Problem 1
Chapter 1, Exercise 33. Imagine a sphere of radius \( a \) filled with negative charge of uniform density, the total charge being equivalent to that of two electrons. Embed in this jelly of negative charge two protons and assume that in spite of their presence the negative charge distribution remains uniform. Where must the protons be located so that the force on each of them is zero? (This is a surprisingly realistic caricature of a hydrogen molecule; the magic that keeps the electron cloud in the molecule from collapsing around the protons is explained by quantum mechanics!)

Problem 2
Chapter 2, Exercise 1. The vector function which follows represents a possible electrostatic field:

\[
E_x = 6xy \quad E_y = 3x^2 - 3y^2 \quad E_z = 0
\]

Calculate the line integral of \( \vec{E} \) from the point \((0, 0, 0)\) to the point \((x_1, y_1, 0)\) along the path which runs straight from \((0, 0, 0)\) to \((x_1, 0, 0)\) and thence to \((x_1, y_1, 0)\). Make a similar calculation for the path which runs along the other two sides of the rectangle, via the point \((0, y_1, 0)\). You ought to get the same answer if the assertion above is true. Now you have the potential function \( \phi(x, y, z) \). Take the gradient of this function and see that you get back the components of the given field.

Problem 3
Chapter 2, Exercise 4. Describe the electric field and the charge distribution that go along with the following potential:

\[
\begin{align*}
\phi &= x^2 + y^2 + z^2 & \text{for} & \quad x^2 + y^2 + z^2 < a^2 \\
\phi &= -a^2 + \frac{2a^3}{(x^2+y^2+z^2)^{1/2}} & \text{for} & \quad a^2 < x^2 + y^2 + z^2
\end{align*}
\]

Problem 4
Chapter 2, Exercise 8. For the cylinder of uniform charge density in Fig. 2.17:

(a) Show that the expression there given for the field inside the cylinder \( (E = 2\pi pa^2/r \text{ outside the cylinder, } E = 2\pi pr \text{ inside}) \) follows from Gauss’ law.

(b) Find the potential \( \phi \) as a function of \( r \), both inside and outside the cylinder, taking \( \phi = 0 \) at \( r = 0 \).

Problem 5
Chapter 2, Exercise 12. The right triangle with vertex \( P \) at the origin, base \( b \), and altitude \( a \) has a uniform density of surface charge \( \sigma \). Determine the potential at the vertex \( P \). First find the contribution of the vertical strip of width \( dx \) at \( x \). Show that the potential at \( P \) can be written as \( \phi_P = \sigma b \ln[(1 + \sin \theta)/\cos \theta] \).
Problem 6
Chapter 2, Exercise 15. Calculate the curl and divergence of each of the following vector fields. If the curl turns out to be zero, try to discover a scalar function $\phi$ of which the vector field is the gradient:

(a) $F_x = x + y; F_y = -x + y; F_z = -2z$
(b) $G_x = 2y; G_y = 2x + 3z; G_z = 3y$
(c) $H_x = x^2 - z^2; H_y = 2; H_z = 2xz$

Problem 7
Chapter 2, Exercise 19. We have two metal spheres, of radii $R_1$ and $R_2$, quite far apart from one another compared with these radii. Given a total amount of charge $Q$ which we have to divide between the spheres, how should it be divided so as to make the potential energy of the resulting charge distribution as small as possible? To answer this, first calculate the potential energy of the system for an arbitrary division of the charge, $q$ on one and $Q - q$ on the other. Then minimize the energy as a function of $q$. You may assume that any charge put on one of these spheres distributes itself uniformly over the sphere, the other sphere being far enough away so that its influence can be neglected. When you have found the optimum division of the charge, show that with that division the potential difference between the two spheres is zero. (Hence they could be connected by a wire, and there would still be no redistribution. This is a special example of a very general principle we shall meet in Chapter 3: on a conductor, charge distributes itself so as to minimize the total potential energy of the system.)

Problem 8
Chapter 2, Exercise 20. As a distribution of electric charge, the gold nucleus can be described as a sphere of radius $6 \times 10^{-13}$ cm with a charge $Q = 79e$ distributed fairly uniformly through its interior. What is the potential $\phi_0$ at the center of the nucleus, expressed in megavolts? (First derive a general formula for $\phi_0$ for a sphere of charge $Q$ and radius $a$. Do this by using Gauss’ law to find the internal and external electric field and then integrating to find the potential.)

Ans. $\phi = 3Q/2a = 95,000$ statvolts = 28.5 megavolts.

Problem 9
Chapter 2, Exercise 26. Use the result for Problem 2.12 to answer this question: If a square with surface charge density $\sigma$ and side $s$ has the same potential at its center as a disk with the same surface charge density and diameter $d$, what must be the ratio $s/d$? Is your answer reasonable?

Problem 10
Chapter 2, Exercise 30. Consider a charge distribution which has the constant density $\rho$ everywhere inside a cube of edge $b$ and is zero everywhere outside that cube. Letting the electric potential $\phi$ be zero at infinite distance from the cube of charge, denote by $\phi_0$ the potential at the center of the cube and $\phi_1$ the potential at a corner of the cube. Determine the ratio $\phi_0/\phi_1$. The answer can be found with very little calculation by combining a dimensional argument with superposition. (Think about the potential at the center of a cube with the same charge density and with twice the edge length.)
Problem 11
Calculate the potential difference of a dipole. Assuming a charge of $+q$ at $z = +d/2$, and a charge of $-q$ at $z = -d/2$, what is the electric potential difference between a point $P$ and $\infty$, where $(r, \theta, \varphi)$ are spherical coordinates for the point $P$? In the dipole approximation, you should consider only $r >> d$. 