18.01A Problem Set 3, Fall 2017
(due in class on Thursday, Sep. 28)

Part I  (30 points)

TB = Simmons; SN = 18.01A Supplementary Notes (all have solutions) The problems marked 'other' are not to be handed in.

**Topic 7** (W, Sep. 20)  More applications: work, average value.
  Read: Class notes topic 7; also see SN: AV; TB: 7.7 to middle p. 247
  Hand in: 4D/3, 5, 8; 4D'/2, 3; 4J 1.

**Topic 8** (R, Sep. 21)  Integration: substitution, trig. integrals, completing the square.
  Read: Class notes topic 8 or 10.2, 10.3, 10.4.
  Hand in: 5B/7, 16; 5C/6, 9, 11; 5D/1, 2, 7, 10.

  Read: TB: 10.6,  SN: F
  Hand in: 5E/3, 5, 6, 10h (complete the square)

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**Coming next**

**Topic 10** (T, Sep. 26)  Integration by parts, numerical integration.
  Read: TB: 10.7, 10.9.

**Topic 11** (R, Sep. 28)  Improper integrals.
  Read: TB: 12.4,  SN: INT

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**Exam:**  (T, Oct. 3)  **Exam 1**  (covers topics 1-9)

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Part II  (90 points)

Directions: Try each problem alone for 20 minutes. If, after this, you collaborate, you must write up your solutions independently.

**Problem 1**  (Topic 7, 30 pts: 10,10,10)
(In this problem I took some problems from the textbook and changed them from English to metric units.)

(a)  Textbook 7.7/3: A spring supporting a railroad car has a natural length of 25 cm. and a force of 2000 Newtons compresses it 1/2 cm. Find the work done in compressing it from 25 cm. to 20 cm. (Give your answer in joules.)

(b)  Textbook 7.7/5: A bucket weighing 5 kg when empty is loaded with 30 kg of sand. Unfortunately there is a hole in the bucket and sand leaks out uniformly at such a rate that two thirds of the sand is lost when the bucket has been lifted 10 meters. Find the work done in lifting the bucket this distance.

(c)  Textbook 7.7/21: A spherical tank of radius $a$ is at the top of a tower with its bottom at a distance $h$ above the ground. How much work is needed to fill the tank with water pumped from ground level?
Problem 2  (Topic 7, 20 pts: 10,10)
In the upper half of the unit disk centered at the origin a random rectangle is inscribed with its base on the $x$-axis and its two upper corners on the unit circle. What is the average area of such a rectangle if it is determined by

(a) picking its lower right corner at random on the $x$-interval $[0,1]$?
(b) picking its upper right corner at random on the first quadrant of the unit circle $(0 \leq \theta \leq \pi/2)$?

Problem 3  (Topic 8, 10 pts)
Use the shell method to find the volume of revolution for the solid formed by revolving the disk $(x-b)^2 + y^2 \leq a^2$ around the $y$-axis. You should assume that $0 < a < b$.

Problem 4  (Topic 8, 15 pts: 10,5)
(a) Use the trigonometric identity $\tan^2 x = \sec^2 x - 1$ to derive the reduction formula

$$\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} \int \tan^{n-2} x \, dx.$$

(b) Use this to integrate $\tan^5 x$.

Problem 5  (Topic 9, 8 pts: 10,5)
A simple model for the spread of an infectious disease is $\frac{dx}{dt} = kx(1-x)$, where $x$ is the fraction of the population with the disease, $1-x$ is the healthy fraction of the population and $k > 0$ is a constant of proportionality. (The model says the rate of spread is proportional to the number of contacts between healthy and sick individuals.)

(a) This is a differential equation which can be solved by ‘separating variables’, i.e.

$$\frac{dx}{x(1-x)} = k \, dt.$$

Integrate both sides of this equation and solve for $x$ as a function of $t$.
(b) What happens in the long run?