 Outside investors

There are an infinity of outside investors. Each of them is endowed with enough cash, so that insurers may potentially raise as many funds as needed on capital markets. However, investments are not restricted to insurance securities. Investors have unlimited access to an alternative investment opportunity providing an exogenous expected rate of return \( \gamma \), which can be viewed as the expected yield on financial markets.

### 2.2 Insurers

There are \( N \) insurers. Each of them starts out with an initial amount of capital \( K \). In our setup, insurers can be viewed as a close-knit team of inside investors of an insurance company who are able to enforce underwriting and portfolio monitoring policies.
Insurers have to make a choice between two business lines, primary (or
direct) insurance, or reinsurance. They also may invest in the alternative
investment opportunity.

2.3 Primary insurance

If an insurer chooses primary insurance, the portfolio she underwrites has
the following features:

• The exogenous legal capital requirement is $I$. In other words, $(I - K)^+$
is the minimum equity that she has to raise in order to be allowed to
operate. This equity ensures that her probability of ruin is low enough.

• Moral hazard is formalized by assuming that insurers may deliberately
not monitor their portfolio in absence of proper incentives to do so,
and that their monitoring effort is not observable by outside investors.
Monitoring consists basically in:

1. Checking that brokers and agents respect the insurer's underwriting
rules, so that risks are in adequation with tariffs.

2. Checking that the estimated cost of outstanding and incurred but
not reported (IBNR) claims is appropriate.

More precisely, each portfolio's technical income (premiums minus losses
and administration costs, say) is a real random variable at $t = 0$ de-
noted by $\bar{x}_H$ if portfolio is monitored and $\bar{x}_L$ if not. $\bar{x}_H$ and $\bar{x}_L$ respec-
tive cumulative functions are denoted $F_H(\cdot)$ and $F_L(\cdot)$.

Monitoring a portfolio entails a non verifiable monitoring cost $C$ paid
by the non reinsured insurer. This monitoring cost is denoted $c$ for
reinsured portfolios.

Throughout the paper, it is assumed that primary insurers have the bar-
gaining power. They make take-it-or-leave-it offers to their outside share-
holders and reinsurers.
2.4 Reinsurance

If an insurer chooses to reinsure primary insurers, she finances them with her initial amount of capital. The reinsurer is able to monitor primary insurers monitoring effort, because she has more auditing abilities and more technical skills than ”sleeping” investors. Nevertheless, reinsurers monitoring effort is not observable by investors and entails a non verifiable cost.

The cost for monitoring a given insurer is denoted $c_R$. It is assumed that this cost is shared between the reinsurers of this given insurer proportionally to their stakes in the insurance company. This assumption will be discussed later. It allows tractable computations and seems in accordance with the practices of the reinsurance market.

The global monitoring cost of a reinsured portfolio is thus $c + c_R$. Note that no particular relationship between $C$ and $c + c_R$ is assumed.

Additional assumptions about terminal income distributions and the shape of contracts are required.

2.5 Income distribution

For any level of equity $J$, let us denote

- $R_H (J) = \int_{-J}^{+\infty} (J + x) dF_H (x)$ the portfolio expected income if the insurer ”behaves”.
- $R_L (J) = \int_{-J}^{+\infty} (J + x) dF_L (x)$ the portfolio expected income if she ”shirks”.
- $\Delta R (J) = R_H (J) - R_L (J)$

The following assumptions are required:

Assumption 1 For any level of equity $J$, non monitored portfolios yield less than the alternative investment and than monitored portfolios. At the minimum level $I$, monitored portfolios yield more than the alternative investment. Formally:

\[
\begin{align*}
\forall J \in [I; +\infty[, (1 + \gamma) J & > R_L (J) \\
\forall J \in [I; +\infty[, R_H (J) - C & > R_L (J) \\
R_H (I) - C & > (1 + \gamma) I
\end{align*}
\]

This prevails also for reinsured portfolios where $C$ becomes $c + c_R$. 

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Assumption 2 $\forall J \geq I, \frac{R_H(J)}{1 - P_H(-J)} \geq \frac{R_L(J)}{1 - P_L(-J)}$.

In words, the conditional to non ruin expected pay-off is higher for monitored portfolios.

As shown later, this assumption ensures that exogenous capital requirement constraint is always binding.

2.6 Contracts design

In HT setup\(^3\), industrial firms incomes are binomial, either nil in case of failure or constant in case of success. Unfortunately, such a modeling would be irrelevant regarding financial institutions. Because of our more general model, we have to face a non trivial contract design issue.

In this paper, we restrict our study to exogenously specified contracts shapes commonly used in practice, stocks and proportional reinsurance:

Assumption 3 Financial contracts are stocks.

In other words, each shareholder’s pay-off is a quota-share of company’s income, proportional to her stake.

Assumption 4 Reinsurance treaties are proportional reinsurance. In other words, reinsurers buy a quota-share of insurance companies gross income.

In the real world, most reinsurance treaties are either proportional or excess of loss treaties\(^4\).

Summarizing, we assume that insurers have access to two alternative tools in order to meet capital requirements or "finance their solvency", capital and reinsurance. This latter is in fact reinsurers capital in our framework. Reinsurers are better informed than investors, and their capital is more expensive precisely because they have to make a monitoring effort whereas uninformed investors do not. The conditions under which insurers don’t need reinsurance are derived in section 3. When such conditions are not satisfied, the equilibrium in the reinsurance market is studied in section 4. The sensitivity of capacity and cession rate to the exogenous variables is studied in section 5.

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\(^3\) as in many papers addressing corporate finance for non financial firms

\(^4\) Excess of loss treaties are equivalent to a direct insurance contract with a fixed deductible per claim.
3 Insurance without reinsurance

At \( t = 0 \), insurers have three options:

1. investing in the outside opportunity
2. undertaking insurance business
3. undertaking reinsurance business

Of course, insurers seek to maximize their expected terminal wealth.

If \( K \geq I \), meeting capital requirements requires no external financing. Thus, all the insurers undertake insurance business and monitor their portfolios by virtue of Assumption 1. Their excess funds \( K - I \) may be invested in the alternative technology.

If \( K < I \), insurers who ought to undertake insurance business need to get an external financing at least equal to \( I - K \).

Both investors and insurers who prefer reinsuring than underwriting primary risks may provide such an external funding. Intuitively, it is obvious that reinsurers capital is more expensive than "uninformed capital", because of the monitoring cost \( c_r \). Insurers expected share maximization is equivalent to minimization of their financing costs\(^5\). Thus, they prefer to restrict their external financing to "uninformed capital".

In this section, we derive the conditions under which such a pure uninformed external financing is possible.

Let us first define:

- \( J \) the insurers total equity
- \( \frac{D}{J} \) the insurers stake in terminal income. In other words, outside investors pay \( J - K \) at date 0 for a \( \frac{J-D}{J} \) stake at date 1.

The insurers who undertake insurance business face the following program:

\(^5\) or equivalently minimizing dilution.
\[
\max_{D,J} \frac{D}{J} R_H(J) - C \\
\text{s.t.} \\
\begin{align*}
J &\geq I \\
\frac{D}{J} R_H(J) - C &\geq \frac{D}{J} R_L(J) \\
\frac{D}{J} R_H(J) - C &\geq (1 + \gamma) K \\
\frac{1 - D}{J} R_H(J) &\geq (1 + \gamma)(J - K)
\end{align*}
\] (CR) is the legal capital requirement constraint.

(II) is the insurer incentive constraint. Her share must be high enough, so that she has incentive to monitor.

(IP) is the insurer participation constraint. Her insurance investment must yield more than \( \gamma \).

(OP) is the outside investors participation constraint. Their investment must yield at least \( \gamma \).

Of course rational insurers won’t pay actually more than \( \gamma \) so as to minimize dilution. Thus, (OP) is binding, which yields:

\[
\frac{d}{dJ} \left( \frac{D}{J} R_H(J) - C \right) = -F_H(J) - \gamma < 0
\]

This entails that direct insurers raise as little equity as possible and that (CR) is binding: \( J = I \).

Note also that, by virtue of Assumption 1, (IP) is satisfied as soon as (OP) is binding.

It turns out from (OP) and (II) that a necessary condition for uninformed financing being sufficient is

\[
K \geq \bar{K} = I - \frac{R_H(I)}{1 + \gamma} \left( 1 - \frac{C}{\Delta R(I)} \right)
\]

"One lends only to the rich." Insurers face a classical credit constraint, even if they are "equity constrained" in our setting. If \( K < \bar{K} \), the existence of moral hazard prevents insurers from raising enough funds from investors, although this resource is potentially unlimited. In this case, some of them may supply another external source of funds to the insurance market by becoming reinsurers. Such a setup is a loss of capacity of the insurance market in the sense that fewer primary portfolios are underwritten. It is studied in next section.
4 Equilibrium in reinsurance market

From now on, insurers who undertake insurance business are termed "direct insurers" whereas those who undertake reinsurance business are "reinsurers".

In this section, we assume that $K < \bar{K}$. Thus both uninformed (investors) and informed (reinsurers) capital coexist. This situation is more realistic than the previous one. It is very rare indeed that an insurance company, even well-off, purchases no reinsurance. Direct insurers new program is described in subsection 4.1. Equilibrium in the reinsurance market is derived in subsection 4.2.

4.1 Direct insurers program

Let us define:

- $J_R$ the amount of reinsurers capital a direct insurer raises
- $D_J$ the reinsurers share in the payoff of a direct insurance company
- $(1 + \beta) = \frac{D_R \times R_H(J)}{J \times J_R}$ the unitary cost of reinsurance capital, considered as exogenous by direct insurers
- $n$ the number of insurers who decide to undertake reinsurance business
- $\alpha = \frac{n}{N-n}$ the "cession rate" of the insurance market, which measures reinsurance business as a proportion of direct insurance business.

Direct insurers face the following program:
\[
\max_{J, J_R, D, D_R} \quad \frac{D}{J} R_H (J) - c
\]
\[
s.t.
\begin{align*}
J & \geq I \quad \text{(CR)} \\
\frac{D}{J} \Delta R (J) & \geq c \quad \text{(IP')} \\
\frac{D}{J} R_H (J) - c & \geq (1 + \gamma) K - \frac{c_R}{\alpha} \quad \text{(IP1)} \\
\frac{D}{J} \Delta R (J) & \geq c_R \quad \text{(RI)} \\
\frac{J - D - D_R}{J} R_H (J) & \geq (1 + \gamma) (J - J_R - K) \quad \text{(OP')} 
\end{align*}
\]

where

(CR) is the legal capital requirement constraint.

(II’) is the direct insurer incentive constraint, with the “reinsured” monitoring cost c.

(IP1) is the first direct insurer participation constraint: direct insurance business expected yield is at least equal to \( \gamma \).

(IP2) is the second direct insurer participation constraint: direct insurance business expected yield is at least equal to reinsurance expected yield.

(RI) is the reinsurers incentive constraint. Their shares must be high enough, so that they are credible monitors.

(OP’) is the outside investors participation constraint, taking into account reinsurers stake.

The following key element allows to characterize the direct insurer financing decision more precisely. Since monitoring is costly, \( \beta \) must exceed \( \gamma \). Consequently, reinsured direct insurers will demand precisely the minimum level of reinsurance which is consistent with proper incentives for reinsurers to monitor. Thus, the level of reinsurance \( J_R \) has an upper limit:

\[
J_R \leq \frac{R_H (J) c_R}{(1 + \beta) \Delta R (J)}
\]

This limit is reached when direct insurers raise both capital and reinsurance. However, if direct insurers equity is high enough, reinsurers financing is sufficient and no uninformed capital is necessary. This situation prevails if

\[
J_R = J - K
\]

In next subsection the equilibrium in reinsurance market is derived in the particular case where \( J_R < J - K \).
4.2 Equilibrium in reinsurance market

Let us study the case $J_R < J - K$, where both outside capital and reinsurance coexist, which is far more realistic.

The reinsurance market clearing equation is:

$$J_R = \alpha K \quad \text{(ER)}$$

A crucial remark is that (IP2) is necessarily binding at the equilibrium in the reinsurance market: Risk neutral insurers have to be indifferent between direct insurance and reinsurance activities.

We get

$$\frac{D_R}{J} = \frac{c_R}{\Delta R(J)} \quad \text{from (RI)}$$

$$\frac{D}{J} R_R(J) = (1 + \beta) K + c - \frac{c_R}{\alpha} = \frac{c_R R_L(J)}{\alpha \Delta R(J)} + c \quad \text{from (IP2)}$$

$J_R < J - K$ can be written $\alpha < \frac{J}{K} - 1$

The inequality in (IP1) becomes

$$\alpha \leq \frac{c_R R_L(J)}{(1 + \gamma) K \Delta R(J)}$$

$(II')$ becomes

$$\alpha \leq \frac{c_R}{c}$$

Thanks to $(OP')$, we finally get $\alpha$ as a solution of

$$\boxed{\alpha^2 + \rho_f \alpha - \rho_R = 0} \quad \text{(P(\alpha))}$$

With the necessary conditions

$$\begin{cases} \alpha < \frac{J}{K} - 1 \\ \alpha \leq \frac{c_R}{c} \\ \alpha \in [0; 1] \end{cases}$$

where
\[ \rho_I = \frac{R_H(J) - c - \frac{c_R R_H(J)}{\Delta R(J)} - (1 + \gamma)(J - K)}{(1 + \gamma)K} = \frac{\frac{\partial}{\partial J} R_H(J) - c - (1 + \gamma)J_R}{(1 + \gamma)K} \]

is the expected surplus of a monitored insurance portfolio divided by the yield of the outside opportunity.

\[ \rho_R = \frac{c_R \frac{R_H(J)}{\Delta R(J)}}{(1 + \gamma)K} = \frac{R_H(J)_{ER} - c_R}{(1 + \gamma)K} \]

is the reinsurers expected share on a monitored portfolio net of monitoring costs paid by them divided by the yield of the outside opportunity.

\( \rho_I \) and \( \rho_R \) may be interpreted as measures of the unitary excess yield created by insurance and reinsurance business respectively.

Note that our trinomial admits one unique positive root

\[ \alpha = \frac{\rho_I}{2} \times \left( \sqrt{1 + \frac{4\rho_R}{\rho_I^2}} - 1 \right) \]

Thanks to Assumption 2, we get the same result as in the non reinsured world:

**Proposition 1** Under Assumption 2, the capital requirement constraint is binding, so that \( J = I \).

Proof. See the Appendix.

Remark. Note however that without Assumption 2, some peculiar distributions of \( \widetilde{x}_H \) and \( \widetilde{x}_L \) might entail that direct insurers have incentive to raise as much equity as possible in order to maximize their terminal wealth, which makes capital adequacy rules irrelevant.

Remark. Note also that Assumption 2 entails that \( \alpha \) increases with respect to \( I^6 \), which is an interesting result. The choice of the capital requirement \( I \) by the regulator may indeed be interpreted as a trade-off between capacity and solvency. Increasing \( I \) improves direct insurers solvency in the sense that their probabilities of ruin decrease but has a negative impact on

\footnote{see the expression of \( \frac{\partial \rho}{\partial J} \) in the proof of Proposition 1.}
market capacity since more reinsurance is bought in order to fulfill capital requirements. Thus, less primary risks are underwritten.

Since $J = I$, we get that direct insurers initial capital endowments must satisfy:

$$K \geq \overline{K} = I - \frac{R_H(I)}{1 + \gamma} \left(1 - \frac{1}{\Delta R(I)} \left(c + c_R \frac{\beta - \gamma}{1 + \beta}\right)\right)$$

Note that

$$\overline{K} < \overline{K} \iff c + c_R \left(\frac{\beta - \gamma}{1 + \beta}\right) < C$$

$c + c_R < C$ is therefore sufficient for reinsurance reducing the direct insurers need for inside capital $K$. This condition means that the joint monitoring of a direct insurer and her reinsurers is more efficient than a single insurer monitoring. However natural this condition seems, it is not a necessary condition in our setup. Reinsurers monitoring effort may bring direct insurers need for inside capital down even if $c + c_R \geq C$.

In the next section, the comparative statics properties of $\alpha$ with respect to the exogenous variables of our modeling are studied.

5 Sensitivity of direct insurers capacity and cession rate to the exogenous variables

In this section, it is assumed that capital requirements, insurers capital endowments and monitoring costs are such that insurers use both capital and reinsurance and that our program admits one unique positive solution $\alpha$. Note that $\alpha$ may be interpreted as the insurance market cession rate and $1 - \alpha$ as its capacity in our framework. Thus, studying how $\alpha$ depends on the exogenous variables, we test if our modeling is in accordance with evidence from the reinsurance market.

According to most reinsurance market observers and practitioners (see e.g. Sigma n° 9/98 and The Worldwide Reinsurance Review November 1999), non life business has been experiencing three major trends during the last years:

- a soft reinsurance market, with falling tariffs and significant underwriting losses for American reinsurance market;
• better underwriting results for direct insurers;
• fall of insurers cession rates.

A fourth feature over the period are the bullish American and European stockmarkets, which is interpreted as an increasing $\gamma$ over the period. In subsections 6.1 and 6.2, we test the accordance of our modeling with the two following stylized facts which compile these empirical features:

1. $\alpha$ increases with respect to reinsurance excess yield, decreases with respect to insurance excess yield.

2. $\alpha$ decreases with respect to financial yields.

Subsection 6.3 deals with a third stylized fact:

3. $\alpha$ increases with respect to informational costs.

5.1 Sensitivity to insurance and reinsurance profitability

Since $\alpha = \frac{\rho_f}{\rho} \times \left( \sqrt{1 + \frac{4\rho_R}{\rho_f^2}} - 1 \right)$, it is obvious that $\alpha (\rho_f, \rho_R)$ decreases with respect to $\rho_f$ and increases with respect to $\rho_R$. Thus, insurance market capacity is all the more important and cession rates all the less important because insurance profitability is high and reinsurance profitability low.

This quite intuitive result is mentioned because of its consistency with our first stylized fact and with the recent strategical choices of the two biggest reinsurers worldwide, Münchener Rück and Swiss Re. They both have publicly announced their choice to restrict their business in traditional non life reinsurance and even to develop direct business through subsidiaries, in a context where $\rho_f$ is increasing and $\rho_R$ decreasing.

5.2 Sensitivity to financial yields

Proposition 2 $\alpha$ increases with respect to $\gamma$.

Proof. See the Appendix.

This result seems in deep contradiction with our second stylized fact.
This contradiction is quite easy to explain. In the real world, insurers are institutional investors since premiums are invested until claims are settled. Their incomes are therefore significantly improved by increases in financial yields. In our setup, insurers incomes are pure technical incomes. Increases in financial yields have no impact on incomes, but make capital more expensive.

This contradiction sheds light on a weakness of our modeling. By ignoring that asset management contributes very significantly to insurers incomes, we are unable to capture the empirical positive impact of financial yields on insurance market capacity.

5.3 Sensitivity to informational costs

$\rho_I$ is a decreasing function of $c$ and $c_R$. $\rho_R$ is an increasing function of $c_R$. Thus, $\alpha$ is an increasing function of $c$ and $c_R$. In other words, $\alpha$ increases with respect to informational costs, our third stylized fact.

It means first that the global monitoring cost of a given risk has a negative impact on the market capacity for this risk, which is not surprising.

It also means that the monitoring cost of a risk has a positive impact on the cession rate of this risk. This is consistent with the fact that reinsurers are particularly involved in risks such as professional or corporate liability or natural events where insurers profitability and also insurers monitoring effort are revealed to investors in the very long term only. As mentioned earlier, this is due either to the volatility or the long tail development of these business lines. For such risks, informed financing by reinsurers who have long term experience is crucial for direct insurers credibility on financial markets.

Cession rates of different business lines in France in 1998 are consistent with this result:

<table>
<thead>
<tr>
<th>Business line</th>
<th>Cession rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car insurance</td>
<td>7%</td>
</tr>
<tr>
<td>Property insurance</td>
<td>10%</td>
</tr>
<tr>
<td>Liability</td>
<td>19%</td>
</tr>
<tr>
<td>Marine &amp; Aviation</td>
<td>46%</td>
</tr>
<tr>
<td>Catastrophic natural events</td>
<td>46%</td>
</tr>
</tbody>
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6 Concluding remarks

Considering reinsurers to be "insurance venture capitalists", we managed to reproduce some stylized features of the reinsurance market which are out of range of pure risk sharing rationales for this business.

A first caveat about our modeling concerns the monitoring cost sharing among reinsurers for a given insurer. The proportional rule seems close to the real world practice: When several reinsurers have stakes in a treaty, their benefits (or losses) are proportional to their stakes. This assumption is also an important source of tractability. Nevertheless, more sophisticated costs sharing rules (e.g. the introduction of fixed costs) might reduce insurance market capacity at the equilibrium by reducing reinsurance business profitability and/or increasing informational costs.

A more classical caveat concerns the role of inside capital. The discussion is basically the same as in the concluding remarks of Holmström and Tirole (1997).

The most important caveat about this paper is the restriction to exogenously specified financial and reinsurance contracts. As shown by Innes (1990), endogenous optimal contracts in our setup are "live or die", and become debt contracts by adding a monotonicity assumption. Our constrained framework leads however to a tractable expression of the cession rate and highlights the relationship between reinsurance cessions and agency problems within insurance corporations.

A positive aspect of our modeling is the irrelevance of the degree of correlation between primary portfolios. Conversely, HT need perfect correlation to the extent that they set up a continuum of firms: Were firms projects not perfectly correlated, diversification would reveal monitoring efforts thanks to the strong law of large numbers. The irrelevancy of portfolio correlations relies in our setup both on risk neutrality and on the equity nature of contracts. Reinsurers would benefit from diversification with debt-like contracts where the portfolios cross pledge each other and thus reduce informational cost like in Diamond (1984).

Finally, viewing reinsurers as specialized financiers of insurance industry might provide interesting tools for analyzing recent "disintermediation" attempts in this sector. A few years ago, as the first "catastrophe bonds\(^7\)"

\(^7\) or non proportional in the reinsurance terminology

\(^8\) Cash-flows of such bonds are conditioned by insurers claims, which assimilates them to securitized reinsurance treaties.
were issued, many observers noticed that such assets were promising, since they simultaneously provided a diversification in non financial risks to underwriters and brought a new capacity to the reinsurance market. Nevertheless, reinsurance securitization seems quite disappointing, in terms of volumes issued, today. This may be explained by the fact that reinsurers investments in the insurance industry are necessary, so that they are credible monitors to reduce moral hazard: because of asymmetric information, reinsurance cannot do without reinsurers. Another fact is consistent with this analysis. In the rare successful issues of such bonds, the definition of the underlying risk was not linked with a particular portfolio but with "pure" variables such as the cost of a hurricane or a hail storm in a given area. This is a way to reduce moral hazard. The drawback is that covers provided by such bonds are not well correlated with insurers portfolios (basis risk).

Our analysis provides another way to achieve such reinsurance securitizations: If a pool of top ranking reinsurers took a significant stake in the issue and announced it publicly, investors could consider that securitized risks are credibly monitored.

References


[14] Sigma n° 9/1998 ”La réassurance mondiale connaît une vague de concentrations”.


7 Appendix

7.1 Proof of proposition 1

I'm done if I prove that direct insurers shares at the equilibrium decrease with respect to $J$.

By definition of $\rho_I$:

$$(1 + \gamma) K \times \rho_I = \frac{D}{J} R_H(J) - c - (1 + \gamma) J_R$$

Or

$$\frac{D}{J} R_H(J) - c = (1 + \gamma) K \times (\rho_I + \alpha) \tag{7.1}$$

And

$$\frac{\partial \alpha}{\partial J} = \frac{1}{2\alpha + \rho_I} \times \left[ \frac{d \rho_R}{d J} \times (1 + \alpha) + \frac{\alpha}{(1 + \gamma) K} \times (\gamma + F_H(-J)) \right]$$

Thus

$$\frac{d}{d J} \left( \frac{D}{J} R_H(J) - b \right) = (1 + \gamma) K \left( \frac{1}{2\alpha + \rho_I} \left( \frac{d \rho_R}{d J} \times (1 + \alpha) + \frac{\alpha}{(1 + \gamma) K} \times (\gamma + F_H(-J) + \gamma) \right) \right)$$

$$= -(\gamma + F_H(-J)) \left( \frac{\alpha + \rho_I}{2\alpha + \rho_I} \right) + (1 + \gamma) K \frac{d \rho_R}{d J} \left( \frac{1 - \alpha - \rho_I}{2\alpha + \rho_I} \right) \tag{7.2}$$

Therefore

$$\frac{d \rho_R}{d J} = c \rho_R R_H(-J) R_L(-J) \left( \frac{1 - F_L(-J)}{R_L(-J)} - \frac{1 - F_H(-J)}{R_H(-J)} \right) \geq 0 \text{ from Assumption 2} \tag{7.3}$$

and $1 - \alpha - \rho_I = 1 - \frac{\beta R}{\alpha} \leq 0$ because $\alpha \leq \rho_R$. \hfill $\blacksquare$

7.2 Proof of proposition 2

Since $\rho_I$ and $\rho_R$ are both decreasing with respect to $\gamma$, the impact of $\gamma$ on $\alpha$ needs further computations.

Note that

$$\frac{\partial \alpha}{\partial \gamma} = \frac{\partial \alpha}{\partial \rho_I} \times \frac{\partial \rho_I}{\partial \gamma} + \frac{\partial \alpha}{\partial \rho_R} \times \frac{\partial \rho_R}{\partial \gamma}$$

22
And

\[
\begin{align*}
\frac{\partial \alpha}{\partial \rho_I} &= -\frac{\alpha}{2\alpha + \rho_I} \\
\frac{\partial \alpha}{\partial \rho_R} &= \frac{1}{2\alpha + \rho_I} \\
\frac{\partial \rho_I}{\partial \gamma} &= \frac{-(1-K)}{(1+\gamma)K} - \frac{\rho_I}{(1+\gamma)} \\
\frac{\partial \rho_R}{\partial \gamma} &= -\frac{\rho_R}{(1+\gamma)}
\end{align*}
\]

Hence,

\[
\frac{\partial \alpha}{\partial \gamma} = \frac{1}{2\alpha + \rho_I} \times (1+\gamma) \left[ \alpha \left( \rho_I + \frac{I}{K} - 1 \right) - \rho_R \right]
\]

Thus, the sign of \( \frac{\partial \alpha}{\partial \gamma} \) depends directly on the comparison between \( \alpha \) and \( \frac{\rho_R}{\rho_I + \frac{I}{K} - 1} \). To compare them, remember that \( \alpha \) is the positive root of \( P(x) = x^2 + \rho_I \times x - \rho_R \). It entails that

\[
\forall x > 0, x < \alpha \iff P(x) < 0
\]

\[
\iff P\left( \frac{-\rho_R}{x} \right) > 0
\]

\[
\iff P(-\rho_I - x) < 0
\]

Thus

\[
P\left( \frac{-\rho_R}{\rho_I + \frac{I}{K} - 1} \right) < 0 \iff P\left( -\rho_I - \frac{I}{K} + 1 \right) > 0
\]

\[
P\left( -\rho_I - \frac{I}{K} + 1 \right) > 0 \iff P\left( \frac{I}{K} - 1 \right) > 0
\]

This last inequality holds since \( \alpha < \frac{I}{K} - 1 \).

Hence

\[
\frac{\rho_R}{\rho_I + \frac{I}{K} - 1} < \alpha
\]

and \( \frac{\partial \alpha}{\partial \gamma} > 0 \), the required result.■