Flight to Quality, Flight to Liquidity, and the Pricing of Risk

Dimitri Vayanos

MIT and NBER

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INTRODUCTION

- Liquidity premia seem to vary substantially over time.

- Clean experiments:
  - Off- vs. on-the run bonds. (Krishnamurthy (2002))
  - Refcorp vs. government bonds. (Longstaff (2002))
  - Municipal vs. government bonds. (Chalmers, Kadlec, and Mozumdar (2002))

- Effects can be quite strong.
  - Variation in spread equals 4-6 times average spread.
  - Variation in relative prices can be up to 15%.
  - Flight to liquidity.

- Effects could be stronger for other asset classes.
  - Corporate bonds.
  - Emerging market bonds.
  - Stocks.
Why Do Liquidity Premia Vary?

• Liquidity premia seem to be high when
  – Interest-rate volatility is high. (Kamara (1994))
  – Consumer confidence is low. (Longstaff (2002))
  – Money flows away from equity funds, into money market funds. (Longstaff (2002))
  – Stock market goes down. (Chalmers, Kadlec, and Mozumdar (2002))

• Variation in liquidity premia seems to be correlated across markets.
  – On-the-run premium is correlated with commercial-paper spread. (Krishnamurthy (2002))
Questions

• Questions:
  – What is economic mechanism driving variation in liquidity premia?
  – How is risk associated to changes in liquidity premia priced?

• Answers could be important for understanding
  – An important component of asset price volatility.
  – An important component of pricing of asset risk. (Especially for illiquid assets.)
This Paper

• Proposes a theory of time-varying liquidity premia.
• Explores its asset-pricing implications.
• Dynamic multi-asset equilibrium model.
Main Assumptions

Transaction costs.

- Exogenous, constant over time, different across assets.

- Illiquid assets \equiv \text{High TC}.

- Time-variation in liquidity premia will not caused by TC.
  
  - Seems realistic: Variation in bid-ask spreads is often small relative to that in liquidity premia.
    
    * Off-the-run bonds.
    * Refcorp bonds. (Longstaff (2002))
Main Assumptions (cont’d)

Stochastic volatility.

- Asset payoffs have stochastic volatility.
- Volatility will drive liquidity premia.
- Seems realistic.
  - Empirical evidence.
  - Anecdotal evidence: Traders value liquidity more at times of “uncertainty.”

Delegated money management.

- Portfolio decisions are made by fund managers.
- When return falls below a threshold, fund is liquidated.
- High volatility ⇒ High liquidation probability ⇒ Short investment horizons ⇒ High liquidity premia.
Results / Empirical Implications

• Liquidity premia (≡ price differentials between assets which are identical except for TC) increase with volatility.
  – Flight to liquidity.

• Market’s effective risk aversion (≡ expected return per unit of variance) increases with volatility.
  – Flight to quality.
  – Risk aversion varies not because of stochastic utility functions, but because of concern of liquidation.

• Betas depend on volatility.
  – Betas of illiquid assets increase with volatility.

• Correlations depend on volatility.
  – Correlations between similar assets increase with volatility.

• Unconditional two-factor CAPM (factors = market, volatility) does not price assets correctly.
  – Understates risk of illiquid assets.

• Implications for price of volatility factor.
MODEL

• Infinite horizon, discrete time. Time between periods is $h$.

Assets.

• Riskless asset, return $r h$. $r$ is exogenous.

• $N$ risky assets.

• Volatility $v_t$ evolves according to

$$v_{t+h} = v_t + \gamma (\overline{v} - v_t) h + \sigma \sqrt{v_t h} \eta_{t+h}.$$ 

• Asset $n$ pays dividend $\delta_{n,t} h$. $\delta_{n,t}$ evolves according to

$$\delta_{n,t+h} = \delta_{n,t} + \kappa (\overline{\delta} - \delta_{n,t}) h + \sqrt{v_t h} \left( \phi_n \zeta_{t+h} + \psi_n \eta_{t+h} + \xi_{n,t+h} \right).$$

  - $\zeta_{t+h}$: systematic shock, independent of $\eta_{t+h}$.
  - $\xi_{n,t+h}$: idiosyncratic shock.

• Supply $S_n$.

• Transaction costs per share $\epsilon_n$.

• Price (average of bid and ask) $p_{n,t}$. 
Fund Management and Liquidation

- Each investor “manages” a fund of size $W_t$.

- Performance-based liquidation:
  - In each period, a fund can be “monitored” by its owners with probability $\mu h$.
  - Fund is liquidated if $W_{t+h} - W_t < -L\sqrt{h}$, for $L > 0$.

- Random liquidation:
  - In each period, a fund can be liquidated with probability $\lambda h$, regardless of performance.

- At liquidation:
  - Investment in each asset is sold in the market.
  - Manager can find employment in a new fund, whose size is equal to old fund’s liquidation value.
Fund Managers

- Infinitely lived, continuum, mass one.
- Manager decides how much to allocate in each asset.
- Exogenous management fee \((aW_t + b)h\).
- Exogenous withdrawal by fund’s owners \((\hat{a}W_t + \hat{b})h\).
- Assumptions: \(a > 0, a + \hat{a} = r/(1 + rh)\).
- Manager maximizes
  \[-E \sum_{k=0}^{\infty} \exp(-\alpha c_{t+kh} - \beta kh)\].
- Consumption is derived from fee.
- Manager can save/borrow in riskless asset.
EQUILIBRIUM

Basic Properties

• Managers buy and hold the market portfolio.
• Price of asset $n$ is
  \[ p_{n,t} = q_n(v_t) + \frac{1 - \kappa h}{r + \kappa} (\delta_{n,t} - \bar{\delta}). \]
• Value function of a manager is
  \[ -\exp[-A[W_t + zw_t + Z(v_t)]] , \]
  where $A \equiv \alpha a$, $z \equiv r/[(1 + rh)a]$, and $w_t$ are manager’s private savings.
• When $h$ goes to zero, functions \( \{q_n(v)\}_{n=1,...,N} \) and $Z(v)$ can be characterized by a system of $N + 1$ ODEs.
  - ODEs are derived from Bellman equation.
  - They are second order.
Preliminaries

- Notation:
  \[ \phi_M \equiv \sum_{n=1}^{N} S_n \phi_n, \]
  \[ \psi_M \equiv \sum_{n=1}^{N} S_n \psi_n, \]
  \[ q_M(v) \equiv \sum_{n=1}^{N} S_n q_n(v), \]
  \[ \epsilon_M \equiv \sum_{n=1}^{N} S_n \epsilon_n. \]

- Number of assets \( N \) becomes large, while \( \sum_{n=1}^{N} S_n \) stays fixed.
  - \( S_n \) goes to zero.
  - Idiosyncratic shocks do not matter.
NO PERFORMANCE-BASED LIQUIDATION

- ODEs for \( \{q_n(v)\}_{n=1,...,N} \) and \( Z(v) \) have linear solution:
  
  - \( q_n(v) = q_{n0} - q_{n1}v \).
  - \( q_M(v) = q_{M0} - q_{M1}v \).
  - \( Z(v) = Z_0 + Z_1v \).

- Properties:
  
  - \( q_{M1} > 0 \): Market goes down when volatility increases.
  - \( \partial q_{n0}/\partial \epsilon_n < 0 \): Liquidity premia are positive.
  - \( \partial q_{n1}/\partial \epsilon_n = 0 \): Liquidity premia are independent of volatility.
CAPM

- ODE for $q_n(v)$ can be written as

$$E_t(R_{n,t+h}) = ACov_t(R_{n,t+h}, R_{M,t+h}) + AZ'(v_t)Cov_t(R_{n,t+h}, R_{v,t+h})$$
$$+ [r + 2\lambda \exp(2A\epsilon_M)] \epsilon_n h,$$

where

- $R_{n,t+h}$: excess return on asset $n$.
- $R_{M,t+h}$: excess return on market portfolio.
- $R_{v,t+h}$: excess return on volatility portfolio.
- Excess returns are per share, and between $t$ and $t+h$.

- Conditional two-factor CAPM, adjusted for transaction costs.
- Linear solution $\Rightarrow Z'(v_t) = Z_1$.
- Taking expectations, we find

$$E(R_{n,t+h}) = ACov(R_{n,t+h}, R_{M,t+h}) + AZ_1 Cov(R_{n,t+h}, R_{v,t+h})$$
$$+ [r + 2\lambda \exp(2A\epsilon_M)] \epsilon_n h,$$

i.e., unconditional two-factor CAPM, adjusted for transaction costs.
Volatility Factor

• Recent research has considered a volatility/liquidity factor.

• Pastor and Stambaugh (2001)
  – Liquidity factor $\equiv$ Price reversals.

• Acharya and Pedersen (2002)
  – Liquidity factor $\equiv$ Aggregate price impact.

• Liquidity factor:
  – Negative risk premium.
  – Significant explanatory power.
  – In our model, aggregate price impact is proportional to volatility $\Rightarrow$ Volatility factor corresponds to PS-AP liquidity factor.

• Ang and Hodrick (2002)
  – Volatility factor.
Volatility Risk Premium

• $Z_1 > 0 \Rightarrow$ Volatility factor carries positive risk premium.
  – Holding market beta constant, investors prefer assets that pay off when volatility is low.
  – Opposite to empirical findings.

• Intuition:
  – Holding wealth constant, investors are better off at times of high volatility.
PERFORMANCE-BASED LIQUIDATION

• Liquidation condition is $W_{t+h} - W_t < -L\sqrt{h}$.

• Probability of this event depends on tails of $\zeta_{t+h}$ and $\eta_{t+h}$.

• Normal distribution is not tractable.

• Power laws are, however, very tractable.
**Power Laws**

- Assume that lower tail of $\zeta_{t+h}$ and upper tail of $\eta_{t+h}$ follow power laws with exponent $b$, i.e.,

\[
\text{Prob}(\zeta_{t+h} < -y) \sim \frac{c_\zeta}{y^b},
\]

\[
\text{Prob}(\eta_{t+h} > y) \sim \frac{c_\eta}{y^b}.
\]

- $b = 2$: Liquidation probability is

\[
\pi(v_t) = \frac{v_t}{L^2} \left[ c_\zeta \frac{\phi_M^2}{(r + \kappa)^2} + c_\eta \left[ \frac{\psi_M}{r + \kappa} + q'_M(v_t)\sigma \right]^2 \right].
\]

Linear in $v_t$.

- $b = 4$: Liquidation probability is

\[
\pi(v_t) = \frac{v_t^2}{L^4} \left[ c_\zeta \frac{\phi_M^4}{(r + \kappa)^4} + c_\eta \left[ \frac{\psi_M}{r + \kappa} + q'_M(v_t)\sigma \right]^4 \right].
\]

Quadratic in $v_t$.

- Equations are valid for $L$ large relative to $v_t$. 
Asset Pricing

- Expected returns are given by

\[
E_t(R_{n,t+h}) = ACov_t(R_{n,t+h}, R_{M,t+h}) + AZ'(v_t)Cov_t(R_{n,t+h}, R_{v,t+h}) \\
+ \mu \left. \frac{\partial \pi(v_t, x)}{\partial x_n} \right|_{x=S} \frac{\exp(2A\epsilon_M) - 1}{A} h \\
+ [r + 2(\lambda + \mu \pi(v_t)) \exp(2A\epsilon_M)] \epsilon_n h.
\]

- Third term is “liquidation” risk premium.
LINEAR LIQUIDATION PROBABILITY

• Solution is still linear.
  
  \[- q_n(v) = q_{n0} - q_{n1}v \]
  
  \[- q_M(v) = q_{M0} - q_{M1}v \]
  
  \[- Z(v) = Z_0 + Z_1v. \]

• New property:
  
  \[- \partial q_{n1}/\partial \epsilon_n > 0: \text{Liquidity premia increase with volatility.} \]
CAPM

- Liquidation risk premium depends on covariance between asset $n$, and market and volatility portfolios.
- Can incorporate liquidation risk premium into CAPM risk premium.
- Conditional two-factor CAPM, with modified risk-aversion coefficients:

$$E_t(R_{n,t+h}) = A_M(v_t)\text{Cov}_t(R_{n,t+h}, R_{M,t+h}) + A_v(v_t)\text{Cov}_t(R_{n,t+h}, R_{v,t+h}) + [r + 2[\lambda + \mu \pi(v_t)] \exp(2A_\epsilon M)] \epsilon_n h.$$ 

- In linear case, modified risk-aversion coefficients $A_M(v_t)$ and $A_v(v_t)$ are constants.
- Taking expectations, we can derive unconditional two-factor CAPM.
Risk-Aversion Coefficients

• For market portfolio:

\[ A_M \equiv A + 2\frac{\mu}{L^2} c_\zeta \frac{\exp(2A\epsilon_M) - 1}{A}. \]

  – Greater than without performance-based liquidation.

• For volatility portfolio:

\[ A_v \equiv AZ_1 + 2\frac{\mu}{L^2}(c_\zeta - c_\eta) \left[ q_{M1} - \frac{\psi_M}{\sigma(r + \kappa)} \right] \frac{\exp(2A\epsilon_M) - 1}{A}. \]

  – Can become negative if \( c_\eta > c_\zeta \) (volatility tails fatter than dividend tails).

  – \( \Rightarrow \) Volatility factor can carry negative risk premium.

  – Holding market beta constant, investors prefer assets that pay off when volatility is high.

  – Intuition: Holding such assets reduces probability of liquidation.
Betas and Correlations

- Conditional betas are constant (independent of volatility) and equal to unconditional betas.
- Same for correlations.
QUADRATIC LIQUIDATION PROBABILITY

• Solution is nonlinear.

• Highest order new term (for large $L$) is quadratic in $v$:

  $$ - q_n(v) = q_{n0} - q_{n1}v - q_{n2}v^2 $$

  $$ - q_M(v) = q_{M0} - q_{M1}v - q_{M2}v^2 $$

  $$ - Z(v) = Z_0 + Z_1v + Z_2v^2. $$

• $\partial q_{n2}/\partial \epsilon_n > 0$: Liquidity premia increase with volatility.
Conditional CAPM

- Conditional two-factor CAPM:

\[
E_t(R_{n,t+h}) = A_M(v_t) \text{Cov}_t(R_{n,t+h}, R_{M,t+h}) \\
+ A_v(v_t) \text{Cov}_t(R_{n,t+h}, R_{v,t+h}) \\
+ [r + 2(\lambda + \mu \pi(v_t)) \exp(2A\epsilon_M)] \epsilon_n h.
\]

- In quadratic case, modified risk-aversion coefficients \(A_M(v_t)\) and \(A_v(v_t)\) depend on \(v_t\).

- For market portfolio:

\[
A_M(v_t) \equiv A + 4\frac{\mu v_t}{\nu L^4} c_\zeta \phi_M^2 \frac{\exp(2A\epsilon_M) - 1}{A}.
\]

  - Market is more risk averse when volatility is high.
  - Stochastic risk aversion is not because of stochastic utility functions, but because liquidation probability is convex in volatility.
Betas

- Conditional betas are stochastic.

- An asset’s market beta increases with volatility if

\[
\frac{1}{2} \left[ \frac{\sigma q_{n1} - \frac{\psi_n}{r+\kappa}}{\sigma q_{M1} - \frac{\psi_M}{r+\kappa}} + \frac{q_{n2}}{q_{M2}} \right] 
\geq \frac{\phi_M \phi_n}{(r+\kappa)^2} \left[ \frac{\psi_M}{r+\kappa} - \sigma q_{M1} \right] \left[ \frac{\psi_n}{r+\kappa} - \sigma q_{n1} \right] \geq \frac{\phi_M^2}{(r+\kappa)^2} \left[ \frac{\psi_M}{r+\kappa} - \sigma q_{M1} \right]^2.
\]

- Holds if asset is more price-sensitive to volatility than average.

- Holds if
  - Asset is more payoff-sensitive to volatility than average. \((\psi_n \text{ small})\)
  - Asset has higher transaction costs than average. \((\epsilon_n \text{ large})\)
Correlations

- Conditional correlations are stochastic.
- For two assets $n_1$ and $n_2$ such that $\phi_{n_1} = \phi_{n_2}$, $\psi_{n_1} = \psi_{n_2}$, $\epsilon_{n_1} = \epsilon_{n_2}$, correlation increases with volatility.
Unconditional CAPM

- Taking expectations in conditional CAPM, we find

\[
E(R_{n,t+h}) = E[A_M(v_t)] \text{Cov}(R_{n,t+h}, R_{M,t+h})
+ E[A_v(v_t)] \text{Cov}(R_{n,t+h}, R_{v,t+h})
+ \text{Cov}[A_M(v_t), \text{Cov}_t(R_{n,t+h}, R_{M,t+h})]
+ \text{Cov}[A_M(v_t), \text{Cov}_t(R_{n,t+h}, R_{M,t+h})]
+ [r + 2(\lambda + \mu E(\pi(v_t))) \exp(2A\epsilon_M)] \epsilon_n h.
\]

- New terms: Covariances between stochastic risk-aversion coefficients and conditional covariances.
  - “Risk-aversion/beta” covariances.
CAPM Pricing

• Can two assets $n_1$ and $n_2$ have same unconditional covariances, but different sum of risk-aversion/beta covariances?
  – Yes.
  – Sum of risk-aversion/beta covariances is greater for $n_1$ if $q_{n_12} > q_{n_22}$.
  – This is equivalent to $\epsilon_{n_1} > \epsilon_{n_2}$.

• Unconditional CAPM understates risk of illiquid assets.

• Intuition:
  – Illiquid assets have high betas at times of high volatility.
  – At these times, market is most risk averse.
Conclusion

This paper:

- Proposes a theory of time-varying liquidity premia.
- Explores its asset-pricing implications.

Basic idea:

- Fund managers face liquidation after poor performance.
- High volatility $\Rightarrow$ High liquidation probability $\Rightarrow$ Short investment horizons $\Rightarrow$ High liquidity premia.

Main contributions:

- Several phenomena associated to crises / volatile times are related.
  - Large liquidity premia. (Flight to liquidity)
  - Increased market risk aversion. (Flight to quality)
  - Increased riskiness of illiquid assets.
  - Increased correlations.
- CAPM understates risk of illiquid assets.
  - These assets become very risky in crises, when market is most risk averse.