No books, notes or calculators. You should be able to do this in about 70 minutes. The real test will be shorter. Remember to study all the material. **Not everything that may be on the test is on the practice exam.**

**Problem 1.** Find the best quadratic approximation to \( f(x) = \frac{\ln x}{x^2} \) for \( x \approx 0 \).

**Problem 2.** Find \( \lim_{x \to 0} \frac{1 - e^{(x^2)}}{\sin^2 x} \).

**Problem 3.** Find the first four non-zero terms in the Taylor series around \( a = 0 \) for the function \( \frac{1}{(1 + x)^2} \).

**Problem 4.** Radioactive material decays exponentially. Assuming consistent units, if you have an amount \( A \) then after a time \( t \) there will be \( Ae^{-kt} \). \( (k \) is called the decay constant.) Suppose a new nuclear storage facility takes in radioactive material at the rate of \( 1 - (t - 1)^2 \) kg/year for its first 2 years of operation. Assume a decay constant of \( k \) and show how to write an integral for the amount of radioactive material at the end of the 2 years. (You don’t have to compute the integral, but you do have to show reasoning.)

**Problem 5.** Compute \( \int_2^3 \frac{(1+\ln x)^7}{x} \, dx \).

**Problem 6.** For this problem we have \( f(x) = x^3 \) and we consider the region between the graph of \( f(x) \) and the \( y \)-axis for \(-1 \leq x \leq 1 \).

(a) Compute the volume of revolution when this region is revolved around the \( y \)-axis. First, you must draw and label a sketch showing how you slice the volume. Include labels.

(b) Write down an integral expressing the arclength of the graph of \( f(x) \). (You do not need to compute the integral.)

**Problem 7.** Consider the function \( F(x) = \int_0^x \sqrt{3 + \sin t} \, dt \). Without attempting to find an explicit formula for \( F(x) \),

(a) Determine whether \( F(x) \) is concave up or concave down in the interval \( 0 < x < 1 \).

(b) Show that \( F(1) \leq 2 \).

(c) Give \( \int_1^2 \sqrt{3 + \sin(2t)} \, dt \) in terms of \( F(x) \). (Notice the factor of 2 in the sin term.)

(d) Let \( G(x) = \int_0^{x^2} \sqrt{3 + \sin t} \, dt \). Compute \( G'(x) \).

**Problem 8.** Consider the first hump of the graph of \( y = \sin x \). For both (a) and (b) you must draw a sketch. Include labels.

(a) Find the average distance from this curve to the line \( y = -1 \).

(b) Find the average distance from this curve to the \( y \)-axis.
Problem 9. Integrate each of the following.

(a) \[ \int \frac{7 - x}{(x - 1)(x^2 + 1)} \, dx. \]

(b) \[ \int \frac{3x^3 + 6x^2 + 2x + 2}{x^2 + 2x} \, dx. \]

(c) \[ \int \frac{1}{(1 + x^2)^2} \, dx. \]

(d) \[ \int_{0}^{1} \frac{x^2}{\sqrt{4 - x^2}} \, dx. \]