The Knowledge Trap: Human Capital and Development Reconsidered*

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Abstract

This paper presents a model where human capital differences, rather than residual productivity differences, can explain several central phenomena in the world economy. In the model, workers choose both the duration and content of their training. A "knowledge trap" occurs where skilled workers avoid narrow, deep training and thus fail, collectively, to embody frontier knowledge. Standard human capital accounting is shown to underestimate the resulting skill differences between rich and poor nations. The theory may explain price, wage and income differences across countries, and suggests novel interpretations of immigrant outcomes, poverty traps, and the brain drain, among other applications.

Keywords: human capital, education, technology, TFP, relative prices, wages, cross-country income differences, international trade, multinationals, poverty traps, migration

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1 Introduction

To explain several central phenomena in economics, from the wealth and poverty of nations to patterns of world trade, economists often rely on large, residual productivity differences. That is, explanations rely on some critical factor of production that is distinct from the contributions of physical and human capital. This paper presents an alternative view: I show how to put “ideas” back into people, presenting a model where human capital can explain many stylized facts about the world economy. One contribution of this paper is to show that current macro-accounting methods can severely underestimate human capital differences across countries. More generally, this paper develops a richer perspective on human capital investment to understand why frontier knowledge may fail to be loaded into the minds of a nation’s workforce.

The model emphasizes cross-country differences in the quality of skilled workers. The results follow from education decisions, which have two dimensions. One is duration - how much time to put in education - which defines whether you become a skilled worker or not. The other is content - what specific knowledge to acquire. In particular, given an investment of time, one might become a "generalist" (e.g. a generalist doctor) with modest knowledge about multiple tasks, or a "specialist" (e.g. a heart surgeon) with deep knowledge at a particular task. Quality advantages emerge in the collective productivity of skilled workers, where specialists working in teams bring greater collective knowledge to bear in production.

The theory thus builds on Adam Smith’s foundational observation that the division of labor can bring high productivity. The twist is to understand why these gains may go unrealized in the educational phase. In the model, any gains from narrowly focused training are traded off against the cost of both finding specialists with complementary skills and coordinating with them in production. This tradeoff may favor breadth over depth for three reasons. First, deep, specialized knowledge may be hard to acquire locally; for example, heart surgery may be hard to learn without guidance from existing heart surgeons. Absent expert instructors, focused training provides less advantage. Second, specialization may be worthwhile only when a sufficient mass of complementary specialists already exists. For example, learning heart surgery is less useful in the absence of anesthesiologists. These
two issues suggest a variety of poverty trap where local deep knowledge is a prerequisite for individuals to willingly and successfully seek deep knowledge. Finally, coordination costs in production may be especially high. For any (or all) of these reasons, a low-productivity "generalist" equilibrium may persist. I call such outcomes a "knowledge trap" because the generalist equilibrium features shallower collective knowledge.

Such quality differences are limited to skilled workers. Importantly, however, the real wages of unskilled workers also rise in rich countries. This is a general equilibrium effect that follows when the output of skilled workers is relatively abundant, making the output of unskilled workers relatively scarce. This scarcity drives up unskilled wages. More precisely, when decisions to become skilled or remain unskilled are endogenous - the duration dimension of education is a choice variable - the wage structure is pinned down in equilibrium so that although quality differences are limited to skilled workers, real income gains are shared equally by skilled and unskilled workers alike.

This equilibrium effect is crucial, because it poses significant challenges to standard human capital accounting methods. The standard approach infers cross-country skill differences from within-country returns to schooling, but in this model the entire wage distribution shifts, so that within-country wage equilibria on their own say nothing about cross-country skill differences. Estimation approaches based on immigrant behavior face similar problems. The wage gains experienced by unskilled workers who immigrate from poor to rich countries need not be explained by technology, as many authors infer; in this model, the wage gains follow simply because unskilled workers are relatively scarce in the rich country. For example, one may ask why taxi drivers earn so much more in rich countries. A natural explanation is that the taxi driver’s clients - skilled workers – have a much higher opportunity cost of their time and hence will pay more for the ride.

In sum, rich countries are rich because they attain deeper collective knowledge among skilled workers. The relative scarcity of low skill means that the real wages of unskilled workers also rise in rich countries, even though such workers have no more skill in rich than

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1The idea that coordination costs of teamwork limit the gains from specialization follows Becker & Murphy (1992). More broadly, the limits to specialization considered in this paper are based on local frictions, rather than on the extent of the market as in Smith (1776).
poor countries. One thus finds a skill-based explanation for cross-country income differences that can also get wages right.

Furthermore, the division of knowledge among skilled workers means that (a) schooling duration is insufficient to assess skill and (b) the productivity of skilled workers is interdependent. Workers are puzzle pieces, who fit together differently in different economies. This richer perspective on skill may inform phenomena such as migrant behavior (why do skilled immigrants often take menial jobs?), the brain drain (why don’t skilled workers move to poor countries, where they are scarce?), and pathways to escape poverty traps, including intriguing roles for multinationals in triggering development, which may inform growth miracles in places like Hyderabad and Bangalore.

This paper is organized as follows. Section 2 introduces the core ideas. Section 3 presents a formal model, clarifying conditions for the existence of "knowledge traps" and their general equilibrium effects. Section 4 discusses several applications and relates them to existing empirical evidence in addition to new evidence about the quality of skilled workers. I show that the model provides an integrated perspective on (i) cross-country income differences, (ii) immigrant labor market outcomes, and (iii) poverty traps, as well as price phenomena, including (iv) why some goods are especially cheap in poor countries and (v) why "Mincerian" wage structures appear in all countries. Section 4 also offers possible insights about the role of multinationals in development and closes by considering generalizations to inform international trade patterns, skill-biased technical change, and income divergence across countries. Section 5 concludes.

Related Literature Many existing papers explore theoretical aspects of the division of labor (e.g. Kim 1989, Becker and Murphy 1992, Garicano 2000). Other papers explore multiple equilibria in human capital (e.g. Kremer 1993, Acemoglu 1996), and still others explore specialization in intermediate goods, i.e. at the firm level, as the source of development failures (e.g. Ciccone and Matsuyama 1996, Rodriguez-Clare 1996, Acemoglu et al. 2006). A key innovation in this paper is to imagine specialization in education as a basis for different organizational forms of labor supply. More precisely, this paper imagines a two-dimensional education decision where both the breadth and duration of education are
endogenous choices. There is thus a division of labor among skilled workers (based on breadth), and a division of labor between skilled and unskilled workers (based on duration).

This theoretical approach allows a reinterpretation of several empirical literatures, especially the "macro-Mincer" approach in the vast development accounting literature (surveyed in Caselli 2005), which attempts to assess the role of human capital in cross-country income differences. These empirical literatures will be discussed in detail below.

The primary contributions of this paper are two-fold. First, I show broadly how standard accounting methods may underestimate the role of education in skill formation. Second, I present a simple model, based on the division of skills, to show why large skill differences may exist and persist across countries, providing a parsimonious interpretation of many stylized facts in the world economy.

2 The Core Ideas

This section provides an introductory discussion of the core ideas in this paper. First, I introduce a "knowledge trap" to show how endogenous educational decisions can produce large cross-country differences in skill. Second, I show how standard macroeconomic accounting methods will misaccount for these skill differences. Section 3 integrates these ideas into a formal model.

2.1 A Knowledge Trap

Modern production in rich countries appears to involve huge amounts of knowledge, from microprocessors to gas turbines, from polymer synthesis to radiation oncology, from computer network security to accounting consistent with the GAAP. It is difficult for an individual to know more than a fraction of this knowledge. In fact, at birth, individuals know none of it. A basic, collective challenge is then how – and whether – economies load this advanced knowledge into people’s minds.

To fix ideas, imagine there are two tasks, $A$ and $B$, which are complementary in the production of a good. For example, the good could be heart surgery, where one task is anesthesiology and the other is the surgery itself. Alternatively, the ultimate output could
be a microprocessor, a key input in modern electronics and computing, or a gas turbine, a key input in energy production, both of which build on knowledge across many complementary tasks.\(^2\)

Now imagine individuals must train to acquire skill, and one must decide how to use an endowment of training time. One might train as a "generalist", developing skill at both tasks. Alternatively, one might focus all their training on one task, becoming especially adept at that task. For simplicity, let training as a generalist produce a skill level \(1\) at both tasks, while training as a specialist produces a skill level \(m > 1\) at one task and \(0\) at the other.

As a simple example, let production be \(Y = \sqrt{H^A H^B}\) when working alone and \(cY\) when pairing with another worker. This Cobb-Douglas production function captures the complementarity between skills, and the term \(c < 1\) represents a coordination penalty from working in a team. Output is per unit of clock-time, and the amount of skill applied to a particular task, e.g. \(H^A\), is the summation of skill applied per unit of clock-time.

In this setting, a generalist working alone does best by dividing his time equally between tasks and earning \(Y = \frac{1}{2}\). A pairing of complementary specialists optimally applies each worker to their specialty, producing \(Y = mc\) for every unit of clock time, or \(\frac{1}{2}mc\) per team member. The specialist organizational form is therefore more productive as long as \(mc > 1\); that is, as long as coordination penalties do not outweigh the benefits of deeper expertise.

A "knowledge trap" occurs when an economy of generalists is a stable equilibrium. In a poor country, this may occur most simply because \(m\) is small. For example, photolithography may be difficult to master without training from an existing expert. Alternatively, photolithography combines microprocessor photolithography (the etching of the processor onto silicon, which draws on material science and optics), microprocessor design (including the instruction set architecture, memory, control and data path design, thermal analysis, etc), and microprocessor software (the assembler, compiler, debugger, etc) all of which draw on very different kinds of knowledge. Turbine production involves the integrated design and manufacture of turbine blades, turbofans, compressors, combustors, control systems, fuel systems, nozzles, et cetera, which draw on disparate and highly specific engineering expertise, including thermodynamics, material science, fluid mechanics, rotational and vibrational dynamics and high-heat electronics. One broadly-trained engineer working alone may be able to produce a simple integrated circuit or even a very simple turbine, but the advanced, highly productive versions (e.g. a low-power Intel Atom processor or a GE90 gas turbine) are not produced by one person. Joseph Palladino of General Electric Aircraft Engines (personal correspondence) estimates that 30-35 different disciplines are required to implement a modern jet engine. Related, MIT currently offers 25 different varieties of Ph.D. within the fields of "Aeronautics and Astronautics". In this paper, I will consider a model with two complementary tasks to focus on the core ideas; generalizations to more tasks would make complementarities more acute.

\(^2\)Microprocessor production combines microprocessor photolithography (the etching of the processor onto silicon, which draws on material science and optics), microprocessor design (including the instruction set architecture, memory, control and data path design, thermal analysis, etc), and microprocessor software (the assembler, compiler, debugger, etc) all of which draw on very different kinds of knowledge. Turbine production involves the integrated design and manufacture of turbine blades, turbofans, compressors, combustors, control systems, fuel systems, nozzles, et cetera, which draw on disparate and highly specific engineering expertise, including thermodynamics, material science, fluid mechanics, rotational and vibrational dynamics and high-heat electronics. One broadly-trained engineer working alone may be able to produce a simple integrated circuit or even a very simple turbine, but the advanced, highly productive versions (e.g. a low-power Intel Atom processor or a GE90 gas turbine) are not produced by one person. Joseph Palladino of General Electric Aircraft Engines (personal correspondence) estimates that 30-35 different disciplines are required to implement a modern jet engine. Related, MIT currently offers 25 different varieties of Ph.D. within the fields of "Aeronautics and Astronautics". In this paper, I will consider a model with two complementary tasks to focus on the core ideas; generalizations to more tasks would make complementarities more acute.
coordination penalties in production may be more severe in poor countries. Hence, poor
countries may feature \( m'c' < 1 \) while a rich country has \( mc > 1 \).

More subtly, an economy of generalists may persist due to thin supply of complementary
specialist types. To see this, imagine you are born into a world of generalists and consider
whether you would want to become a specialist instead. The best you could do as a lone
specialist would be to pair with an existing generalist. In such a pairing, the specialist
focuses on the task in which they have expertise, the generalist on the other, and the
optimal output is \( Y = \sqrt{mc} \). The generalist would have to be paid at least their outside
option, \( \frac{1}{2} \), to willingly join the specialist in such a team. The most income the specialist
could earn is therefore \( \sqrt{mc} - \frac{1}{2} \), which itself must exceed \( \frac{1}{2} \) for a player to prefer training as
a specialist. Hence the generalist equilibrium is stable to individual deviations if \( \sqrt{mc} < 1 \).
We thus have a potential trap: for any coordination penalty in the range \( \frac{1}{m} < c < \frac{1}{\sqrt{m}} \)
mutual specialization is more productive and yet the generalist equilibrium is stable.\(^3\)

I call these specialization failures a "knowledge trap" because skilled workers in the
generalist equilibrium have shallow knowledge. This doesn’t mean that they have little
education. For example, the generalist doctor knows something about both anesthesiology
and surgery – not to mention oncology, infectious disease, psychiatry, ophthalmology, etc.
Learning something about all these different subjects may require a lot of education. But
this generalist doctor will likely be far less productive than a set of specialists who work
together. The specialists may have no more schooling per person, but they have much
deeper knowledge about individual tasks, so that the collective body of knowledge across
the specialists may be far greater. Quality differences thus follow here from the content
dimension of education. To see why potentially large quality differences will not be detected
by standard human capital accounting methods, we must further consider the duration
dimension of education, which we turn to next.

\(^3\)This type of knowledge trap would be resolved by mutual specialization in complementary tasks, and
one may ask why this coordination problem isn't resolved naturally in the market, especially by firms. The
implicit assumption is that educational decisions are primarily made prior to the interactions of individuals
and firms, so that firms cannot coordinate major educational investments but rather make production
decisions given the skill set of the labor force. This seems a reasonable characterization empirically, since
skilled workers (engineers, lawyers, doctors, etc.) typically train for many years in educational institutions
that are distinct from firms, before entering the workforce. In this sense, it then falls to other institutions
to solve this type of coordination problem. These issues will be discussed further in Section 4.
2.2 Human Capital and Wages

A large literature has concluded that schooling variation across countries is too small to explain cross-country income differences (see Caselli 2005 for a survey). This inference is primarily drawn using the "macro-Mincer" approach, which attempts to compute human capital stocks from data on the wage-schooling relationship (e.g. Klenow and Rodriguez-Clare 1997, Hall and Jones 1999). If workers are paid their marginal products, then the wage gain from schooling can inform how schooling influences productivity. Wage-schooling relationships are usually taken to follow the log-linear, i.e. "Mincerian", form (Mincer 1974),

\[ w(s) = w(s')e^{r_m(s-s')} \]  

(1)

where \( s \) is schooling duration, \( w(s) \) is the wage, and \( r_m \) is the percentage increase in the wage for an additional year of schooling.\(^4\)

To see how such within-country wage relationships can be misused in inferring cross-country skill differences, consider first that these wage structures emerge as a local equilibrium when labor supply is endogenous. In particular, define a worker’s lifetime income as

\[ y(s) = \int_{s}^{\infty} w(s)e^{-rt}dt \]  

(2)

where individuals earn no wage income during their \( s \) years of training and face a discount rate \( r \). If in equilibrium workers cannot deviate to other schooling decisions and be better off, then for any two schooling levels

\[ y(s) = y(s') \]

and therefore (1) follows immediately with \( r_m = r \).\(^5\) The log-linear wage structure follows through arbitrage. Individuals become skilled by investing time in education, which means giving up wages today in exchange for higher wages later. In this simple setting, the rate

\(^4\)Such log-linear wage-schooling relationships have been estimated in many countries around the world (see Psacharopolous 1994).

\(^5\)This arbitrage argument follows in the spirit of Mincer (1958). Integrating (2) gives \( y(s) = \frac{1}{r}w(s)e^{-rs} \) so that \( y(s) = y(s') \) implies \( w(s) = w(s')e^{r(s-s')} \). Equivalently, (1) follows if workers choose schooling duration to maximize lifetime income. That is, with \( s^* = \arg \max y(s) \) we have \( w'(s^*) = rw(s^*) \) which is just the log-linear wage structure expressed as a marginal condition.
of return on a foregone dollar of wage income is pinned down by the expected return on investment - i.e. the discount rate.\footnote{Here the interest rate and the return to schooling are equivalent. A richer model would introduce other aspects, such as ability differences, progressive marginal income tax rates, out-of-pocket costs for education, and finite time horizons which could drive the return to schooling above the real interest rate. See Heckman et al. (2005) for a broader characterization of lifetime income.} Quality differences in education won’t appear in the wage data, because educational duration decisions reallocate workers endogenously to ensure this equilibrium rate of return.

Now consider how one can interpret skill from wages. Imagine that there are two goods, good 1 (e.g. haircuts) produced by unskilled workers with no education and good 2 (e.g. surgery) that requires $S$ years of training to perform. Let preferences be the same in all countries and demand for each good be downward sloping. Lastly, imagine as above that skill, $h$, and time, $L$, are the only inputs to production, so that $x_1 = h_1L_1$ and $x_2 = h_2L_2$. The marginal product for each good is then $w_1 = p_1h_1$ and $w_2 = p_2h_2$, and we have
\[ h_2 = \frac{p_1}{p_2} h_1 e^{rS} \] where $w_2/w_1 = e^{rS}$ follows from income arbitrage as above.

To compare skill across countries, standard accounting methods assume that unskilled workers have the same innate skill, $h_1$, in all economies and estimate the skill of the educated as
\[ h_2 = h_1 e^{rS} \]
But this method for estimating $h_2$ is clearly problematic. As just shown in (3), one must also confront relative prices ($p_1/p_2$), which are well known to differ substantially across countries.\footnote{Such relative price differences are large and motivate the need for purchasing power parity (PPP) price corrections when comparing real incomes across countries.} And it is easy to see how ignoring relative prices might substantially understate human capital differences. Under the innocuous assumptions that poor countries are relatively abundant in low skill and that demand is downward sloping, $p_1/p_2$ will be relatively small in poor countries. Hence the skill gains from education ($h_2/h_1$) must be adjusted upwards in rich countries relative to poor countries. Failing to account for these price differences will dampen cross-country skill differences compared to the case where we assume prices are the same everywhere.\footnote{The standard accounting method assumes that the output of different skilled workers are perfect sub-
These observations suggest that skill differences might play a more important role in the world economy than a large literature has suggested. The following section presents a general equilibrium model, integrating the quality differences of knowledge traps with endogenous schooling duration decisions. Section 4 then details several applications and reconsiders established empirical evidence from the model’s perspective.

3 The Knowledge Trap Economy

Imagine a world where workers are born, invest in skills, and then work, possibly in teams. They can work in one of two sectors. One sector requires only unskilled labor, and output is insensitive to the education level of the worker. Output in the other sector depends on formal education.

The key decision problem for the individual is what skills to learn. Skill type is chosen to maximize expected lifetime income. Once educated, the worker enters the labor force and produces output, which occurs efficiently conditional on the education decisions made and the ability to form appropriate teams. The educational decision is thus the key to the model.

3.1 Environment

There is a continuum of individuals of measure \( L \). Individuals are born at rate \( r > 0 \) and die with hazard rate \( r \), so that \( L \) is constant. Individuals are identical at birth and may either start work immediately in the unskilled sector or invest \( S \) years of time to undertake education. If they choose to educate themselves, they may develop skill at two tasks, A and B. We denote an individual’s skill level \( h = \{h_A, h_B\} \). An individual may choose to become a "generalist" and learn both skills, developing skill level \( h = \{h, h\} \). Alternatively, one may focus on a single skill and develop deeper but narrower expertise, attaining skill level \( h = \{mh, 0\} \) or \( h = \{0, mh\} \) where \( m > 1 \).

In this case \( p_1 = p_2 \) (effectively, there is one good only). Under this assumption, one could estimate \( h_2/h_1 \) based purely on \( u_2/u_1 \). However, this assumption is unrealistic if we believe that worker types are less than perfect substitutes. More realistically, any number of high school students are unlikely to successfully perform angioplasty, assemble a jet engine, or write a contract consistent with the UCC. Different types of workers produce different types of goods that face downward sloping demand. Hence, skill endowments will matter in making inferences about human capital. This will be discussed further in Section 4.
3.1.1 Timing

For the individual, the sequence of events is:

1. The individual is born.

2. The individual makes an educational decision, becoming one of four types of workers,

   (a) Type U workers ("unskilled") undertake no education, $s^U = 0$, and have skill level $h^U = \{0, 0\}$.

   (b) Type G workers ("generalists") undertake $s^G = S$ years of education and learn both tasks, developing skill level $h^G = \{h, h\}$.

   (c) Type A workers ("A-specialists") focus $s^A = S$ years on task A, developing skill level $h^A = \{mh, 0\}$.

   (d) Type B workers ("B-specialists") focus $s^B = S$ years on task B, developing skill level $h^B = \{0, mh\}$.

3. The individual enters the workforce.

   (a) Unskilled workers (type U) go to work immediately in the unskilled sector.

   (b) Skilled workers (types G, A, B) enter the skilled sector after $S$ years and may choose to work alone or pair with other skilled workers.

      i. Unpaired skilled workers randomly meet other unpaired skilled workers with hazard rate $\lambda$.

      ii. If paired and your partner dies (at rate $\rho$), then you become unpaired again.

3.1.2 Preferences

Expected utility is given by

$$U^k = \int_0^\infty u(C^k(t))e^{-rt}dt$$

where $u(C)$ is increasing and concave. The effective rate of time preference is given by $r$, the hazard rate of death, which is equivalent to the discount rate.\textsuperscript{9} This equivalence implies

\textsuperscript{9}There is no physical capital in this model, so there is no rental rate of capital. However, there are loans, since players are born with no wealth and therefore those in school must borrow to consume. We
that an individual’s consumption does not change across periods, by the standard Euler equation.\textsuperscript{10}

Let preferences across goods be

$$C^k(x_1, x_2) = (\gamma x_1^p + (1 - \gamma) x_2^p)^{1/p}$$

(4)

where $x_1$ is the good produced by the unskilled sector, $x_2$ is the good produced by the skilled sector, and $\varepsilon = \frac{1}{1 - p}$ is the elasticity of substitution between goods, which we assume is finite.

### 3.1.3 Income

The expected present value of lifetime income for a worker of type $k$ is

$$W^k = \int_{s^k}^{\infty} rV^k e^{-\tau \delta} d\tau$$

(5)

where $s^k \in \{0, S\}$ is the duration of education. Time subscripts are suppressed because we will focus on steady-state equilibria. $V^k$ is the value of being a type $k$ worker at the moment your education is finished, which is the expected value of being an unpaired worker of type $k$. For unskilled workers, $rV^U = w_1$, where $w_1$ is the wage earned from producing the unskilled good. For skilled workers we have

$$rV^k = w_2^k + \lambda \sum_{j \in \Omega^k} \Pr(j) (V^{kj} - V^k)$$

(6)

The flow value of being unpaired, $rV^k$, equals the wage from working alone, $w_2^k$, in the skilled sector plus the expected marginal gain from a possible pairing. You meet other unpaired skilled workers at rate $\lambda$, and the unpaired skilled worker is type $j$ with probability $\Pr(j)$. We assume a uniform chance of meeting any particular unpaired skilled worker, so that

$$\Pr(j) = L^j_p / L_p$$

(7)

imagine a zero-profit competitive annuity market where individuals hand over rights to their future lifetime income, $W$, upon birth in exchange for a payment, $a$, every period. This payment must be $a = rW$ by the zero profit condition. Therefore, the rate of interest on loans is the same as the hazard rate of death.

\textsuperscript{10}The Euler equation is $\frac{du(C)}{u(C)} = r - r = 0$, so that $u(C)$ and hence $C$ are constant with time.
where $L^j_p$ is the measure of workers of type $j$ who are unpaired and $L^j_p = \sum_j L^j_p$. You accept the match if $V^{kj} \geq V^k$ and reject otherwise, which defines the "acceptance set", $\Omega^k \subset \{G, A, B\}$, the set of types that a player of type $k$ is willing to match with. If you reject, you remain in the matching pool. If you accept, you leave the matching pool and earn $V^{kj}$, which is defined

\[ rV^{kj} = w^{kj}_2 - r \left( V^{kj} - V^k \right) \]  

The flow value of being paired, $rV^{kj}$, is equal to the wage you receive in this pairing, $w^{kj}_2$, less the expected loss from becoming a solo worker again, which occurs when your partner dies (with probability $r$).

Paired workers split the value of their joint output by Nash Bargaining, dividing the joint output such that

\[ w^{kj}_2 = \arg \max_{w^{kj}} \left( V^{kj} - V^k \right)^{1/2} \left( V^{jk} - V^j \right)^{1/2} \]  

Meanwhile, a solo worker earns the total value of his output when working alone.

### 3.1.4 Output

Sector 1 produces a simple good, $x_1$, with unskilled labor and with no advantage to skill in tasks A or B. Each worker in sector 1 produces with the technology

\[ \frac{x_1}{z} \]

per unit of clock time.

Sector 2 produces a good where skill at tasks A and B matters. Workers in sector 2 may work alone or with a partner, with the production function

\[ x_2 = zc(n) \left( H_A^\alpha + H_B^\alpha \right)^{1/\alpha}, \quad H_k = \sum_{i} t^k_i h^k_i \]  

where $\sigma = \frac{1}{1-\alpha}$ is the elasticity of substitution between the two skills and we assume $\sigma \leq 1$.

\[ \lambda \Pr(j) L^k_p = \lambda \left( \frac{L^k_p}{L_p} \right) L^k_p = \lambda \left( \frac{L^k_p}{L_p} \right) L^k_p = \lambda \Pr(k) L^k_p \]  

11 Note that this specification guarantees that the aggregate rate at which type $k$ people bump into type $j$ people ($\lambda \Pr(j)L^k_p$) is the same as the rate at which type $j$ people bump into type $k$ people ($\lambda \Pr(k)L^j_p$). Specifically,

\[ \lambda \Pr(j)L^k_p = \lambda \left( \frac{L^k_p}{L_p} \right) L^k_p = \lambda \left( \frac{L^k_p}{L_p} \right) L^k_p = \lambda \Pr(k)L^j_p \]
so that both inputs are necessary for positive production. The term $c(n) \in [0, 1]$ captures the coordination penalty from working in a team of size $n \in \{1, 2\}$. Without loss of generality set $c(1) = 1$ and $c(2) = c$. The time devoted by individual $i$ to task $k$ is $t_i^k$, and members of a team split their time across tasks to produce maximum output.

### 3.2 Equilibrium

An equilibrium is a decision by each worker that maximizes her utility given the decisions of other workers. The choice involves (a) maximizing lifetime income, and (b) maximizing utility of consumption given this lifetime income. We look at equilibria where all players of skilled type $k$ have the same matching policy $\Omega^k$ that is constant with time.

It is convenient to define the equilibrium in terms of aggregate variables. Let $L_k^k$ be the measure of living individuals who have chosen to be type $k$, and let $L_q$ be the measure of workers actively producing the good of type $q$. Let $X_q^S$, $X_q^D$, and $p_q$ respectively be the total supply, total demand, and price of good $q$.

**Definition 1** A steady-state equilibrium consists of $W_k^k$, $V_k^k$, $C_k^k$, $L_k^k$ for all worker types $k \in \{U, G, A, B\}$; $V_k^{kj}$, $\Omega_k^k$, $L_k^k$ for all skilled worker types $j \in \{G, A, B\}$; and $L_q$, $X_q^S$, $X_q^D$, $p_q$ for each good $q \in \{1, 2\}$ such that

1. **(Income maximization: Choice of worker type)** $W_k^k \geq W_j^k \forall k \in \{U, G, A, B\}$ such that $L_k^k > 0, \forall j \in \{U, G, A, B\}$

2. **(Income maximization: Matching policy)** $j \in \Omega_k^k$ for any $j \in \{G, A, B\}$ such that $V_k^{kj} \geq V^k, \forall k \in \{G, A, B\}$

3. **(Consumer optimization)** $C_k(x_1, x_2) \geq C_k(x_1', x_2') \forall x_1, x_2, x_1', x_2'$ such that $p_1x_1 + p_2x_2 \leq rW_k^k$ and $p_1x_1' + p_2x_2' \leq rW_k^k, \forall k \in \{U, G, A, B\}$

4. **(Market clearing)** $X_q^D = X_q^S \forall q \in \{1, 2\}$

5. **(Steady-state)** $L_k^k$ is constant $\forall k \in \{U, G, A, B\}$ and $L_k^k$ is constant $\forall k \in \{G, A, B\}$

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12 The CES production function in (10) is used for simplicity. The theory can be developed from a more general production function, $x_2 = c(n)f(H_A, H_B)$, where $f(H_A, H_B)$ is a symmetric, constant returns to scale function. Gross complements ($\sigma \leq 1$) provides substantial tractability but is not a necessary condition for the main results (see also Footnote 39).
We will further focus on equilibria in the "full employment" setting, where $\lambda \to \infty$.

### 3.3 Analysis

We analyze the equilibria in this model in two stages. First, we focus on the skilled sector. We investigate two different equilibria that can emerge in the organization of skilled labor, a "generalist" equilibrium and a "specialist" equilibrium. Second, we introduce the unskilled sector and demand to close the economy.

#### 3.3.1 Organizational Equilibria in the Skilled Sector

The value of being a skilled worker of type $k$ at the moment one’s education is complete is, from (6) and (8),

$$V^k = \frac{1}{r} \left( w_2^k + \frac{\lambda}{2r} \sum_{j \in \Omega^k} \Pr(j)w^{kj} \right)$$

so that the value of being a type $k$ worker depends on (a) the wage you earn if you work alone, $w_2^k$, (b) the wage you can earn in pairings you are willing to accept, $w^{kj}$, and (c) the rate such pairings occur, $\lambda \Pr(j)$. To solve this model, we consider the wages and pairings that can be supported in equilibrium.

The equilibrium definition requires that no individual be able to deviate and earn higher income. Hence we must have $W^k = W$ for all active worker types in any equilibrium and therefore, by (5),

$$V^k = V \text{ for all } k \in \{G, A, B\}$$

That is, each type of skilled worker must have the same expected income upon finishing school. If one type did better than the others, an individual would switch to become this type.

This common value, $V$, means that in any equilibrium individuals have the same outside option when wage bargaining. Defining $x^{kj}_2$ as the maximum output individuals of type $k$ and $j$ can produce when working together, it then follows from Nash Bargaining, (9), that in any accepted pairing $V^{kj} = V^{jk}$ and

$$w^{kj}_2 = \frac{1}{2} \rho_2 x^{kj}_2$$

14
so that in equilibrium a worker team splits its joint output equally. Meanwhile, if skilled
workers work alone, then they earn the total product, so that

\[ w_2^k = p_2 x_2^k \]  \hspace{1cm} (13)

where \( x_2^k \) is the maximum output an individual of type \( k \) can produce when working alone.

These results lead to a limited set of matching behaviors that can exist in equilibrium.

**Lemma 1** (Matching Rules) In equilibrium, matching behavior is either

\[ \{\Omega_A, \Omega_B, \Omega_G\} = \{\{B\}, \{A\}, \emptyset\} \text{ or } \{\Omega_A, \Omega_B, \Omega_G\} = \{\{B, G\}, \{A, G\}, \{A, B\}\} \]

**Proof.** See appendix. ■

This result states in part that types never match with themselves. This is intuitive
because matching with one own’s type provides no productivity advantage but incurs co-
ordination costs. The lemma also states that a specialist is always willing to match with
the other specialist type in equilibrium. This is intuitive because an AB pairing produces
the highest wages. A second, intuitive equilibrium property follows from the symmetry
between specialists and their desire not to be unemployed.

**Lemma 2** (Balanced Specialists) In equilibrium, \( L^A = L^B \).

**Proof.** See appendix. ■

This lemma limits the class of possible equilibria. If \( L^s \) is the total mass of skilled
workers, then we can distinguish three potential equilibria: (1) a "generalist" equilibrium
where \( \{L^A, L^B, L^G\} = \{0, 0, L^s\} \); (2) a "specialist" equilibrium where \( \{L^A, L^B, L^G\} = \{\frac{1}{2}L^s, \frac{1}{2}L^s, 0\} \); and (3) a "mixed" equilibrium where \( \{L^A, L^B, L^G\} = \{L', L', L^s - 2L'\} \) for
some \( L' \) such that \( 0 < L' < \frac{1}{2}L^s \).

**Proposition 1** (Knowledge Trap) With full employment, where \( \lambda \rightarrow \infty \), a "generalist"
equilibrium exists iff \( x_2^{AG} \leq 2x_2^G \) and a "specialist" equilibrium exists iff \( x_2^{AB} \geq 2x_2^G \). With
full employment, any "mixed" equilibrium limits to the "generalist" equilibrium. For some
parameter values, both a generalist and specialist equilibrium can exist. These equilibria are
summarized in Figure 1.
Proof. See appendix. ■

Figure 1: The Knowledge Trap

The intuition for these results is straightforward. As $\lambda \to \infty$, workers meet at such a high rate that they match instantaneously in equilibrium and are never unemployed. Hence skilled workers choose matches based simply on wages. In the "generalist" case, skilled workers earn $w_2^G = p_2x_2^G$. If a player deviates to be a specialist, say type A, then the best he can do is pair with an existing generalist and earn $p_2x_2^{AG} - w_2^G$. Hence, a world of generalists is an equilibrium iff $p_2x_2^{AG} - w_2^G \leq w_2^G$, or

$$x_2^{AG} \leq 2x_2^G.$$ 

In the "specialist" case, skilled workers produce in teams and earn a wage $w_2^{AB} = \frac{1}{2}p_2x_2^{AB}$. If a player deviates to be a generalist, then he could either (a) work alone and earn $w_2^G$ or (b) pair with an existing specialist and earn $p_2x_2^{AG} - w_2^{AB}$. The latter option cannot be worthwhile. In particular, since $x_2^{AG} < x_2^{AB}$, deviating to be a generalist only to pair with a specialist is not better than remaining as a specialist in the first place. We therefore only need consider the first case, where the deviating generalist works alone.

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13 With full employment, the deviating player captures the joint output net of the other player's outside wage. With finite $\lambda$, the possibility of unemployment further affects the wage bargain - see Appendix.
Hence, this world of specialists is an equilibrium iff \( w_2^G \leq w_2^{AB} \), or

\[ x_2^{AB} \geq 2x_2^G \]

These existence conditions can be rewritten in terms of the model’s exogenous parameters, using the production functions, where the condition for specialist stability, \( x_2^{AB} \geq 2x_2^G \), is simply \( mc \geq 1 \), and the condition for generalist stability, \( x_2^{AG} \leq 2x_2^G \), is \( mc \leq \left( \frac{2}{1+m-c} \right)^{\frac{1}{n-1}} \). The equilibria are plotted in Figure 1.

A country where coordination costs are low (i.e. high \( \epsilon \)), or the skill gains from narrow training are large (i.e. high \( \theta \)) will tend towards the specialist equilibrium. A country where coordination costs are high or gains from focused training are modest will tend towards the generalist equilibrium. The failure to develop deep specialists could therefore be viewed as institutional problems, where the important policy parameters are \( m \) and \( c \), as will be discussed below. There are also, however, regions of the parameter space where different equilibria may emerge even if \( m \) and \( c \) are the same, providing the possibility of multiple, pareto-ranked equilibria. In general, a country with specialized skilled workers is \( mc \) times more productive than an economy with generalist skilled workers. Moreover, the ratio of income between generalist and specialist equilibria is potentially unbounded even where both are stable.

**Corollary 1 (Gains from Specialization)** Output in the skilled sector is \( mc \) times larger in a "specialist" equilibrium than in a "generalist" equilibrium. Moreover, the range of potential combinations \( mc \) where both a generalist and specialist equilibria exist is unbounded from above.

**Proof.** See appendix. ■

Note the important roles of (1) coordination costs and (2) task complementarity in supporting a sub-optimal generalist equilibrium. Deviating to become a specialist only to pair with an existing generalist is less appealing when coordination costs are high (i.e. smaller \( c \)) or complementarities of tasks are high (i.e. smaller \( \sigma \)). With sufficient coordination costs or complementarity, \( m \) (and hence \( mc \)) can become unboundedly large, so that the generalist case is stable even though the specialist organization produces unboundedly higher income.
For example, with Leontief task aggregation ($\sigma = 0$), $mc$ can be unboundedly large for arbitrarily small coordination costs.

Lastly, note the role of a "thick market" problem for supporting a robust generalist equilibrium despite large $mc$. The generalist equilibrium is stable to the extent that finding a complementary specialist type is challenging were you to deviate yourself. With finite $\lambda$, the generalist equilibrium is stable to trembles where positive masses of specialists appear, because the search friction impedes easy matching. The convenient case of "full employment", where $\lambda \to \infty$, is the limit of trembling hand perfect equilibria.\footnote{In the limit, the model still features a "needle in a haystack" friction where, although search is extremely rapid ($\lambda \to \infty$) there are so many workers (a continuum) that one cannot expect to find a particular worker in finite time.}

### 3.3.2 The Equilibrium Economy

Given the possible organizational equilibria in the skilled sector, we now consider the influence of this organizational equilibrium on the economy at large. Denote with the superscript $n$ the organizational equilibrium in the skilled sector, where $n = G$ defines the "generalist" outcome and $n = AB$ defines the "specialist" outcome. The equilibrium in the skilled sector will influence the endogenous outcomes in both the skilled and unskilled sectors, including labor allocations, prices, and wages.

The first result concerns wages.

**Lemma 3 (Log-linear Wages).** In any full employment equilibrium

$$w_2^n = w_1^n e^{rS}$$

\begin{equation}
\label{eq:log-linear-wages}
(14)
\end{equation}

**Proof.** See appendix. \blacksquare

This functional form follows from (a) exponential discounting and (b) the opportunity cost of time. Through endogenous decisions to become skilled or unskilled, an identical Mincerian wage structure emerges regardless of the organizational equilibrium in the skilled sector.

Given this wage relationship, we can now pin down prices. In equilibrium, workers in
each sector are paid

\[ w^n_1 = p^n_1 z \]
\[ w^n_2 = p^n_2 z A^n \]

where skilled workers’ productivity depends on their organizational equilibrium,

\[ A^n = 2^{1-\gamma} h \times \left\{ \begin{array}{ll}
1, & n = G \\
mc, & n = AB
\end{array} \right. \]

Therefore, using the wage ratio, the price ratio on the supply side is determined as a function of exogenous parameters\(^{15}\)

\[ \frac{p^n_1}{p^n_2} = A^n e^{-rS} \tag{15} \]

Now consider the demand side to close the model. With CES preferences, aggregate demands are such that

\[ \frac{X^n_1}{X^n_2} = \left( \frac{\gamma}{1-\gamma} \right) \left( \frac{p^n_1}{p^n_2} \right)^{-\varepsilon} \]

Market clearing implies \( p^n_1 X^n_1 = w^n_1 L^n_1 \) and \( p^n_2 X^n_2 = w^n_2 L^n_2 \) so that labor allocations are also pinned down given relative prices

\[ \frac{L^n_1}{L^n_2} = \left( \frac{\gamma}{1-\gamma} \right)^{\varepsilon} \left( A^n e^{-rS} \right)^{1-\varepsilon} e^{rS} \tag{16} \]

where \( L^n_q \) is the measure of people actively working in sector \( q \).\(^{16}\)

Real income per-capita, \( y^n = Y^n / L \), is also pinned down given relative prices\(^{17}\)

\[ y^n = z \left( \gamma^\varepsilon + (1-\gamma)^\varepsilon \left( A^n e^{-rS} \right)^{\varepsilon-1} \right)^{\frac{1}{1-\varepsilon}} \tag{17} \]

and we can define human capital’s contribution to output as \( H^n = y^n L / z \).\(^{18}\)

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\(^{15}\)The price ratio is determined entirely by the supply side because both the skilled and unskilled sectors exhibit constant returns to scale.

\(^{16}\)There are also a number of students who are training in sector 2 and not yet active workers. Given the hazard rate of death \( r \), we have \( e^{rS} L^n_2 \) people currently training and working in sector 2, so that total labor supply is \( L = L^n_1 + e^{rS} L^n_2 \).

\(^{17}\)Real national income \((Y^n)\) is given by \( p^n Y^n = w^n_1 L_1 + w^n_2 L_2 \), where the aggregate price level is \( p^n = \left( \gamma^\varepsilon \left( p^n_1 \right)^{1-\varepsilon} + (1-\gamma)^\varepsilon \left( p^n_2 \right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \). Real per-capita income \((y^n = Y^n / L)\) is

\[ y^n = \frac{w^n_1}{p^n} \left( \frac{L^n_1}{L} + \frac{w^n_2 L^n_2}{w^n_1 L} \right) = \frac{w^n_1}{p^n} \left( \frac{L^n_1}{L} + \frac{w^n_2}{w^n_1} \right). \]

Thus average per-capita income is equivalent to the real wage in the low-skilled sector. This follows in equilibrium because workers’ net present value of lifetime wage income is equivalent at birth. We can alternatively write this in terms of sector 2 wages, since \( w^n_1 = e^{-rS} w^n_2 \).

\(^{18}\)Note that the model, which considers two final goods in consumption, is equivalent to a model
4 Applications and Discussion

This section examines several applications of the model. A primary application uses general equilibrium reasoning to show why human capital can play a much larger role in the world economy than the standard calibration method suggests. A series of further applications show that "knowledge traps" per se may provide a parsimonious interpretation of several stylized facts in the world economy.

4.1 Wages, Prices, and Labor Allocations

When people choose to be highly educated, any excessive wage gains to the highly-educated can be arbitrated away by an increase in the supply of such workers. In the model, this choice problem generates the log-linear "Mincerian" wage structure and pins the skilled wage premium to the interest rate, as in (14).19

The key implication is that two countries can have vastly different mappings between schooling duration and skill, and yet have identical wage returns to schooling in equilibrium. In fact, skill differences are hidden by the wage structure. It is prices and labor supply that shift to ensure the equilibrium wage-schooling relationship.

In particular, from (15), prices adjust in the model such that

$$\frac{p_{1AB}}{p_{2AB}} = \frac{p_{1}}{p_{2}} \quad (18)$$

This result says that the low skilled-good will be cheaper in the poor country. This feature of the equilibrium may be appealing, as it provides a Balassa-Samuelson effect in relative prices (e.g. Harrod 1933, Balassa 1964, Samuelson 1964). The model may thus inform a central observation in development, which is that certain goods are relatively cheap in poor countries, an effect that motivates the need for PPP price corrections when comparing real

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19 Note that this simple perspective suggests a positive correlation between interest rates and returns to schooling across countries. In fact, the literature finds both (a) higher interest rates in poor countries (e.g. Banerjee and Duflo 2005) and (b) higher rates of return to schooling in poor countries (Psacharopoulos 1994). Also see footnote 6.
income across countries. The knowledge trap model provides an endogenous basis for this phenomenon, where low-skilled goods (e.g., haircuts) are relatively cheap in a poor country because low skill is relatively abundant there.

Meanwhile, from (16) and (18), labor supply adjusts such that

\[
\frac{L_1^{AB}/L_2^{AB}}{L_1^G/L_2^G} = (mc)^{1-\varepsilon}
\]

(19)

Together, the equilibrium price and labor supply adjustments in (18) and (19) decouple the wage returns to schooling from the skill-gains from schooling. These results pose substantial challenges to the standard macro-Mincer calibration method, which we turn to next.

4.2 Calibration Evidence

Many analyses have concluded that human capital plays a relatively modest role in explaining the wealth and poverty of nations, leaving residual variation in total factor productivity as a major explanation (see Caselli 2005 for a review). This conclusion is reached using the "macro-Mincer" method to account for human capital (Klenow and Rodriguez-Clare 1997, Hall and Jones 1999). In this method, each economy’s human capital is calculated as the labor supply at each level of education, weighted by the average wage at that education level; i.e., in this paper’s notation, \(H_{Mincer}^n = L_1^n + e^{\varepsilon S}L_2^n\). The returns to education are taken as \(e^{\varepsilon S}\), and countries differ in their human capital to the extent that they have more or less educated workers. To see why this method is problematic, first consider a simple example.

**Example 1** With Cobb-Douglas aggregation (\(\varepsilon = 1\)), it follows from (19) that \(L_1^{AB}/L_2^{AB} = L_1^G/L_2^G\), so that the labor allocation does not vary with the skill gains from education, \(mc\).

Therefore, the macro-Mincer human capital stock calculation, \(H_{Mincer}^n = L_1^n + e^{\varepsilon S}L_2^n\), would

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20 Classic explanations for this price phenomenon imagine exogenous cross-country differences in technology (Balassa 1964, Samuelson 1964) or factor endowments (Bhagwati 1984).

21 Note that the model considers price differences between a final good completely produced through skilled labor and a final good completely produced through unskilled labor. In looking at microeconomic price data, one would consider input mixes of skilled and unskilled labor away from these extremes, which would attenuate the observed price differences in final goods. In a generalization of the model, the observable price effect would appear such that goods that use unskilled labor relatively intensively would be relatively expensive in rich countries.

22 Some calibrations also allow \(e^{\varepsilon S}\) to vary across countries, based on observed educational returns. To focus on the core methodological issue, the following theoretical results will abstract from variation in \(r\). Calibration evidence discussed below will incorporate such variation as well. See also footnote 6.
not vary with the skill gains from education. Mincerian accounting would therefore suggest no role for human capital, even should human capital explain unboundedly large income differences across countries.23

Figure 2: Equilibrium Wage-Schooling Relationships

The general intuition can be stated as follows. With downward sloping demand for different labor classes, countries that are very good at producing high skill will find that goods produced by low-skill workers are scarce, which drives up low-skilled wages. In particular, with relative wages pinned down by the discount rate, as in (14), workers allocate themselves so that the percentage wage gains for skilled and unskilled workers rise or fall in equal proportion. Wages are Mincerian in each country, but this within-country equilibrium does not inform human capital differences across countries. Rather, the wage-schooling relationship shifts vertically depending on the skilled equilibrium. This is shown in Figure 2.

23Moreover, a regression of per-capita income on average schooling duration would also show no relationship. With Cobb-Douglas preferences (\( \varepsilon = 1 \)) the average schooling in a population is

\[
s^n = S\frac{L^n}{L} = (1 - \gamma)Se^{-\gamma S}
\]

a constant independent of which equilibrium is attained. For average schooling to be positively associated with income (which it is), we require the elasticity of substitution between skilled and unskilled labor to be greater than 1. Then countries with high quality skilled-labor (i.e. specialization) will see an endogenous increase in the supply of such skilled workers.
2 for the Cobb-Douglas case, in which price adjustments fully offset productivity differences, requiring no labor adjustment.

Because Mincerian accounting rules out the scarcity effect on unskilled wages, it will in fact systematically understate human capital differences across countries given the observed allocations of labor. Define the ratio of actual human capital differences across countries to the Mincerian calculation of these differences as

\[ R_H = \frac{H^{AB}/H^G}{H_{Mincer}^{AB}/H_{Mincer}^G} \]

**Lemma 4** (Mincer as Lower Bound) \( R_H \geq 1 \) for all \( \varepsilon \in [0, \infty] \). Moreover, \( \lim_{\varepsilon \to 1} R_H = \infty \) for a given labor allocation \( L_1^G/L_1^{AB} \neq 1 \).

**Proof.** See appendix. \( \blacksquare \)

This lemma states that Mincerian human capital accounting is only a lower-bound on the actual human capital differences across countries. The lemma further says that the magnitude of the underestimate may be arbitrarily large, depending on the elasticity of substitution between skilled and unskilled labor. The reasoning follows from (19). For example, fixing the observed labor allocation, \( (L_1^{AB}/L_2^{AB}) / (L_1^G/L_2^G) < 1 \), reducing \( \varepsilon \) towards 1 calls for greater \( mc \), which makes for a larger human capital difference between these countries.\(^{24}\) Put another way, once educational attainment is seen as a choice problem, it is natural to ask why so many more workers seek higher education in rich countries. The larger supply of such workers is reconciled in equilibrium by larger skill gains from schooling. As \( \varepsilon \) falls, the human capital differences must increase to compensate if we are to explain the observed supply of skilled workers.

It is clear that the elasticity of substitution between skilled and unskilled labor becomes a key parameter in assessing the role of human capital. The literature suggests values of \( \varepsilon \in [1, 2] \).\(^{25}\) Figure 3 provides estimates of \( R_H \), fixing the observed labor allocation and exploring the effect of \( \varepsilon \). I use Barro-Lee educational attainment data to describe the labor allocations, counting the unskilled, \( L_1^n \), as those with no more than secondary school education and comparing labor allocations in the United States to the mean labor allocation

\(^{24}\)In practice, we see \( (L_1^{AB}/L_2^{AB}) / (L_1^G/L_2^G) < 1 \), which is consistent with \( \varepsilon > 1 \).

\(^{25}\)See, e.g., the review by Katz and Autor (1999).
in the poorest 10% of countries. Taking the Katz and Murphy (1992) estimate of $\varepsilon = 1.4$, we see that macro-Mincer calibration would understate human capital differences by a factor of 2.7. As $\varepsilon$ falls to 1.2, human capital differences would be understated by a factor of 7.5.

Figure 3: Actual vs. Macro-Mincer Calculations of Human Capital Differences

This simple calibration can be pushed further. In particular, one may ask whether residual TFP differences are still needed to explain cross-country income differences once these skilled productivity differences are accounted for. In fact, Caselli and Coleman (2006) provide such a calibration, using realistic values of $\varepsilon$, although their interpretation is different. Caselli and Coleman (2006) calibrated separate productivity terms for skilled ($s$) and unskilled ($u$) workers across countries using the production function

$$y = k^\alpha \left[ (A_u L_u)^\rho + (A_s L_s)^\rho \right]^{\frac{1}{\rho - 1}}$$

which is the analogue of (4) in this paper with the addition of physical capital, $k$. They find an enormous productivity advantage of skilled workers ($A_s$) in rich countries while the productivity of unskilled workers ($A_u$) is no higher there, leaving little need for residual TFP differences. This calibration is closely consistent with the predictions of the knowledge

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26 The educational attainment data comes from Barro and Lee (2001). The poorest countries are determined using Penn World Tables v. 5.6 (Summers and Heston 1991). These are the same data sources used in Caselli and Coleman (2006), which is discussed below.

27 They calculate $L_u$ and $L_s$ by aggregating workers within lower schooling ranges ($L_u$) and upper schooling ranges ($L_s$) with perfect substitutes assumed within each range.

28 In their preferred specification, Caselli and Coleman further argue that unskilled workers are actually less productive in rich than poor countries. That is, with low enough $\varepsilon$, the productivity of the educated
trap model: enormous productivity advantages in rich countries that are limited to skilled workers.\textsuperscript{29} By contrast, Caselli and Coleman argue, citing the Mincerian-based evidence, that the skilled productivity differences are too large to be explained as human capital, and therefore view $A_s$ as a residual productivity advantage. However, a knowledge trap can support the massive productivity advantages that such a calibration suggests, and the Mincerian evidence, as seen through Lemma 4, does not in fact limit the human capital differences across countries. Since the productivity gains appear limited to the educated, it seems natural to imagine that education itself provides some critical advantage. I will next consider evidence from immigration, to help assess whether these differences in skilled workers’ productivity really are independent of the individual worker’s human capital.

4.3 Immigrant Wages and Occupations

An alternative approach to assessing human capital’s role in cross-country income differences is to examine what happens when workers trained in poor countries are placed in rich countries. If human capital differences were critical, it is argued, then such workers should experience significant wage penalties in the rich country’s economy. Noting that immigrants from poor to rich countries earn wages broadly similar to workers in the rich country, authors have concluded that human capital plays at most a modest role in explaining productivity differences across countries (Hendricks 2002). However, this estimation approach as implemented faces the same issue as standard accounting approaches, by limiting the effect of scarce labor supply.\textsuperscript{30}

must rise so much across countries that it overexplains income differences and thus requires TFP to be negatively correlated with income. More generally, their result for unskilled workers is sensitive to the calibration parameters and how one defines "unskilled worker". If one classifies such workers as having less than high school or less than college-level education, then unskilled workers in their calibration become mildly more productive in rich countries. What appears highly robust about their specification is that skilled workers have enormous productivity advantages in rich countries, as is consistent with the knowledge trap model.

Another important calibration is Mannelli and Sheshadri (2005), who estimate human capital by considering it as an endogenous choice variable. They find large quality differences in human capital across countries that, once accounted for, require little or no TFP differences. Their estimation suggests large advantages in the quality of education in rich countries even at entrance to primary school. This skill advantage at very low-education levels differs from the "knowledge trap" approach, which emphasizes difference that are limited to the highly skilled and differs from the Caselli and Coleman calibration, where skill differences exist only among those with more education. Mannelli and Sheshadri’s imputed quality differences at all skill levels follow because their model does not allow for the relative price effects that occur when skilled and unskilled workers produce different intermediate or final goods.

The main estimates in Hendricks (2002) assume workers output at different skill classes are perfect substitutes thus eliminating any effect of scarcity on the wages of the unskilled. To the extent calibrations
The knowledge trap model predicts that low-skilled immigrants, who are the majority of immigrants, will enjoy (a) much higher real wages than they left behind and (b) face no wage penalty in the rich economy vis-a-vis other unskilled workers. Indeed, why would education matter for the uneducated, working as taxi drivers, retail workers, and farm hands? Wage gains follow naturally when the low-skilled immigrant moves to a place where his labor type is relatively scarce. The over-riding role of scarcity, rather than productivity, for unskilled workers is corroborated by the calibration discussed above. The potentially more informative implications of the knowledge trap model lie among skilled immigrants.

**Corollary 2 (Immigrant Workers)** An unskilled worker who migrates from a poor to a rich country will earn a higher real wage. The skilled generalist who migrates from a poor to a rich country will work in the unskilled sector and earn the unskilled wage, which may provide more or less real income than staying at home.

**Proof.** See Appendix. ■

Skilled immigrants, as generalists, are unable to find local specialists willing to team with them. Moreover, they won’t work alone; the specialized equilibrium of the rich country raises the low-skilled wage enough to make unskilled work a more enticing alternative to the immigrant generalist than using his education. Hence, for example, we can see immigrant Ph.D.’s who drive taxis.

Friedberg (2000) demonstrates that the source of education does matter to immigrant wages, but the literature does not appear to have looked explicitly at higher education. Descriptive facts can be assembled however using census data. I divide individuals in the 2000 U.S. Census into three groups: (1) US born, (2) immigrants who arrive by age 17, and (3) immigrants who arrive after age 30. The idea is that those who immigrated by age 17 likely received any higher education in the United States, while those who immigrated after age 30 likely did not.

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[31] The data and methods are detailed further in the Appendix.
Figure 4a shows two important facts. First, controlling for age and English language ability, the location of higher education appears to matter. Among highly educated workers, those who immigrate after age 30 experience significant wage penalties, of 50% or more. Meanwhile there is no wage penalty if the immigrant arrived early enough to receive higher education in the United States. Second – and conversely – the location of high-school education does not matter. Wages do not differ by birthplace or immigration age for workers with an approximately high-school level education. Hence, the location of education matters for high skill workers but not so much for low skill workers, as the "knowledge trap" suggests.\(^{32}\)

Figure 4b considers related evidence based on occupation type. To construct this graph, each occupation in the census is first categorized by the modal level of educational attainment for workers in that occupation. For example, taxi drivers typically have high school degrees, physicians typically have professional degrees, and physicists typically have Ph.D.’s. The figure shows the propensity for workers with professional or doctoral degrees to work in different occupations. We see that US born workers and early immigrants have extremely similar occupational patterns. However, late immigrants with professional or doctoral degrees have a much smaller propensity to work in occupations that rely on such degrees. Instead, they tend to shift down the occupational ladder into jobs that require only college degrees and even, to a smaller extent, into occupations typically filled by those with high school or less education. This pattern is further reflected in Figure 3a, which shows that late immigrants with professional or Ph.D. degrees earn average wages no better than a locally educated college graduate.

This evidence is consistent with the "knowledge trap" model but inconsistent with a pure technology story, in which the location of education would not matter. More broadly, the evidence is consistent with the idea that human capital differences across countries exist primarily among the highly educated, as suggested independently by the calibration

\(^{32}\)Note also that immigrants with high school or less education have extremely similar wage outcomes regardless of immigration age. This further suggests that early-age immigrants are an adequate control group for late-age immigrants, highlighting that differing labor market outcomes only occur at higher education levels. Lastly, it is clear that very-low education immigrants (e.g. primary school) do significantly better than very-low educated US born workers. Such limited education is very rare among the US born and likely reflects individuals with developmental difficulties, which may explain that wage gap.
discussed above.

4.4 Poverty Traps and Education

The theory in this paper emphasizes that educational choices are not sufficiently described by schooling duration. Rather, the content of education - the division of knowledge across workers - is critical. If workers organize themselves to learn narrow, deep knowledge, and work in teams, then they can access the knowledge frontier. But workers may organize as generalists, avoiding reliance on teamwork and failing to embody frontier knowledge. This section further considers challenges to collective skill improvement from the perspective of the model.33

4.4.1 The Quality of Higher Education

Income differences across countries may persist if countries are in different regions of Figure 1. Countries with $mc < 1$ will have shallow knowledge and remain in poverty. This may occur if acquiring deep skills is hard in poor countries ($m$ is small). One can think of $m$ as a policy parameter, where, for example, $m$ increases through public investment in higher education. Small $m$ also follows naturally if knowledge acquisition is limited by local access to others with deep skill - i.e. expert teachers. For example, becoming skilled at protein synthesis will be difficult without access to existing skilled protein synthesists: their lectures, advice, the ability to train in their laboratories, etc. In this setting, we can imagine a simple, further type of knowledge trap. If we write $m^n$, where $m^G < m^{AB}$, then countries that start in the generalist equilibrium will remain there if $m^G c < 1$.

Escaping such a trap involves importing skill from abroad to train local students or sending students abroad and hoping they will return. Both approaches face an incentive problem however, since those with deep skills will earn higher real wages by remaining in the rich country. The model thus suggests a "brain drain" phenomenon.

Corollary 3 (Brain Drain) Once trained as a specialist in the rich country, one will prefer

33 The following discussion focuses on poverty-inducing mechanisms that follow from the supply of human capital. Coordination costs (the parameter $c$) may also be important, especially should coordination costs be more severe in poorer countries. Lastly, market size may be an important limiting factor in specialization. This last possibility may also bear consideration in confronting data but is not incorporated in the model for focus and brevity.
to stay.

Proof. See appendix. ■

Specialists in rich countries prefer to stay because they can work with complementary specialists there and thus earn higher wages. Hence students who migrate to the U.S. for their Ph.D.’s face real wage declines if they go home - even though they are scarce at home. Related, it is clear that students from rich countries do not migrate to developing countries for their education, even though university and living expenses are considerably lower. This may further suggest that the quality of education is low.34

This result suggests that wage subsidies or other incentives may be required to attract skilled experts to the poor country and improve local training.

4.4.2 The Coordination of Higher Education

Even if poor countries can produce high-quality higher education, there is still an organizational challenge. Countries may be in the middle region of Figure 1, facing the same parameters \( m \) and \( c \) but sitting in different equilibria. Here a country cannot escape poverty without creating thick measures of specialists with complementary skills.35 This may be hard. Any intervention must convince initial cohorts of students to spend years in irreversible investments as specialists, which would be irrational if complementary specialists were not expected. Hence we need a "local push".36,37 Yet it is not obvious what institu-

34 I thank Kevin Murphy for pointing this out.
35 One could alternatively construe the "trap" as being a deterministic function of the initial conditions, where a sufficient mass of specialists of each type creates a stable, high income state, while insufficient supply of specialists creates a stable, low income state.
36 Note that the type of trap allowed in the model differs from poverty traps that envision aggregate demand externalities (e.g. Murphy et al., 1989). Rather, knowledge traps can be overcome locally, when workers achieve greater collective skill. A challenge for aggregate demand models is that many poor economies are quite open to trade or have large GDP on their own despite low per-capita GDP, so it is unclear that aggregate demand is a credible obstacle. Meanwhile, booms are often local, whether it is city-states like Hong Kong or Singapore, or cities within countries, like Bangalore, Hyderabad, and Shenzhen, which have led growth in India and China. Yet such booms are also rare, which suggests that local coordination problems are themselves not trivial to overcome.
37 Some authors see such coordination failures as easily solved due to trembling hand type arguments (e.g. Acemoglu 1997). However, there are several reasons to think that small "trembles" are unlikely to undo a generalist equilibrium. First, we are considering many years of education for an individual, so that a "tremble" must be rather large. Second, while we consider two tasks for simplicity, there may be \( N > 2 \) tasks needed for positive output, which would then require simultaneous trembles over many specialties. Third, with greater search frictions in the market (smaller \( \lambda \)), trembles must occur over a large mass of workers. Fourth, in tradeable sectors, one must leap to the skill equilibrium of the rich countries to compete internationally - small skill trembles won’t suffice.
tions have the incentives or knowledge to coordinate such a push. A firm may have little incentive to make these investments when students can decamp to other firms. Public institutions may not produce the right incentives either. Developing deep expertise requires time, so that the fruits of educational investments may not be felt for many years, depressing the interest of public leaders (or firms), who may have short time horizons. Even if local leaders wish to intervene, it may be challenging to envision the set of skills to develop, especially if there are many required skills and deep knowledge does not exist locally. These difficulties suggest a need for "visionary" public leaders. They also suggest an intriguing role for multinationals in triggering escapes from poverty.

4.4.3 Multinationals and Poverty Traps

Intra-firm trade can allow for production teams that span national borders, and I discuss here how a multinational can play a unique role in helping countries escape poverty.

**Corollary 4** (*Desirable Cheap Specialists*) A firm of specialists in a rich country would hire specialists in poor countries, if they could be found.

**Proof.** See appendix. ■

This result follows because the skilled wage in the poor country is held down by the Mincerian wage equilibrium, making a specialist there attractive. Hence, production would shift to incorporate a skilled specialist in the poor country if such a type existed. But now we have a cross-border coordination problem. A multinational will only be able to find these specialists if they exist in sufficient measure, and no one in the poor country will want to become such a specialist unless the multinational will be able to find them.

The interesting aspect is that a multinational allows the local educational institutions to avoid producing all required specialities locally. The multinational provides the complementary worker types from abroad. For example, in Hyderabad, governor Naidu both subsidized a vast expansion in engineering education and personally convinced Bill Gates

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38 Contracts may help here, but labor contracts that prevent workers from departing a firm (i.e., in an extreme form, slavery) are typically illegal. Labor market frictions may allow firms to do some training if frictions give the firm some monopsony power (e.g. Acemoglu and Pischke 1998). Still, it is clear that Ph.D.’s are produced in educational institutions, not in firms.
to employ these workers in Microsoft's global production chain, so that computer programmers in Hyderabad now team with other skilled specialists in advanced economies. Here, the "visionary" leader need not recreate Microsoft but simply produce a sufficient quantity of one specialist type that Microsoft will hire. To the extent that a thick supply of this specialist type triggers complementary specialization locally, the local economy may escape from the trap broadly.

4.5 Generalizations: Trade, Growth, and Skill Bias

The emphasis on the division of labor among skilled workers also suggests natural generalizations to international trade and growth contexts, with the possibility to inform (i) comparative advantage, (ii) skill-biased technical change and (iii) cross-country income divergence.

First, knowledge traps may provide a useful perspective on comparative advantage. The factor endowment model of trade, Heckscher-Ohlin, explains why Saudi Arabia exports oil but is famously poor at predicting trade flows based on capital and labor endowments – the so-called "Leontief Paradox" (Leontief 1953, Maskus 1985, Bowen et al. 1987, etc.). International trade analysis, much like cross-country income analysis, has therefore relied on substantial residual productivity terms to explain the empirical patterns (e.g. Trefler 1993, 1995, Harrigan 1997).

With knowledge traps, the rich country has a comparative advantage in the skilled good while the poor country has a comparative advantage in the low-skilled good. Yet these comparative advantages - based in the division of labor - won’t appear in standard models.


40 With only two types of specialists, the emergence of one type in the poor country triggers the emergence of the other, and the poor country will become rich. With more than two specialist types, the emergence of one type may not inspire the local creation of the other types. Here, a multinational can continue to employ a narrow type of skilled specialists in one country without triggering a general escape from poverty. Here we will see both outsourcing and persistently "cheap engineers".

41 In terms of the model, we can consider two small open economies who can trade both goods 1 and 2. With world prices, $p_1/p_2$, such that $p_{1A}^{1B} / p_{2A}^{1B} < p_{1A} / p_{2A} < p_{1A}^{1B} / p_{2A}^{1B}$ the country in the generalist equilibrium exports the low-skilled good (1) while the country in the specialist equilibrium exports the high skilled good (2).
calculations of labor endowments. Knowledge traps may thus suggest a human-capital interpretation of residual productivity terms, where rich countries are net exporters of skilled goods not simply because they have more skilled workers, but because their skilled workers have so much more collective skill.

The emphasis on the division of labor among skilled workers also suggests natural generalizations to a growth context. First, consider the apparent empirical tendency toward skill-biased technical change (see, e.g. Katz and Autor 1999). If growth is associated with the creation of new ideas and a consequent expansion of frontier knowledge, which can be modeled as an increase in $\mu$, then growth is intrinsically skill-biased. Second, to the extent that workers in poor countries, organized for general knowledge, do not access this deepening set of ideas, cross-country divergence in per-capita income becomes the natural outcome empirically, as is the usual case (Jones 1997, Pritchett 1997). In fact, advanced economies appear to accumulate an enormous amount of codified knowledge. As one measure, the ISI Web of Science indexes 1.3 million science and engineering journal articles published by US knowledge workers from 2001 to 2005. Moreover, microevidence suggests that these advanced ideas are increasingly accessed by specialized skilled workers working in teams (Jones 2009, Wuchty et al. 2007). Divergence follows naturally to the extent that such new knowledge is economically important and yet workers in poor countries, for any of the reasons detailed above, do not learn these specialized ideas.

5 Conclusion

This paper offers a human-capital based interpretation of several phenomena in the world economy and therefore a possible guide to core obstacles in development. The model shows how differences in the organization of labor may (a) persist across economies yet

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42 For example, the degree of specialization won’t appear in designations like "professional" or "highly educated" worker, which can explain why attempts to save Heckscher-Ohlin through finer-grained classifications of labor endowments have failed (e.g. Bowen et al. 1987).

43 In an endogenous growth framework, some fraction of skilled workers would produce productivity enhancing ideas that lead to growth in $\mu$. Such accumulation of knowledge may require innovators to become more specialized along the growth path, so that the number of tasks at the frontier (2 in this model) becomes endogenous and increases with time. See Jones (2009) for such a growth model, as well as empirical evidence that knowledge workers in the U.S. become increasing specialized with time and work in larger teams.
(b) not appear in the wage structure. Together, these ideas show how standard human capital accounting may severely underestimate cross-country skill differences. Building from a simple conception of human capital, the model may provide an integrated perspective on cross-country income differences, relative wages, price differences, migrant behavior, poverty traps, and other phenomena in a way that appears broadly consistent with important facts.

By expanding the human capital decision to incorporate the content of education, rather than simply its duration, the theory may substantially amplify human capital’s role in understanding development. The division of labor provides a natural mechanism for individuals to solve the basic problem that frontier knowledge in an economy is too much for any one individual to know. By suggesting specific mechanisms that disrupt collective specialization, the theory further suggests tangible, micro-empirical avenues for future work. Accounting directly for the content dimension of education will not be easy. While schooling duration is relatively straightforward to observe, differences in the content of education and the division of knowledge are harder to define and measure. This paper suggests that building such data, while difficult, is an important area for future work.

This paper also speaks directly to a long-running debate over the roles of "human capital" and "technology" in explaining income differences across countries. I close by further considering this distinction. Much existing literature imagines human capital and ideas as distinct inputs into a production function and, using macro-Mincer accounting, suggests a modest role for human capital, pushing education toward the periphery in understanding key issues in development. What is called technology, the residual, has consequently occupied a central position and is often imagined as a set of techniques, methods, facts, models, et cetera that impact production. At root, this paper attempts to reconfigure this debate and, in some sense, sidestep it. This paper shows how human capital may play a central role. At the same time, this paper embraces the critical importance of ideas. People are born with empty minds, and education is seen as the process of acquiring knowledge. Rather than conceiving technology as a distinct, disembodied input to production, this paper imagines that ideas must first be embodied in people. It is thus the emphasis on embodiment, rather than the role of "ideas", that distinguishes this paper from other approaches. In this
perspective, technological progress, the expansion of the set of ideas, may well drive economic development, but here too the effects of knowledge will likely be felt – and understood – not in contest with human (or physical) capital, but through its embodiment into the people and machines that actually produce things.
6 Appendix

Proof of Lemma (Matching Rules)

Proof. The lemma follows from five intermediate results.

(1) Workers are never willing to match with their own type \((k \notin \Omega^k \forall k)\).

In equilibrium, all skilled types have some \(V > 0 \). A type \(k \) never matches with type \(k \) if \(V^{kk} < V \). For As or Bs, the joint output when teaming with one’s own type is zero. Hence \((a)\) implies \(V^{AA} = V^{BB} = \frac{1}{2}V < V \). Therefore, neither As or Bs will match with their own type. For Gs, \((a)\) implies \(V^{GG} = \frac{1}{2}w_2^{GG}/r + \frac{1}{2}V \). Noting that \(V \geq w_2^G/r \) (G’s income if he never matches, from \((6)\)) and that \(w_2^{GG} < w_2^G \) (GG matches provide no skill advantage but incur a coordination penalty), it follows that \(V^{GG} < V \). Hence no type will match with her own type.

(2) Type \(k \) is willing to match with type \(j \) iff type \(j \) is willing to match with type \(k \) \((k \in \Omega^j \iff j \in \Omega^k)\).

A type \(k \) is willing to match with type \(j \) if \(V^{kj} \geq V \). With the Nash Bargaining Solution and common \(V \) in equilibrium, it follows that \(V^{kj} = V^{jk} \). Hence \(k \in \Omega^j \iff j \in \Omega^k \).

(3) As are willing to match with Gs iff Bs are willing to match with Gs \((G \in \Omega^A \iff G \in \Omega^B)\).

As are willing to match with Gs if \(V^{AG} \geq V \). In equilibrium, \(V^{AG} = V^{BG} \). This follows from \((8)\) because with (a) common \(V \) and (b) \(x_2^{AG} = x_2^{BG} \), Nash Bargaining implies \(w_2^{AG} = w_2^{BG} \). Hence, \(V^{AG} \geq V \iff V^{BG} \geq V \), so that As are willing to match with Gs iff Bs are willing to match with Gs.

(4) If an A or B is willing to match with Gs, then the A or B is also willing to match with the complementary specialist type \((G \in \Omega^A \Rightarrow B \in \Omega^A \) and \(G \in \Omega^B \Rightarrow A \in \Omega^B)\).

If As are willing to match with Gs, then \(V^{AG} \geq V \) and \(w_2^{AG} = \frac{1}{2}p_2x_2^{AG} \). But \(w_2^{AB} = \frac{1}{2}p_2x_2^{AB} \geq \frac{1}{2}p_2x_2^{AG} = w_2^{AG} \) and hence, from \((8)\), \(V^{AB} \geq V^{AG} \). Hence A will also be willing to match with Bs: \(G \in \Omega^A \Rightarrow B \in \Omega^A \). A symmetric argument demonstrates that \(G \in \Omega^B \Rightarrow A \in \Omega^B \).

(v) As and Bs must match \((\Omega^k \neq \emptyset) \) for \(k = A, B\).

This result follows because tasks A and B are gross complements in production. Hence, As or Bs who work in isolation do not produce positive output and earn no income.\(^{44}\)

With these five properties, the only remaining, possible equilibrium matching policies are \(\{\Omega^A, \Omega^B, \Omega^G\} = \{\{B\}, \{A\}, \{\emptyset\}\} \) or \(\{\Omega^A, \Omega^B, \Omega^G\} = \{\{B, G\}, \{A, G\}, \{A, B\}\} \).

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\(^{44}\)Gross complements, \(\sigma \leq 1 \), is a (strong) sufficient condition for this result but is not necessary. If \(\sigma > 1 \), then positive production becomes possible when a specialist works alone. Nevertheless, it can be shown that, with \(\sigma > 1 \), As and Bs still prefer to match in equilibrium so long as \(c > (1/2)^{1/r} \), i.e. matching occurs as long as coordination costs are not too severe \((c \) is not too small\) or the elasticity of substitution between tasks is not too great \((\sigma \) is not too large\). The paper focuses on the case of \(\sigma \leq 1 \) to enhance tractability, brevity and intuition.
Proof of Lemma (Balanced Specialists)

Proof. (I) First consider the case where $L^A > 0$ and $L^B > 0$.

1. In equilibrium $V^A = V^B$. Let $\{\Omega^A, \Omega^B, \Omega^G\} = \{\{B, G\}, \{A, G\}, \{A, B\}\}$. Equating $V^A = V^B$ using (11) implies $0 = [\Pr(A) - \Pr(B)] \left[ \frac{w_2^{AB}}{w_2^G} + \frac{1}{\lambda} \Pr(G) \left( w_2^{AB} - w_2^G \right) \right]$. Hence $\Pr(A) = \Pr(B)$ in equilibrium. If, alternatively, $\{\Omega^A, \Omega^B, \Omega^G\} = \{\{B\}, \{A\}, \emptyset\}$, it follows directly from $V^A = V^B$ using (11) that $\Pr(A) = \Pr(B)$.

2. Next we show that $\Pr(A) = \Pr(B)$ implies $L^A = L^B$. The probability of meeting a worker of type $j$ is $\Pr(k) = \frac{L^k}{L_p}$. To analyze $L_p^k$, the mass of type $k$ workers who are unmatched, note that workers enter and leave the matching pool by four routes. Workers enter the matching pool either because (a) they finish their studies or (b) their partner dies. Workers exit the pool either by (c) dying themselves or (d) pairing with other workers. These flows are defined as follows.

(a) There are $L^k$ people in the population of type $k$. In steady state, they are born at rate $rL^k$ and survive to their graduation with probability $e^{-rs}$. The rate at which new graduates enter the matching pool is therefore $rL^ke^{-rs}$.

(b) There are $L^k e^{-rs} - L_p^k$ type $k$ workers currently matched in teams. Since workers die at rate $r$, the rate of reentry into the matching pool is $r \left( L^k e^{-rs} - L_p^k \right)$.

(c) Type $k$ workers in the matching pool die at rate $rL_p^k$.

(d) Type $k$ workers in the matching pool match other unpaired workers at rate $\lambda L_p^k \sum_{j \in \Omega^b} \Pr(j)$.

Summing up these routes in and out of the matching pool, we have

$$\dot{L}_p^k = 2rL^k e^{-rs} - 2rL_p^k - \lambda L_p^k \sum_{j \in \Omega^b} \Pr(j)$$ \hspace{1cm} (20)

In steady-state, $\dot{L}_p^k = 0$, which implies that $L_p^k = \left[ 1 + \frac{1}{\lambda} \sum_{j \in \Omega^b} \Pr(j) \right]^{-1} e^{-rs} L^k$. The ratio of probabilities for an A and B meeting is therefore

$$\frac{\Pr(A)}{\Pr(B)} = \frac{1 + \frac{1}{\lambda} \sum_{i \in \Omega^b} \Pr(i) L^A}{1 + \frac{1}{\lambda} \sum_{i \in \Omega^a} \Pr(i) L^B}$$ \hspace{1cm} (21)

It then follows directly, given the allowable matching rules defined by Lemma 1, that $\Pr(A) = \Pr(B)$ implies $L^A = L^B$.

(II) Second, consider the case where $L^A > 0$ and $L^B = 0$.

We rule this case out by contradiction. Since As earn zero if they work alone, As must match in equilibrium. Hence an equilibrium with $L^A > 0$ and $L^B = 0$ would require $L^G > 0$ with As and Gs matching. In equilibrium, common $V$ then implies from (11) that

$$rV = \frac{\lambda \Pr(G) \left[ \frac{p_2 x_2^{AG}}{1 + \frac{1}{\lambda} \Pr(G)} \right]}{1 + \frac{1}{\lambda} \Pr(G)}$$ \hspace{1cm} (22)

Now consider a player who deviates to type B. This player could choose to match only
with Gs and earn the same $V$. Hence, when meeting an A, the B deviator would have no worse outside option than $V$. Hence, if B chose to match with an A, $w^{BA} \geq \frac{1}{2}p_2x_2^{AB}$.

Hence if the B deviator chose to match with As or Gs then

$$rV_B \geq \frac{\lambda}{2}\Pr(A)\frac{1}{2}p_2x_2^{AB} + \frac{\lambda}{2}\Pr(G)\frac{1}{2}p_2x_2^{AG} \geq \frac{1}{2}\Pr(A) + \frac{1}{2}\Pr(G) > rV$$

where the strict inequality follows because $x_2^{AB} > x_2^{AG}$. Therefore, by contradiction, there is no equilibrium with $L^A > 0$, $L^B = 0$. By a symmetric argument there is no equilibrium where $L^A = 0$, $L^B > 0$.

Hence in equilibrium the model must feature $L^A = L^B$. 

Proof of Proposition (Knowledge Traps)

Proof. Consider the "generalist", "specialist", and "mixed" cases in turn.

(I) The "generalist" case, where $\{L^A, L^B, L^G\} = \{0, 0, L^A\}$.

In this case,

$$rV = w^G$$

where $w^G = p_2x_2^G$.

Now consider whether an (infinitesimal) individual would deviate to a specialist type, say type $A$. The type $A$ worker earns $w^A = 0$ when working alone. Hence from (11)

$$rV^A = \left[\frac{\lambda}{2\eta} / (1 + \frac{\lambda}{2\eta})\right] w^A$$

where $w^A = \frac{1}{2}p_2x_2^A - \frac{1}{2}r(V - V^A)$ from the Nash Bargaining Solution. Solving these to eliminate $w^A$ gives

$$rV^A = \frac{\lambda}{2 + \frac{1}{2\eta}} (p_2x_2^A - w^G)$$

Workers won’t deviate if $rV \geq rV^A$, or (after some algebra)

$$x_2^A \leq 2x_2^G \left(1 + \frac{2r}{\lambda}\right)$$

If this condition holds, the "generalist" case is an equilibrium. With full employment, $\lambda \to \infty$, the "generalist" case is an equilibrium iff $x_2^A \leq 2x_2^G$.

(II) The "specialist" case, where $\{L^A, L^B, L^G\} = \{L^A, \frac{1}{2}L^B, \frac{1}{2}L^B, L^G\}$.

In this case,

$$rV = \frac{\lambda}{2 + \frac{1}{2\eta}} w^{AB}$$

where $w^{AB} = \frac{1}{2}p_2x_2^{AB}$.

If a player deviates to type $B$ and chooses $\Omega^B = \{G\}$, then $rV_B = \frac{\lambda}{2}\Pr(G)w^{BG}$. Nash Bagaining implies $w^{BG} = \frac{1}{2}p_2x_2^{BG} - \frac{1}{2}r(V - V^B)$. With $V$ given in (22), and noting $x_2^{BG} = x_2^{AG}$, it then follows that $rV_B - rV = 0$. In this setting, deviating to be a player of type $B$ provides the same income as the existing As receive.
The "specialist" case is an equilibrium if \( rV \geq rV^G \). If you deviate to be a generalist and don’t match with specialists, then \( rV^G = w_2^G = p_2x_2^G \). If you do match with specialists, then \( rV^G = (w_2^G + \frac{\lambda}{2}w_2^{GA})/(1 + \frac{\lambda}{2r}) \), where \( w_2^{GA} = \frac{1}{2}p_2x_2^{AG} - \frac{1}{2}r(V - V^G) \) from the Nash Bargaining Solution.

Assuming Gs match with As and Bs the condition that \( rV \geq rV^G \) is therefore (after some algebra)

\[
x_2^{AB} \geq \left( \frac{\lambda}{\lambda + 4r} \right) \left( \frac{1}{1 + \frac{\lambda}{2r}} \right) \left( \frac{2 + \frac{\lambda}{2r}}{2 + \frac{\lambda}{2r}} \right) \left( \frac{2r + \frac{\lambda}{2r}x_2^{AG}}{2r} \right)
\]

Assuming alternatively that Gs do not match, the condition that \( rV \geq rV^G \) is

\[
x_2^{AB} \geq \left( 1 + \frac{4r}{\lambda} \right) 2x_2^G
\]

So the condition for the specialist case to be an equilibrium is

\[
x_2^{AB} \geq 2x_2^G \max \left[ 1 + \frac{4r}{\lambda}, \frac{2 + \frac{\lambda}{2r}}{2 + \frac{\lambda}{2r}} \right] \left( \frac{2r + \frac{\lambda}{2r}x_2^{AG}}{2r} \right)
\]

As \( \lambda \to \infty \), the specialist case is an equilibrium if \( x_2^{AB} \geq \max \left[ 2x_2^G, x_2^{AG} \right] \). Noting that \( x_2^{AB} > x_2^{AG} \), the binding condition can therefore only be \( x_2^{AB} \geq 2x_2^G \) with full employment.

(III) The "mixed" case, where \( \{L^A, L^B, L^G\} = \{L', L', L^s - 2L'\} \). There are two sub-cases: (i) Gs do not match with As and Bs and (ii) Gs do match with As and Bs (see Lemma 1).

(i) If Gs do not match, then the equivalence of \( rV \) across worker types in equilibrium requires, using (11), that

\[
\frac{\lambda}{2r}Pw_2^{AB} = w_2^G
\]

where \( P = \Pr(A) = \Pr(B) \), \( w_2^G = p_2x_2^G \), and with the Nash Bargaining Solution \( w_2^{AB} = \frac{1}{2}p_2x_2^{AB} \).

(ii) If Gs do match, then the equivalence of \( rV \) across worker types in equilibrium requires that

\[
\frac{\lambda}{2r} \left[ Pw_2^{AB} + (1 - 2P)w_2^{AG} \right] = \frac{w_2^G + \frac{\lambda}{2}2Pw_2^{GA}}{1 + \frac{\lambda}{2r} \left[ 1 - P \right]}
\]

where \( w_2^{AB} \) and \( w_2^G \) are as in (i) and, with the Nash Bargaining Solution, \( w_2^{AG} = \frac{1}{2}p_2x_2^{AG} \).

Deviating to another worker type has no effect on payoffs, since players are infinitesimal. These cases thus exist as equilibria if (a) a player would not change her matching policy and (b) there exists a \( P \in [0, 1/2] \) that satisfies equality of income between specialists and generalists.

Comparing a Gs payoff when he doesn’t match with the payoff when he does (the RHS of equations (23) and (24)), it is clear that \( x_2^{AG} \geq 2x_2^G \) is necessary for G to match in equilibrium, and \( x_2^{AG} \leq 2x_2^G \) is necessary for G not to match in equilibrium. Rearranging
(23), we can define an equilibrium value \( P^* \) as
\[
P^* = \frac{2r}{\lambda(\frac{x^A}{x^B} - 1)}
\]
where \( P \in [0, 1/2] \) is necessary for an equilibrium to exist. Thus the "mixed" case where Gs do not match is an equilibrium iff \( x^A_G \leq 2x^G_2 \) (Gs do not want to match), \( x^A_G \geq 2x^G_2 \) \((P^* \geq 0)\), and \( \lambda \geq 4r \left[ \frac{1}{2} \frac{x^A_G}{x^B_2} \right]^{-1} \) \((P^* \leq 1/2)\).

As \( \lambda \to \infty \) (full employment), \( P^* \to 0 \), so that this "mixed" equilibrium converges towards the "generalist" equilibrium.

If G does match in equilibrium, then rearranging (24) produces a quadratic in P, with either 0, 1, or 2 roots such that
\[
\hat{P} = -\frac{2r}{\mu(\frac{x^A}{x^B} - 1)} \pm \sqrt{\left( \frac{2r}{\mu(\frac{x^A}{x^B} - 1)} \right)^2 + 8\frac{2r}{\mu^2(\frac{x^A}{x^B} - 1)}(\frac{2r}{\mu^2} + 1 - \frac{x^G}{x^B_2})}
\]
(25)
The "mixed" case where Gs do match with As and Bs is an equilibrium iff \( x^A_G \geq 2x^G_2 \) (Gs match with As and Bs) and \( \hat{P} \in [0, 1/2] \). It can be shown that as many as 2 such equilibria are possible for some parameter values.

As \( \lambda \to \infty \) (full employment), it follows directly from (25) that \( \hat{P} \to 0 \), so that any such "mixed" equilibrium also converges towards the "generalist" equilibrium.

Proof of Corollary (Gains from Specialization)

Proof. Output per specialist is \( \frac{1}{\mu} p_x x^A = p_x m c^\frac{1}{\mu-1} z h \) and output per generalist is \( p_x x^G = p_x^G \frac{1}{\mu} z h \), so that the ratio of these outputs is \( \frac{p_x x^G}{p_x^G} = \mu^\frac{1}{\mu-1} \). Hence the first part. For the second part, recall that the condition for the generalist equilibrium to be stable is \( x^A_G \leq 2x^G_2 \) with full employment. Using the production function (10), this condition is equivalently written in terms of underlying parameters as \( mc \leq \left( \frac{2}{1+m} \right)^{-\frac{1}{\mu-1}} \). Recalling that tasks A and B are gross complements in production \((\sigma \leq 1)\), it follows that \( \lim_{m \to \infty} \left( \frac{2}{1+m} \right)^{-\frac{1}{\mu-1}} = \infty \). Hence the maximum possible \( mc \) for which generalists exist in a stable equilibrium is unbounded from above.

Proof of Lemma (Log-Linear Wages)

Proof. Given that individuals have the same choice set at birth and maximize income, they must be indifferent across career choices so that \( W^k = W \) for all worker types. With full employment, this income arbitrage means from (5) that
\[
\int_0^\infty w^n_1 e^{-rt} dt = \int_s^\infty w^n_2 e^{-rt} dt
\]
(26)
where \( w_1^m = r V^U \) is the wage paid in the unskilled sector and \( w_2^m = r V \) is the wage paid in the skilled sector. Integrating (26) gives \( w_2^m = w_1^m e^{r s} \).

**Proof of Lemma (Mincer Accounting as Lower Bound)**

**Proof.** In the model, \( H^{AB}/H^G = Y^{AB}/Y^G \). Skilled workers are \( mc > 1 \) times more skilled in the AB case than the G case. From (17) and (16), we write

\[
\frac{H^{AB}}{H^G} = \frac{L_1^{AB} \left( 1 + e^{r S L_1^{AB}/L_1^G} \right)^{\frac{1}{1+\epsilon}}} {L_1^G \left( 1 + e^{r S L_1^G/L_1^G} \right)^{\frac{1}{1+\epsilon}}}
\]

(27)

Recalling that \( H_{\text{Mincer}}^m = L_1^m + e^{r S L_2^m} \), we can manipulate (27) to write the ratio of the true human capital ratio to the Mincerian calculation, \( R_H = \frac{H^{AB}/H^G}{H_{\text{Mincer}}^m/H_{\text{Mincer}}^m} \), as

\[
R_H = \left( \frac{1 + e^{r S L_1^{AB}/L_1^G}}{1 + e^{r S L_1^G/L_1^G}} \right)^{\frac{1}{1+\epsilon}}
\]

Consider the case where \( \epsilon \in [1, \infty] \). From (19), \( L_2^{AB}/L_1^{AB} \geq L_2^G/L_1^G \), with strict inequality if \( \epsilon > 1 \). Given the observed labor allocations, it follows that \( \lim_{\epsilon \to \infty} R_H = 1 \) and that \( R_H \) declines in \( \epsilon \). Further \( \lim_{\epsilon \to \infty} R_H = 1 \). Hence, \( R_H \geq 1 \) given \( \epsilon > 1 \).

Consider the case where \( \epsilon \in [0,1] \). From (19), \( L_2^{AB}/L_1^{AB} \leq L_2^G/L_1^G \), with strict inequality if \( \epsilon < 1 \). Given the observed labor allocations, it follows that \( \lim_{\epsilon \to 1} R_H = \infty \) and that \( R_H \) increases in \( \epsilon \). Further \( \lim_{\epsilon \to 0} R_H > 1 \). Hence, \( R_H > 1 \) given \( \epsilon \leq 1 \).

In sum, over \( \epsilon \in [0, \infty] \) it follows that \( R_H \geq 1 \). Moreover, for a given labor allocation \( L_1^G/L_1^{AB} \neq 1 \), \( \lim_{\epsilon \to 1} R_H = \infty \). ■

**Proof of Corollary (Immigrant Workers)**

**Proof.** The low-skilled immigrant earns a higher real wage by moving to the rich country because, from (17)

\[
\frac{w_1^{AB}/p^{AB}}{w_1^G/p^G} = \frac{y^{AB}}{y^G} > 1
\]

Hence an unskilled worker who migrates from a poor to a rich country will earn a higher real wage.

Now consider skilled immigrants.

Note first that the skilled generalist who migrates will never team with a specialist in the rich country. Rather, he would always prefer to work alone, since he must give up too much of the joint product to convince a specialist to partner with him. In particular, he would earn \( p_2^{AB} x_2^G \) alone, while in a team (with full employment) he would earn \( p_2^{AB} \left( x_2^{AG} - \frac{1}{2} x_2^{AB} \right) \), and there are no parameter values where \( x_2^G < x_2^{AG} - \frac{1}{2} x_2^{AB} \). To see this, write this condition
as $1 < x_2^{AG} / x_2^G - \frac{1}{2} x_2^{AB} / x_2^G$. Note that $\frac{1}{2} x_2^{AB} / x_2^G = mc$ and that $x_2^{AG} / x_2^G$ can be no greater than $mc + c$.\(^{16}\) Hence the condition is equivalently $1 < c$, which contradicts the assumption of the model that there are coordination costs in production, $c < 1$.

Next, note that working alone as a generalist in the rich country is never preferred to staying in the poor country. In either country, the generalist produces $x_2^G$ units of output per unit of time. Given that this good is relatively expensive in the poor country (i.e. recall that $p_2^G / p_1^G = mc (p_2^{AB} / p_1^{AB})$), the real income is higher working as the generalist in the poor country.

Lastly, note that the generalist may still prefer to migrate and work in the unskilled sector. This occurs when the real wage gain across countries for unskilled work $\frac{w_2^{AB} / p_2^{AB}}{w_1^G / p_1^G}$ (see above) is larger than the real wage gain locally for skilled work, $e^{rs}$, which is more likely the greater the income differences between the countries; for example, the greater the gains from specialization, $mc$.

In sum, skilled generalists may or may not be better off migrating to rich countries, but if they do they will work in the unskilled sector. ■

**Proof of Corollary (Brain Drain)**

**Proof.** The specialist who moves to the poor country will earn a wage $w_2' = p_2^G (x_2^{AG} - x_2^G)$. Since the poor country is in a generalist equilibrium, we must have $x_2^{AG} \leq 2x_2^G$ which implies that $w_2' \leq p_2^G x_2^G = w_2^G$. Hence, the skilled worker who moves from the rich to the poor country will earn a wage no greater than the skilled worker wage in the poor country. Now note that skilled workers receive a higher real wage in the rich country than the poor country because, from (14) and (17),

$$\frac{w_2^{AB} / p_2^{AB}}{w_1^G / p_1^G} = \frac{y_2^{AB}}{y_2^G} > 1$$

Hence, specialists in the rich country will prefer to stay. ■

**Proof of Corollary (Desirable Cheap Specialists)**

**Proof.** Think of the firm as a specialist in the rich country. He earns $w_2^{AB} = \frac{1}{2} p_2^{AB} x_2^{AB}$. If he can alternatively form a cross-border team by locating an (off-equilibrium) specialist in the poor country, then he can earn at least $w_2 = p_2^{AB} x_2^{AB} - p_2^{AB} x_2^G$, where he need provide the specialist in the poor country no more than $x_2^G$, the going rate for generalists in that country. Hence, hiring a specialist in the poor country makes sense iff $x_2^{AB} - x_2^G \geq \frac{1}{2} x_2^{AB}$ or $x_2^{AB} \geq 2x_2^G$, which is just the condition for specialists to exist in the first place in the rich country. ■

**Data and Analysis for Figure 4**

\(^{16}\) This follows because $x_2^{AG} / x_2^G$ is increasing in $\sigma$, attaining a maximum $x_2^{AG} / x_2^G = mc + c$ as $\sigma \rightarrow \infty$. 41
Data on wages and occupations is taken from the 1% microsample of the 2000 United States census, which is available publicly through www.ipums.org. There are 2.8 million individuals in this sample, including 320 thousand individuals who immigrated to the United States.

The wage-schooling relationships in Figure 4a are the predicted values from the following regression

\[
\ln w_i = \alpha + \beta MALE + Age_{fe} + English_{fe} + Group_{fe} + Education_{fe} + Group_{fe} \times Education_{fe} + \varepsilon_i
\]

where \( w_i \) is the annual wage, \( MALE \) is a dummy equal to 1 for men and 0 for women, \( Age_{fe} \) are fixed effects for each individual age in years, \( English_{fe} \) are fixed effects for how well the individual speaks English (the IPUMS "speakeng" variable which has 6 categories), \( Education_{fe} \) are fixed effects for highest educational attainment (the IPUMS "educ99" variable, which has 17 categories) and \( Group_{fe} \) are fixed effects for three different groups: (1) US born, (2) immigrants who arrive by age 17, (3) immigrants who arrive age 30 or later. Figure 4a plots predicted values from this regression, plotting the log wage against educational attainment for each of the three groups. For comparison purposes, the predicted values focus on males between the ages of 30 and 40 who speak English at least well.

To construct Figure 4b, the modal educational attainment is first determined for each of the 511 occupational classes in the data (using the IPUMS variable "occ"). Occupations are then grouped according to modal educational attainment. For example, lawyers are grouped with doctors as typically having professional degrees, and taxi drivers are grouped with security guards as typically having high school degrees. For each of the three groups defined for the \( Group_{fe} \) above, Figure 4b shows the propensity of individuals with professional or doctoral degrees to work in occupations with the given modal educational attainment.

References


\[47\text{Integrated Public Use Microdata Series (Steven Ruggles, Matthew Sobek, Trent Alexander, Catherine A. Fitch, Ronald Goeken, Patricia Kelly Hall, Miriam King, and Chad Ronnander. Integrated Public Use Microdata Series: Version 3.0 [Machine-readable database]. Minneapolis, MN: Minnesota Population Center [producer and distributor], 2004.)}\]


Figure 4a: Do Skilled Immigrants Experience Wage Penalties?  
The Wage-Schooling Relationship

Figure 4b: Do Skilled Immigrants use their Education?  
Typical Educational Level of Occupation for Workers with Professional or Doctoral Degrees