Problem 0: Please read the course notes through chapter 6.

Problem 1 (I&D 2.25 mod)

A plane wall of thickness $2L = 40$ mm and thermal conductivity $k = 5$ W/mK experiences uniform volumetric heat generation at a rate of $q''' [W/m^3]$, while convection heat transfer occurs at both of its surfaces ($x = -L, +L$), each of which is exposed to a fluid of temperature $T_\infty = 20$ °C. Under steady-state conditions, the temperature distribution in the wall is of the form $T(x) = a + bx + cx^2$, where $a = 82$ °C, $b = -210$ °C/m, $c = -2 \times 10^4$ °C/m$^2$, and $x$ is in meters. The origin of the $x$-coordinate is at the midplane of the wall.

a) Sketch the temperature distribution as a function of $x$.

b) If the temperature profile was not known, provide the differential equation that you need to solve with the appropriate boundary conditions.

c) What is the volumetric rate of heat generation $q''' [W/m^3]$ in the wall?

d) Determine the surface heat fluxes, $q_x(-L)$ and $q_x(+L)$. How are these fluxes related to the heat generation rate?

e) What are the heat transfer coefficients for the surfaces at $x=-L$ and $x=+L$?

f) Obtain an expression for the heat flux distribution, $q_x(x)$. Is the heat flux zero at any location? Explain any significant features of the distribution.

g) If the source of the heat generation is suddenly turned off ($q''' = 0$), what is the rate of change of energy stored in the wall immediately afterwards?

h) What temperature will the wall eventually reach with $q''' = 0$? How much energy must be removed by the fluid per unit area of the wall [J/m$^2$] to reach this state? The density and specific heat of wall material are 2600 kg/m$^3$ and 800 J/kgK, respectively.
Problem 2 (I&D 5.29 mod)

Thermal stress testing is a common procedure used to assess the reliability of an electronic package. Typically, thermal stresses are induced in soldered or wired connections to reveal mechanisms that could cause failure and must therefore be corrected before the product is released. As an example of the procedure, consider a multi-chip module with an array of silicon chips \((\rho_{ch}=2300 \text{ kg/m}^3, c_{ch}=710 \text{ J/kgK}, k_{ch}=150 \text{ W/mK})\) joined to an alumina substrate \((\rho_{sb}=4000 \text{ kg/m}^3, c_{sb}=770 \text{ J/kgK}, k=40 \text{ W/mK})\) by solder balls \((\rho_{sd}=11,000 \text{ kg/m}^3, c_{sd}=130 \text{ J/kgK}, k_{sd}=80 \text{ W/mK})\). Each chip of width \(L_{ch}\) and thickness \(t_{ch}\) is joined to a unit substrate section of width \(L_{sb}\) and thickness \(t_{sb}\) by solder balls of diameter \(D\).

A thermal stress test begins by subjecting the multichip module, which is initially at room temperature, to a hot fluid stream and subsequently cooling the module by exposing it to a cold fluid stream. The process is repeated for a prescribed number of cycles to assess the integrity of the soldered connections. As a first approximation, assume that there is negligible heat transfer between the components (chip/solder/substrate) of the module in the subsequent analysis.

a) Assuming the heat transfer coefficient, \(h\), is the same for all of the surfaces, justify that a lumped capacitance analysis can be used. The dimensions for the components are \(L_{ch}=15 \text{ mm}, t_{ch}=2 \text{ mm}, L_{sb}=25 \text{ mm}, t_{sb}=10 \text{ mm}, D=2 \text{ mm}\), and \(h=50 \text{ W/m}^2\text{K}\) is the characteristic heat transfer coefficient of the air stream.

b) Obtain expressions for the thermal time constant for each component and numerically evaluate the three time constants. There is heat transfer to all surfaces of a chip, but to only the top surface of the substrate. You can assume that there is no reduction in surface area due to contact between a solder ball and the chip or substrate.

c) Compute and plot the temperature histories \((T(t))\) of the three component for the heating portion of the cycle with \(T_i=20^\circ\text{C}\) and \(T_\infty=80^\circ\text{C}\).

d) At what time does each component experience 99% of its maximum possible temperature rise (i.e., \((T-T_i)/(T_\infty-T_i)=0.99)\)?

e) If the maximum stress on a solder ball corresponds to the maximum difference between its temperature and that of the chip or substrate, when will this maximum occur?

f) To reduce the time required to complete a stress test, a dielectric liquid can be used in lieu of air to provide a larger heat transfer coefficient of \(h=200 \text{ W/m}^2\text{K}\). What is the corresponding savings in time for each component to achieve 99% of its maximum possible temperature rise?
Two copper balls of diameter 2 cm are suspended in an evacuated chamber by a long fine wire as shown above. The fine wire is of negligible mass and thermal conductance. The internal surface of the vacuum chamber and the balls are highly reflective leading to very low surface emissivities for both the balls and the vacuum chamber.

Between the balls is a 1 cm long glass fiber of diameter 0.001 m. Please neglect the mass effects of the glass in your analysis. The thermal conductivity of glass is 1 W/m-K and of copper is 400 W/mK. The density of copper is 8933 kg/m³ and the specific heat capacity is 385 J/kgK.

The balls are initially at temperatures of 425 K (ball 1) and 325 K (ball 2). Because of thermal interactions, the states of the two balls “run-down” to equilibrium over time.

a) Please develop a “Biot-like” argument to show that the temperature gradients in the glass fiber (and not the thermal gradients in the balls) dominate the dynamics of the problem.
b) Please show that the sum of the temperatures of ball 1 and ball 2 does not vary with time. And consequently, what is the final temperature for ball 1 and ball 2?
c) Please develop a differential equation for the temperature of ball 1 as a function of time and the temperature difference between ball 1 and ball 2.
d) By developing the analogous equation to the one you developed in part c for ball 2 and combining it with the equation you developed in part c, please determine how much time passes before the temperature of ball 1 reaches 390 K.
Problem 4 (M 2.47mod n)
On the flight of Apollo 12, plutonium oxide (Pu$^{238}$O$_2$, $k = 4$ W/mK) was used to generate
electrical power. Heat is generated uniformly throughout a spherical “nut” of the oxide through
the loss of kinetic energy from alpha particles emitted by the plutonium. Thermoelectric devices
(mounted on the surface of the sphere) were then used to generate electricity using the plutonium
oxide sphere as their high temperature heat source. The thermal limits on the thermoelectric
materials do not allow the surface of the oxide sphere to be hotter than 200°C. The limits on the
plutonium oxide, however, allow temperatures as high as 1750°C.

a) Please write the heat equation in a form relevant to determining the temperature
distribution in the plutonium oxide. Explain your reasoning in developing this ODE
from the PDE derived in class. What are the boundary conditions at the origin?

b) Please solve your ODE. What is the functional dependence of the temperature on the
radius?

c) What is the maximum allowable volumetric heating rate if the sphere is 4 cm in
diameter?

d) What is the electrical power that is generated assuming the thermoelectrics have a
thermal efficiency of 4%?

Suppose that the plutonium oxide nut were placed in an inert spherical sheath of conductivity $k$.
The inner radius of the sheath is $r_i$ and the outer radius is $r_o$.

e) How does the equation you developed in part a change to model the temperature
distribution in this sheath?

f) Please find an expression for the temperature distribution in the sheath in terms of the
two surface temperatures.

g) Please find an expression for the total heat transfer through the sheath in terms of the
two surface temperatures.

h) Please find an expression for the thermal resistance ($\Delta T/\dot{Q}_{tot}$) of this sheath in terms
of $k$, $r_i$, and $r_o$. What is the corresponding expression if the sheath where a cylindrical
one around a cylindrical pipe?