The Mean Lifetime and Speed of Cosmic-Ray Muons

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Muons are produced in the upper atmosphere as a result of the collisions of incoming high-energy particles from outer space with atmospheric particles. These muons then travel through the atmosphere at high energies and are thus measured to exist for a much longer time than their mean lifetime at rest. This experiment measures both the mean lifetime of muons at rest as well as the mean speed through times of flight between fixed distances. Once the mean speed and the mean lifetime are found, the time that the muon is measured to travel through the atmosphere before decaying can be compared to the mean lifetime at rest and checked for consistency with the predictions of special relativity.

Before the year 1900, physicists believed that any massive particle could achieve any speed. Then, after the Michelson-Morley interferometry experiment failed to provide conclusive evidence for the existence of a luminiferous ether, Einstein [1] concluded in his theory of special relativity that light does not have a rest frame but it has the same speed in all frames of reference, and this speed is the upper limit for any particle. In addition, in special relativity, massive particles would require infinite energy to travel at the speed of light, so they will not be able to even match that speed. This also leads to the effects of length contraction and time dilation, in which time intervals measured between events for an object in its rest frame become larger for an observer who sees the object moving, and lengths become similarly shrunken according to a moving object.

Muons are produced by the collisions of cosmic rays, which are high-energy protons, neutrons, and helium nuclei originating from outer space [2], with particles in the upper atmosphere. The muons themselves are also unstable, so they exist for only a finite amount of time. Furthermore, in its rest frame, the lifetime of a muon is much shorter than the time we observe a muon to travel through the entire atmosphere. The mean lifetime and speed are the characteristics of these fundamental particles that are measured here. They also provide a test of the validity of special relativity through the observation of time dilation between the mean lifetime at rest and the time of flight through the atmosphere, though this analysis is not performed here.

I. MEAN LIFETIME

I.1. Theoretical Basis

Cosmic rays collide with atmospheric particles to produce pions, which decay into muons. These muons subsequently undergo decay into an electron, a muon neutrino, and an electron antineutrino: \( \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \). This decay process is mediated by W bosons, which are the force carriers of the electroweak interaction. The mean decay time \( \tau = \frac{102\text{sec}\cdot m_\mu}{cG_F} \) depends only on the fundamental constants \( h \) and \( c \), the muon mass, and \( G_F \), which is a coupling constant calculated from electroweak theory for beta decay processes like this. [3]

Muon decay events have been found to be independent and uncorrelated, and their rarity allows them to be described by a Poisson process. The Poisson distribution \( P(n; \frac{\tau}{t}) = \frac{1}{n!}(\frac{\tau}{t})^ne^{-\tau} \) describes the number of decay events \( n \) in a time interval \( t \) for a mean rate \( \tau^{-1} \). The waiting time between two decay events is \( P(t' < t) = 1 - P(t' > t) = 1 - P(0; \frac{\tau}{t}) = 1 - e^{-\frac{\tau}{t}} \).

From this, the probability density of waiting for a time \( t \) between decay events is \( p(t) = \frac{\tau}{t}e^{-\frac{\tau}{t}} \), and the mean lifetime, being the reciprocal of the mean decay rate, is exactly \( \tau \). This experiment measures counts rather than direct probabilities, but the counts for each waiting time together are exponentially distributed in the waiting times as well: \( N(t) = N_0e^{-\frac{\tau}{t}} \). Hence, by fitting the count data to an exponential function, \( \tau \) can be determined, and so could the mass \( m_\mu \), though the latter quantity is not determined in this analysis.

I.2. Apparatus and Method

![FIG. 1. Schematic of the decay time measurement apparatus (adapted from [3])](image)

The apparatus for the mean lifetime measurement is shown in figure 1. A power source supplies high voltages to two photomultiplier tubes (PMTs) sitting on top of a scintillator. When a muon enters the scintillator, it hits...
the electrons in the scintillator to produce photons which enter the PMTs. The muon hits the atoms in the scintillator, losing energy until it is brought to rest. It remains at rest until it decays, at which point the decay products collide with the scintillator electrons to produce more photons going towards the PMTs. In each PMT, the incoming photon strikes the cathode, releasing electrons through the photoelectric effect. These electrons travel up the applied voltage and strike more electrodes called dynodes along the way to release more electrons; all these electrons then strike the anode at the end to produce a measurable signal. The signals from the PMTs are then filtered through constant-fraction discriminators (CFDs) which filter out noise from actual high-energy muon decay signals by filtering all signals below a set threshold amplitude. These are then sent through a coincidence circuit, which takes signals in from the two CFDs and sends out a signal only if they are separated in time by less than a certain amount set in the circuit. Muon decay events are rare, the proportion of signals that are noise rather than decay events are assumed to be small, and muon signals are sent through both PMTs at the same time, so if the coincidence circuit produces an output, it is most likely (though not certainly) a muon event being sent through both PMTs at the same time. Because these signals are registered by the coincidence circuit as having no time separation, the time-amplitude converter (TAC), which converts the time separation between two signals into a single voltage signal of amplitude proportional to that time interval, will see all the incoming pairs of signals as having no time separation unless a delay is added to one of the signals. The ‘START’ signal is delayed rather than the ‘STOP’ signal because the TAC produces its output by triggering upon the ‘START’ signal and ending upon the next ‘STOP’ signal, so the next ‘STOP’ signal after the muon entry ‘START’ signal should be the muon decay signal. The TAC then outputs to a multiple-channel analyzer (MCA) connected to a computer, which maps the voltage amplitudes into bins and counts the number of events in each such bin.

The high voltage settings were 1721 V for the left PMT and 1787 V for the right PMT. The TAC range was 200 µs with a multiplier of 100. A delay of the equivalent of 9 feet of extra cable was added. Using a time calibrator which sends square pulses of a known period into the TAC, the time calibration was found to be 1280 ns per 129 bins, where the MCA maps TAC to map as voltage bins, the time calibration was delayed rather than the ‘STOP’ signal because the delay is added to one of the signals. The ‘START’ signal is delayed rather than the ‘STOP’ signal because the TAC produces its output by triggering upon the ‘START’ signal and ending upon the next ‘STOP’ signal, so the next ‘STOP’ signal after the muon entry ‘START’ signal should be the muon decay signal. The TAC then outputs to a multiple-channel analyzer (MCA) connected to a computer, which maps the voltage amplitudes into bins and counts the number of events in each such bin.

The mean of the last 3/4 of the data is subtracted from all bins. Then, to ensure Gaussian statistics in the count rates for bins, the counts per bin near the tails must be higher, and this can be achieved by combining 64 original bins into 1 new bin. The bins of nonpositive counts, like the ones for low time intervals stemming from the effect of the delay line, are removed, as is the first new bin with a positive count due to the effect of the negative counts from many surrounding bins artificially lowering the count value. Finally, due to the persistence of noise even in the tails, the last half of the new bins are removed, yielding the newly-binned data set shown in the graph on the right side.

I.3. Results

In figure 2, the graph on the left is of the original data with 2048 bins excpeting those with zero counts converted into times. It exhibits a fluctuating tail of positive mean due to thermal noise signals at those long time intervals swamping the muon decay events, so the mean lifetime \( \tau \) of the data is subtracted from all bins. Then, to ensure Gaussian statistics in the count rates for bins, the counts per bin near the tails must be higher, and this can be achieved by combining 64 original bins into 1 new bin. The bins of nonpositive counts, like the ones for low time intervals stemming from the effect of the delay line, are removed, as is the first new bin with a positive count due to the effect of the negative counts from many surrounding bins artificially lowering the count value. Finally, due to the persistence of noise even in the tails, the last half of the new bins are removed, yielding the newly-binned data set shown in the graph on the right side.

As shown in figure 3, the logarithm of the counts is plotted against the time intervals, as this can be fit to a line of the form \( \ln(y) = \ln(a) - bx \) from \( y = ae^{-bx} \). From this fit, the mean lifetime \( \tau = 2197 \pm 23 \) ns, and the fit quality is given by a reduced \( \chi^2/\nu = 1.47 \) for \( \nu = 13 \).

The left of figure 4 shows the direct nonlinear fit of the count data to an exponential \( y = ae^{-bx} \). From this, the mean lifetime \( \tau = 2195 \pm 10 \) ns, which shows a slight shift in the mean but also a decrease in the relative and absolute error in the mean compared to the linearized fit. The fit is characterized by a reduced \( \chi^2/\nu = 1.38 \) for \( \nu = 13 \).

The right of the figure shows the nonlinear fit of the count data to an exponential combined with a constant vertical shift of the form \( y = ae^{-bx} + z \). From this, the mean lifetime \( \tau = 2193.45 \pm 0.33 \) ns, so the sharp decrease in the relative error implies that much of the error in the previous determination of \( \tau \) stemmed from the lack of a vertical shift. This fit has a reduced \( \chi^2/\nu = 1.43 \) for \( \nu = 12 \).

These last two fits can be compared through an F-test to see if the difference is significant [4]. The statistic \( F = \frac{\chi^2_{\nu_1}/\nu_1}{\chi^2_{\nu_2}/\nu_2} = 1.04 \) in this case, where \( \nu_1 = 12 \) and \( \nu_2 = 13 \).
The test is whether $3.17 \leq F \leq 3.153 = 0.95$ because by convention the test statistic $F$ should lie outside those bounds for it to be considered a statistically significant result. It does not, so the difference in the reduced $\chi^2$ between the two fits is not significant.

Finally, the difference between the means of the exponential and shifted exponential fits is 1.6 ns, and this provides a bound for the systematic error from using different fitting methods. Calculating the overall mean and uncertainty by weighting the individual mean by a function of the individual uncertainties, the overall mean lifetime $\tau = 2193.5 \pm 0.3_{st} \pm 1.6_{sy}$ ns lies within 2σ of the accepted mean lifetime $\tau = 2197.0$ ns.

II. TIME OF FLIGHT

II.1. Theoretical Basis

Muons are ultimately produced from the collision of very high-energy cosmic rays with atmospheric particles, so they travel at a speed comparable to that of light. This implies that the time the muon experiences traveling through the atmosphere becomes dilated by the Lorentz factor $\gamma$ to become a time that a laboratory can measure; conversely, the distance traveled as measured in the laboratory becomes contracted by the factor $\gamma$ to become the distance traveled that the muon experiences in its rest frame. Given that all measurements are made from the frame of the laboratory, the speed of the muon is simply the ratio of a given distance in the laboratory to the measured time interval to travel that distance.

II.2. Apparatus and Method

The time-of-flight apparatus, as shown in figure 5, is similar to that of the mean lifetime apparatus. However, instead of having a single plastic scintillator, there are two plastic scintillating paddles so that the muon will trigger when it starts and stops traveling the spatial separation of the paddles. The scintillator in each plastic paddle is powered by a high voltage supply. The signal from the top paddle is sent into one CFD, while the signal from the bottom paddle is delayed before being sent into the other CFD. The delay is present because the top paddle signal is the ‘START’ signal for the TAC, so the ‘STOP’ signal that comes immediately after should arise from the same muon flight event through the bottom paddle; this is guaranteed by the addition of the delay. Without the delay, if the muon were to hit the bottom paddle closer to the rest of the apparatus than it did the top paddle, the bottom paddle ‘STOP’ signal would arrive before the top paddle ‘START’ signal, which is undesirable for finding the time interval that a muon traveled between the paddles. The CFD outputs feed into the TAC, which as before outputs a voltage pulse of amplitude proportional to the time interval between the ‘START’ and ‘STOP’ events, being the time of flight between the paddles, to the MCA.

The high voltage settings were 1720 V both paddles. The TAC range was 100 µs with a multiplier of 1. A delay of the equivalent of 32 feet of extra cable was added. The time calibration worked in the same way as with the lifetime measurement, and yielded 20 bins per nanosecond, where the MCA maps over 2048 bins. Twelve measurements of the flight time were performed, with data collection times running from 20 minutes for the shortest separations of the paddles to overnight (approximately 12 hours) for the longest such separations.

II.3. Results

Muons do not travel between the two paddles in a perfectly vertical path, but instead exhibit variations in the zenith angle upon entry. The empirical distribution of
FIG. 6. Left: sample raw flight time count data (separation \( y = 250 \pm 1 \text{ cm} \)) peak region fitted to a Gaussian; right: plot of times of flight versus mean slant distance

particle count intensities with respect to the zenith angle \( \theta \) is \( I(\theta) = I_\nu \cos^2(\theta) \) [5], where \( I_\nu \) depends on the properties of the atmosphere and \( \theta \) varies between 0 for a vertical path between paddles and \( \frac{\pi}{2} \) for a horizontal path parallel to a paddle. Furthermore, the effective area of the paddle measuring muon traversals is modulated as \( A(\theta) = A \cos(\theta) \). Given a measurement time \( t_T \), integrating over azimuthal angles \( \varphi \) between 0 and \( 2\pi \) and normalizing such that the total probability over all \( \theta \) is 1 yields a probability density \( p(\theta) = 4 \cos^3(\theta) \sin(\theta) \). This is the probability density that, over all \( \varphi \) and for any integration time \( t_T \), area \( A \), and intensity factor \( I_\nu \), the muon travels between the paddles at a zenith angle \( \theta \). Performing a Monte Carlo simulation for different origin coordinates at the top paddle and and different angles \( \theta \) weighted by this probability density for \( \theta \) yields a mean slant distance \( D \) for a given vertical paddle separation \( y \).

For the count dataset for each paddle separation, a flight time must be determined. Each raw dataset has 2048 bins. From this, the 300 bins in the peak region surrounding the peak value were chosen, had the bins of zero counts removed, and were fitted to a Gaussian profile; an example of this is shown in the left of figure 6. The right of the figure shows the times of flight plotted against the mean slant distance corresponding to each vertical paddle separation.

The mean slant distances were corrected for the finite times of the propagation of light through the paddle scintillators with indices of refraction around 1.5 [6]. From these, the mean slant distances that differed from the corresponding paddle separations by more than 5% were discarded because of our relative lack of confidence in the ability of this phenomenological model to deal with larger deviations. This affected the three data points from only the two shortest separations. From this remained 9 pairs of mean slant distances and corresponding flight times.

The way to find the muon speed is through \( v = \frac{D}{t_0 - t_d} \) where \( D \) is the mean slant distance, \( t_0 \) is the corresponding measured time of flight, and \( t_d \) is the constant delay time. A delay of 32 feet of cable corresponds to 48 ns given the speed of electrons in a wire. However, other cable length differences in the apparatus may contribute more delay, so the delay time must be taken as another fit parameter. The speed and delay time can be found by minimizing \( \chi^2 \) for the fit of speeds to a constant ratio, and the uncertainties on each are found by holding the value of the other parameter constant where \( \chi^2 \) is minimized and moving to that minimum value of \( \chi^2 \) added to 1 [4]. The result of this is shown in figure 7. From this, \( t_d = 52.986 \pm 0.044 \text{ ns} \), and \( v = (3.102 \pm 0.024) \times 10^{10} \text{ cm} \text{s}^{-1} \), with the minimum reduced \( \chi^2 = 1.39 \). This implies an additional delay of about 5 ns that cannot be determined directly from the configuration of the system.

This new value and error for \( t_d \) can again be propagated into the equation for the error on \( v \), and then an average of each \( v \) for each \( D \) weighted by the error on each \( v \) [4] can be taken to yield \( v = (3.094 \pm 0.025) \times 10^{10} \text{ cm} \text{s}^{-1} \). For a typical muon momentum \( p = 1 \text{ GeV} \) and its mass \( m = 105.668 \text{ MeV} \) [7], the accepted speed \( v = 2.981 \times 10^{10} \text{ cm} \text{s}^{-1} \), arising from \( p = (1 - \gamma^{-2})^{-\frac{1}{2}} m \). The result calculated from the data thus differs from the accepted value by 4.5\( \sigma \).

III. CONCLUDING REMARKS

The mean lifetime of the muon was found to be within 2\( \sigma \) of the accepted value, and the speed was found to be within 4.5\( \sigma \) of the speed given for a typical value of muon momentum. From the mean lifetime, the mass of the muon can be calculated. From the speed of the muon, the time that the muon takes to traverse the atmosphere before decaying can be calculated and compared to the mean lifetime to check for consistency with the predictions of special relativity.


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