The Zeeman and Hyperfine Splittings of Mercury Emission Spectra

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The Zeeman and hyperfine splittings of the emission spectra from mercury vapor lamps are investigated through the use of a Fabry-Perot interferometer. In the absence of a magnetic field, a low-intensity peak near the major 5461 Å emission line can be observed; this yields a hyperfine shift of frequency 12.70 ± 0.19 GHz, consistent with the $^4\text{He}$ Hg emission shift of 12.21 GHz. In the presence of different magnetic fields, the major green line is observed to split into three peaks; from this, the electron charge to mass ratio is found to be $(2.17 ± 0.29) \cdot 10^{11} \, \text{C/kg}$. Each yellow line in the 5769 Å and 5790 Å doublet splits similarly into a triplet, and together they yield a mean charge to mass ratio of $(2.88 ± 0.37_{\text{st}} ± 0.04_{\text{st}}) \cdot 10^{11} \, \text{C/kg}$.

In 1881, Michelson observed the hyperfine energy splitting of hydrogenic atoms; these energy differences are around two orders of magnitude smaller than fine structure energy splittings. Pauli was able to explain this in 1924[1] as being the result of the nucleus having a magnetic moment. In the mid-19th century, Faraday attempted to observe the effect of applying a magnetic field to a sodium vapor lamp on its emission spectrum. Zeeman was able to successfully carry this through nearly half a century later[2] with better equipment; he was able to see single lines polarized as predicted by Lorentz. This experiment aims to observe the hyperfine transition near the 5461 Å green emission line of a mercury vapor lamp in the absence of a magnetic field (hereafter simply called “field”). In addition, this experiment aims to observe the Zeeman splittings of the 5461 Å green line as well as the 5769 Å and 5790 Å yellow doublet lines of a mercury vapor lamp in the presence of a field and use these to find the charge to mass ratio $\frac{q}{m}$ for the electron. These are done by using a Fabry-Perot interferometer (hereafter abbreviated “FP”).

I. THEORETICAL BASIS

I.1. Energy Splittings

The electron and nucleus both have magnetic moments owing to their angular momenta; the electron has spin and orbital angular momenta, while the nucleus approximately has only spin. The nuclear magnetic moment is typically quite small in comparison to that of the electron due to its much larger mass, so the interaction can be approximated as being just between the total electron magnetic moment and the external field. The interaction energy is given by $E = -\mu \cdot B$, where the electron magnetic moment is

$$\mu_J = -\mu_B g_J m_J$$

(2)

where the Bohr magneton $\mu_B$ contains $\frac{\hbar}{m}$, $m_J$ is the spin-orbit magnetic quantum number, and the Landé g-factor is related to the 3 electron angular momentum quantum numbers by

$$g_J \approx \frac{3}{2} + \frac{(s+1)s - (l+1)l}{2(j+1)j}.$$  

(3)

**FIG. 1.** Green line Zeeman splitting allowed transitions

The splitting of parent states by the Zeeman effect is shown in figure 1 for the green line; the diagrams are similar for each of the yellow doublet lines. Selection rules[4] require that transitions only occur for $\Delta m_J \in \{0, \pm 1\}$. Note that $g_J$ can also change for transitions between parent states. The definition $\epsilon \equiv \frac{E}{\mu_B B}$ means that a given transition will have $\Delta \epsilon = \Delta(g_J m_J)$. The transition at 5790 Å between the $^1P_1$ and $^1D_2$ states has three emission lines each with threefold degeneracy due to $g_J$ being 1 for both parent states, and with $\Delta \epsilon = 1$ between neighboring lines; the low- and high-energy lines...
are $\sigma$ polarized, while the mid-level line is $\pi$ polarized. For the 5461 Å and 5769 Å transitions between the $^3P_2$ & $^3S_1$ and $^3D_2$ & $^3P_1$ states, respectively, $g_J$ changes between parent states, so the two $\sigma$ and one $\pi$ polarized lines split further into groups of three lines. The 5461 Å line has $\Delta \epsilon = \frac{1}{2}$ for neighboring lines within the $\sigma$ or $\pi$ groups and $\Delta \epsilon = \frac{3}{2}$ for the middle lines of the $\sigma$ and $\pi$ groups; the 5769 Å line has $\Delta \epsilon = \frac{1}{6}$ for neighboring lines within a group and $\Delta \epsilon = \frac{5}{6}$ for the middle lines of neighboring groups.

### I.2. Fabry-Perot Interferometry

The heart of this experiment lies in the FP, as in figure 2 (left). It consists of two plane mirrors that are close together. Light refracts through the first mirror with an angle depending on its wavelength. This light then reaches the second mirror, at which point it may be refracted again in a wavelength-dependent manner or it may be reflected back to the first mirror. Some light may bounce between the mirrors several times before being transmitted beyond the second mirror, leading to phase differences between other parts of the light that bounced back and forth more or fewer times. These phase differences lead to interference beyond the second mirror, and as the light source is fairly far from the observer, the interference patterns form circular fringes.

In particular, if the mirror separation is $D$, then constructive interference for a wavelength $\lambda$ occurs when $2D \cos(\theta) = n\lambda$ where $n \in \mathbb{Z}$. To make lines of wavelengths $\lambda$ and $\lambda + \Delta\lambda$ of the same interference order $n$ coincide requires a change in the separation given by $\delta D = \frac{n\Delta \lambda}{2 \cos(\theta)}$, while to make lines of wavelength $\lambda$ of neighboring orders $n$ and $n + 1$ coincide requires a separation change $\Delta D = \frac{\lambda}{2 \cos(\theta)}$. Defining $R = \frac{\delta D}{\Delta D}$ and using the faraway point-source approximation to set $\cos(\theta) \approx 1$, then

$$\left| 2 \cos(\theta) \right| = \frac{R}{2D}$$

is the equation for the free spectral range (FSR) of the interferometer, which shows the maximum range of wavelengths of a given interference order that can be displayed free of overlap from adjacent orders\cite{5} when $R = 1$.

The Zeeman splitting is, in a typical experimental setup, less than the FSR, so $R$ represents the ratio of the Zeeman splitting to the FSR. Given that the energy splitting is $\Delta E = 2\pi \hbar c \Delta \left( \frac{1}{\lambda} \right)$, turning to equations and constants in Zeeman theory yields

$$\frac{2\pi c}{D} \Delta \epsilon = q \frac{m}{m} B$$

as the desired relation from which $\frac{q}{m}$ can be extracted.

### II. APPARATUS AND METHOD

A schematic of the apparatus is shown in figure 2 (right). Mounted between two large water-cooled magnets is a mercury vapor lamp; each has its own power supply, and together they are placed adjacent to the optical table. In a line on the optical table are, in order of increasing distance from the lamp, an NRC field lens with focal length $40 \pm 0.5 \text{ cm}$ for focusing the light, an interference filter to let only a small range of wavelengths into the FP, a Burleigh FP, and a telescope with focal length $43.5 \pm 0.5 \text{ cm}$. The FP has the mirror near the lamp fixed, while the mirror farther away can be moved and rotated with respect to the closer mirror through three micrometers that can be manually adjusted. Furthermore, that mirror has piezoelectric crystals for finer adjustment than what the micrometers can provide; these are operated electrically by a Burleigh FP controller, which can control each crystal individually and also output an overall bias voltage to all three if desired. Furthermore, the sensitivity of each crystal to the FP controller output can be changed through the trim controls. The FP controller can output a sawtooth ramp voltage whose amplitude and duration can be set on that same controller as well; this is one input for the Rigol digital oscilloscope. The telescope has an eyepiece to view the interference image, though there is also sufficient room between the FP and the telescope to look in directly. The telescope is connected to a photomultiplier tube, which takes in light through a small aperture in the telescope in order to create a cascade of electrons through a series of electrodes that eventually produces a measurable electrical signal; the photomultiplier has its own power source. This signal is then sent through a PAR amplifier, and that amplified signal is the other input for the oscilloscope.

The lamp was generally set to DC rather than AC for each measurement as the image stabilized better after running for about 15 minutes on AC and then being switched to DC. The amplifier settings were all unchanged at their defaults. The photomultiplier voltage supplied was 900 V. The trim knobs on the FP controller were all at their default positions, meaning they were each turned about 43° of the range away from the far counterclockwise position. The FP controller ramp duration was always 5 s. The oscilloscope had a horizontal...
The calibration curve was used to characterize the green line in the absence of a field, though the micrometer reading stayed at 10.658 ± 0.005 mm throughout the experimental session. After removing the lens, the brightness of the image was recovered without noticeably affecting the contrast by increasing the lamp current to 20 mA.

## III. EMISSION ABSENT FIELD

### III.1. Mirror Separation

![FIG. 3. Fitting green line for mirror separation and hyperfine shift calculations (top-left); Highest field green line raw data (top-right); Same data fitted to 3 peaks (bottom-left); Same data compared with lowest field green line data (bottom-right).]

The micrometer reading corresponding to the mirrors touching was determined as part of this experiment. As described above, an interferogram of the green line was taken at two different micrometer readings such that in each, a large peak and a small peak would be visible in each FSR, and two such copies would be visible within a voltage ramp. For each interferogram, each pair of adjacent small and large peaks was fitted to a sum of two Gaussians with a vertical offset as seen in the top-left of figure 3 using the gradient search algorithm, as this provided more consistent convergence in fitting. The means of adjacent small and large peaks were subtracted to find the separation of the two features in an FSR, and because each interferogram had two peak separations each with its own uncertainty, these separations were combined as a weighted mean. Each interferogram had an FSR represented by the difference in the means of the two large peaks in a ramp. The ratio $R$, as defined before, was calculated by dividing the weighted mean peak separation by the FSR width. There were two values of $R$, each corresponding to a different micrometer reading. Hence, a linear fit $D_m = aR + D_0$ was performed, where $a = \frac{\Delta x}{\Delta R}$ comes from the FSR equation. $D_m$ is the micrometer reading, and $D_0$ is the desired micrometer reading when the mirrors touch each other. From this, the zero separation of the mirrors corresponds to a micrometer reading of 7.047 ± 0.055 mm.
III.2. Hyperfine Splittings

In the absence of a field, the hyperfine splitting of the green line should be visible. Indeed, there is a small, relatively flat peak to the left of the large peak, and this mere existence of an additional feature within the FSR is all that is necessary to determine of the mirror zero position. However, if this is in fact a hyperfine line, then it should be separated from the main peak by the correct amount. The FSR equation gives the wavenumber separation between the main and hyperfine lines as \( \frac{\Delta \lambda}{\lambda} = \frac{1}{D} \) where \( R \) is once again the ratio of the separation between the adjacent small and large peaks to the separation between the two large peaks in the ramp. Using \( D = 3.611 \pm 0.055 \text{mm} \) for the mirror separation while taking \( \lambda = 5461 \text{Å} \) to be exact gives the frequency shift (obtained by multiplying the wavenumber shift by the speed of light) as \( \frac{\Delta \nu}{\nu} = 12.70 \pm 0.19 \text{GHz} \). The main line is largely due to emission by \(^{198}\text{Hg} \), and there is a hyperfine line\(^6\) due to \(^{201}\text{Hg} \) with a frequency shift magnitude of 12.21 GHz. The measured shift differs from this reference shift value by 2.5 times the uncertainty on the measured shift, while the relative uncertainty is only 1.5% of this measured shift, so it is safe to call this feature the \(^{201}\text{Hg} \) hyperfine emission line corresponding to the transition between the 6s6p\(^3\)P\(_2\)(\( f = \frac{5}{2} \)) and 6s7s\(^3\)S\(_1\)(\( f = \frac{5}{2} \)) states.

IV. ZEEMAN SPLITTINGS

Similar to the analysis of the mirror zero position, oscilloscope traces as in figure 3 (top-right) of the green and yellow lines were taken when a set of Zeeman triplets along with a copy of at least one of those peaks became visible within a ramp duration; further Zeeman splittings for applicable lines could not be observed. Each of the peaks in a major Zeeman triplet were fitted to a sum of three Gaussians as seen in figure 3 (bottom-left) with a vertical offset using the Levenberg-Marquardt algorithm for the green lines, as \( \chi^2 \) was more stable when the initial fit parameter estimates changed, or using the gradient search algorithm for the yellow lines, as this provided better convergence. For a given Zeeman triplet, the separation between the left and middle peaks was found, as was the separation of the middle and right peaks; together these yielded an uncertainty-weighted mean of the two separations. Meanwhile, copies of a single peak were fitted using the gradient search algorithm again for better convergence. The representation of the FSR was the difference between the copied peak locations. The ratio of the average Zeeman separation to the FSR is \( R \). The green and each of the two yellow lines have different values of \( \Delta \epsilon \), and \( D \) changed between the green and yellow measurements, so in each case the list of \( y \)-values to be linearly fitted becomes \( \frac{2 \pi c}{\nu} R \), where \( R \) and \( D \) have uncertainties but \( c \) and \( \Delta \epsilon \) are assumed to be exact, and the list of \( x \)-values is simply the list of corresponding fields applied (in teslas), with \( \frac{2}{\nu} \) being the slope.

IV.1. Green

Eight interferograms were taken for the green lines, each corresponding to a different field. The fit of \( \frac{2 \pi c}{\nu} R \) yields \( \frac{2}{\nu} = (2.17 \pm 0.29) \cdot 10^{11} \text{C} \). The accepted value for \( \frac{2}{\nu} \) is \( 1.76 \cdot 10^{11} \text{C} \), so the difference is about 1.5 times the uncertainty on the calculation, and the relative error is 13%. However, looking at the linear fit graph in figure 4 (left) brings some problems to light. The main issue is that the points do not fall easily into a line; they appear to fit better to a convex increasing curve. This is evidenced by \( \Delta \epsilon = 5.12 \) for only 8 data points being fit to a 2-parameter function. It seems like the lower \( x \) points should have a higher \( y \) and the higher \( x \) points should have a lower \( y \) so that the points can be more easily connected by a line. This could be done by changing \( D \) or \( \Delta \epsilon \) individually for points; because both \( D \) and \( \Delta \epsilon \) lie in the denominator of the \( y \)-values being fitted, either one or both would have to increase as a function of the field for the higher \( x \) points to decrease in \( y \)-value more than the lower \( x \) points do.

Comparing interferograms of green lines for the minimum and maximum fields as in figure 3 (bottom-right) illustrates the problem better. The peaks of the two graphs are aligned around \( t = 2 \text{s} \). These are copies of peaks that are around \( t = -1 \text{s} \) for the low field point and \( t = -0.5 \text{s} \) for the high field point. This indicates that the FSR is different between the two measurements, yet the wavelength is unchanged. Although \( D \) was reported to be unchanged for all the green line Zeeman measurements, it is possible that it did change by accident. To rectify this, the high field interferogram can be stretched so that its FSR matches that of the low field interferogram. However, increasing the FSR means decreasing \( D \), which would in fact cause the high field points in the linear fit to take on even higher \( y \)-values, yielding even less linearity. Furthermore, in all these fits, it has been assumed that if only 3 rather than 9 Zeeman peaks can be observed, then the 3 major Zeeman peaks are caused by the middle peaks in the minor Zeeman triplets, allowing for \( \Delta \epsilon \) to be constant for a given wavelength. However, it is apparent that the low field \( \sigma \) peaks come from more outer minor peaks than do the high field \( \sigma \) peaks, implying that \( \Delta \epsilon \) is smaller for higher fields than for lower fields. This too would cause the \( y \)-values to grow more for the high field than for the low field, which again gives less linear behavior. Hence, these two situations can be disregarded as candidates for possible systematic error. Some other systematic effect must have been causing the apparent change in the FSR as well as the apparent non-linearity of the points being fitted to a line.

IV.2. Yellow

The interferograms of the yellow doublet displayed two lines, the left line displaying a larger amplitude than the
right line, and each splitting into major Zeeman triplets. Again, 8 interferograms were taken through oscilloscope traces, each one corresponding to a different field, when the Zeeman triplet was observed on the oscilloscope to have split sufficiently. However, upon further analysis, the interferogram corresponding to the lowest field was observed to not have split fully for either doublet peak, yielding large errors in the peak fitting procedure: this field was most likely too low compared to the other fields to fully effect Zeeman splitting, so that interferogram was discarded. In each of the remaining seven interferograms, regions of three peaks for each doublet were fitted to a sum of three Gaussians with a vertical offset in a manner similar to that of the green line analysis. In addition, copies of an individual peak were fitted to a single Gaussian with a vertical offset in order to find the FSR. The rest of the analysis is identical to that of the green lines in finding $\frac{\Delta \nu}{\nu}$. However, it was also found that the ramp voltage started out flat for a little while before sloping up. This did not affect the smaller doublet line which appeared in the middle of the ramp, but it did cause the Zeeman peak separations in the larger doublet line to become artificially lowered as the ramp quickly accelerated from flat to sloped. Only five of the seven interferograms had an additional copy of the larger doublet line splitting into a major Zeeman triplet fully within a single ramp, so the fitting for finding $\frac{\Delta \nu}{\nu}$ from the larger doublet line only used data from those five interferograms.

\[ \frac{\Delta \nu}{\nu} = (2.88 \pm 0.47) \times 10^{11} \frac{C}{kg}, \text{ with } \Delta \nu = 2.71 \text{ for the 5790 Å line as in figure 4 (right).} \]

The relatively large uncertainty can be explained in large part by a problem with the digital oscilloscope that was causing large swathes of interferograms to become much more granular. This made fitting to find peak locations much more difficult for the yellow doublet lines than for the green line, and the error analysis was found to be rather sensitive to the uncertainties on the peak locations. The closeness of the values and uncertainties of $\frac{\Delta \nu}{\nu}$ for both doublet lines implies that the deviation of the measured values from the accepted value can probably be attributed to a single source. This source of error could again be the poorer performance of the oscilloscope, as the peaks being fitted had means that were slightly more separated than an estimate by eye would yield for the peak locations. The difference is on the order of the ratio of the measured to accepted $\frac{\Delta \nu}{\nu}$, though this is not investigated further. Taking the weighted mean of the two yellow values and using the difference as a lower bound for systematic error yields $\frac{\Delta \nu}{\nu} = (2.88 \pm 0.37_{st} \pm 0.01_{sy}) \times 10^{11} \frac{C}{kg}$, which differs from the accepted value by 2.9 times the overall uncertainty.

\[ \frac{\Delta \nu}{\nu} = (2.88 \pm 0.01) \times 10^{11} \frac{C}{kg}, \text{ with } \Delta \nu = 2.71 \text{ for the 5790 Å line.} \]

V. CONCLUDING REMARKS

The Zeeman effect was observable in the green and yellow emission lines of a mercury vapor lamp through splitting of lines into triplets in the presence of a magnetic field. Further Zeeman splitting into 9 lines for the 5461 Å and 5769 Å lines was not observed. In the absence of a field, hyperfine splitting was observed for the green line corresponding to the $^{201}\text{Hg}$ emission. Finally, using a digital oscilloscope appears to compromise the data in this experiment, so use of an analog oscilloscope is more highly recommended.

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