Lecture Note 2

Agency Theory, Part II:

Getting What You Pay For

Business history is littered with firms that got what they paid for. At the H.J. Heinz Company, for example, division managers received bonuses only if earnings increased from the prior year. The managers delivered consistent earnings growth by manipulating the timing of shipments to customers and by prepaying for services not yet received. At Dun & Bradstreet, salespeople earned no commission unless the customer bought a larger subscription to the firm’s credit-report services than in the previous year. In 1989, the company faced millions of dollars in lawsuits following charges that its salespeople deceived customers into buying larger subscriptions by fraudulently overstating their historical usage. In 1992, Sears abolished the commission plan in its auto-repair shops, which paid mechanics based on the profits from repairs authorized by customers. Mechanics misled customers into authorizing unnecessary repairs, leading California officials to prepare to close Sears’ auto-repair business statewide.

In each of these cases, employees took actions to increase their compensation, but these actions were seemingly at the expense of long-run firm value. At Heinz, for example, prepaying for future services greatly reduced the firm’s future flexibility, but the compensation system failed to address this issue. Similarly, at Dun & Bradstreet and Sears, although short-run profits increased with the increases in subscription sizes and auto repairs, the long-run harm done to the firms’ reputations was significant (and plausibly much larger than the short-run benefit), but the compensation system again ignored the issue. Thus, in each of these cases, the cause of any dysfunctional behavior was not pay-for-performance per se, but rather pay-for-performance based on an inappropriate performance measure. (Baker, Gibbons, and Murphy, 1994)

1. Introduction

In discussing the classic agency model (in “Agency Theory, Part I: Pay for Performance”) we made many simplifying assumptions. In some cases we claimed that relaxing a particular assumption would not alter the main message of the basic model. For example, we concluded that steeper slopes create stronger incentives. We did this for the case of a risk-neutral Agent, but the conclusion would continue to hold if we enriched the basic model to allow the Agent to be risk-averse. Under the new assumption of risk-aversion there would also be an additional consideration: steeper slopes would impose more risk on the Agent. The expositional advantage of the basic model is that the connection between the contract’s slope and the strength of the Agent’s incentive is illustrated as clearly as possible—there really is nothing else going on in the basic model!
Other assumptions in the basic model are much more important—if these assumptions change then the spirit of the analysis changes dramatically. In this note we change an assumption of this latter kind: instead of assuming that the Agent’s contribution to firm value can be observed by the Principal and the Agent (and also by a court if necessary to enforce the compensation contract) we now assume that the only performance measures that can be observed by the Principal and the Agent (and a court) are distortionary, in a sense described below.

It is important to appreciate just how strong an assumption is being made in the classic agency model. In some employment relationships, such as those involving team production, even the Agent may be hard pressed to identify his or her contribution to firm value. In other contexts, the Agent and the Principal may both observe the Agent’s contribution but a court may be unable to do so. (As discussed in Lecture Note 1, think of the Agent’s contribution as not just the number of units produced but also their quality, whether they were produced in a timely fashion, and so on. A court may be able to count the number of units produced, but it is unlikely to be able to document all the details that insiders observe for free.) Finally, and perhaps most frequently, the Agent may know how good a job he or she did, but the Principal (not to mention a court) may not. For example, the Principal may know what was done but not what could have been done.

In this note we analyze a static model in which the Agent’s contribution to firm value (y) cannot be measured and so cannot be used as a performance measure in an incentive contract. In the classic agency model we often thought of the Principal and Agent as a firm’s shareholders and CEO, respectively. If the Agent is the CEO then measuring the dollars flowing into the Principal’s pocket (i.e., the change in shareholder wealth) may provide a useful (if noisy) measure of the Agent’s contribution to firm value. But suppose instead that the Agent is a division manager. In this case the firm’s stock performance may bear little relation to the manager’s performance. The division’s accounting earnings may be a superior measure in some respects (being more focused on the manager’s activities and less subject to firm- and market-wide shocks), but earnings lack the forward-looking character of stock prices, and a single division’s earnings ignore the firm-wide consequences of the manager’s actions (thus potentially inducing managers to “stay in their silos”).

If the Agent’s contribution to firm value cannot be measured then one might instead base a contract on an alternative performance measure, such as the number of units produced (with no adjustment, or at best limited adjustment, made for quality, timely
delivery, the conditions of production, and so on). This note analyzes such contracts in a static setting akin to the classic agency model, focusing on the distortions that such contracts induce.¹

2. An Informal Introduction to “Multi-Task” Incentive Problems²

In this section I discuss models in which firms get what they pay for. The key innovation in these models is to reject a strong but unremarked assumption in the classic agency model, where y is sometimes labeled “output,” as though it could easily be measured. This label is misleadingly simple: in the classic model y reflects everything the principal cares about, except for wages (that is, the principal's payoff is y - w). Therefore, I henceforth call y the Agent's “total contribution to firm value,” to emphasize that it encompasses all the Agent's actions (including mentoring, team production, and so on) and all the effects of these actions (both long- and short-run). In many settings, it is very difficult to measure synergies or sabotage across Agents and/or very difficult to predict the long-run consequences of an Agent's actions based on the observed short-run contribution. To analyze such settings, I impose the following assumption throughout the rest of the lecture notes in this course: no contract based on y can be enforced in court, including but not limited to the linear contract w = s + by.

Of course, even when contracts based on y are not available, other contracts can be enforced in court. Such contracts are based on alternative performance measures—such as the number of units produced, with limited adjustment made for quality, timely delivery, and so on. Let p denote such an alternative performance measure; the wage contract might then be linear, w = s + bp. As in the classic agency model, a large value of b will create strong incentives, but now the Agent's incentives are to produce a high value of p, not of y. But the firm does not directly benefit from increased realizations of measured performance, p; rather, the firm benefits from increased realizations of the Agent’s total contribution to firm

¹ In “Repeated Games, Part II—Subjective Performance Evaluation” we take a different approach to this problem: we use a dynamic model to discuss “relational contracts” that are enforced by the parties’ concerns for their reputations rather than by the power of a court. In this dynamic model the Agent’s contribution to firm value is observed by the Agent and the Principal but not by a court. In this sense, the Agent’s performance can be subjectively assessed but not objectively measured. The model explores the extent to which the Principal can credibly promise to pay a bonus based on such a subjective assessment of the Agent’s contribution to firm value. We will see that such subjective bonuses are very important in many settings, both literally (e.g., in investment banking) and by analogy (e.g., in promotion decisions).

² This section draws on Gibbons (1998), Holmstrom and Milgrom (1991), and Baker (2000).
value, \( y \). The essence of the incentive problem is this divergence between the Agent’s incentives to increase \( p \) and the firm’s desire for increases in \( y \).

To investigate this idea a bit more formally, consider first a simple extension of the classic agency model: \( y = a + \varepsilon \) and \( p = a + \phi \). In this case the contract \( w = s + bp \) creates incentives to increase \( p \) and the induced action also increases \( y \). But now suppose that there are two kinds of actions (or “tasks”) that the Agent can take, \( a_1 \) and \( a_2 \). In this setting, the contract \( w = s + bp \) creates incentives that depend on the bonus rate \( b \) and on the way the actions \( a_1 \) and \( a_2 \) affect the performance measure \( p \). For example, if \( y = a_1 + a_2 \) and \( p = a_1 \) then a contract based on \( p \) cannot create incentives for \( a_2 \) and so misses this potential contribution to \( y \). Alternatively, if \( y = a_1 \) and \( p = a_1 + a_2 \) then a contract based on \( p \) creates an incentive for the Agent to take action \( a_2 \), even though \( a_2 \) is irrelevant to the Agent’s total contribution to firm value. Finally, in an extreme case such as \( y = a_1 + \varepsilon \) and \( p = a_2 + \phi \), the contract \( w = s + bp \) creates no value at all.

The general theme of these “multi-task” examples is that it is no use creating strong incentives for the wrong actions. That is, if attaching a large bonus rate \( b \) to the performance measure \( p \) would create strong but distorted incentives then the optimal bonus rate may be quite small. We now investigate these issues more formally.

All of the examples above are special cases of the following two-task model (which itself can easily be extended to include more actions and other enrichments). Compared to the classic agency model, the chief departure here is the introduction of a fourth element of the model: in addition to the technology of production, the contract, and the payoffs, we now also require a technology of performance measurement.

Suppose that the technology of production is \( y = f_1 a_1 + f_2 a_2 + \varepsilon \), the technology of performance measurement is \( p = g_1 a_1 + g_2 a_2 + \phi \), the contract is \( w = s + bp \), and the payoffs are \( y - w \) to the Principal and \( w - c(a_1, a_2) \) to the Agent. To keep things simple, assume that
\[
c(a_1, a_2) = \frac{1}{2} a_1^2 + \frac{1}{2} a_2^2,
\]
but notice that this assumption rules out the potentially important case where the actions “compete for the Agent’s attention” (i.e., increasing the level of one action increases the marginal cost of the other).

The timing of events in this model is essentially the same as in the classic agency model, except that it is modified to incorporate the new distinction here between \( y \) and \( p \):
1. The Principal and the Agent sign a compensation contract \( w = s + bp \) (which we take to be linear for the reasons discussed in Lecture Note 1—namely, analytical simplicity and constant incentives).

2. The Agent chooses actions \( (a_1 \text{ and } a_2) \) but the Principal cannot observe these choices.

3. Events beyond the Agent’s control \((\epsilon \text{ and } \phi)\) occur.

4. The actions and the noise terms determine the Agent’s contribution to firm value \((y)\) and measured performance \((p)\).

5. Measured performance is observed by the Principal and the Agent (and by a Court, if necessary).

6. The Agent receives the compensation specified by the contract, as a function of the realized value of \( p \).

In this setting, the (risk-neutral) Agent chooses the actions \( a_1 \text{ and } a_2 \) to maximize the expected payoff \( E(w) - c(a_1, a_2) \) and so must solve the following problem:

\[
\max_{a_1, a_2} s + b (g_1 a_1 + g_2 a_2) - \frac{1}{2} a_1^2 - \frac{1}{2} a_2^2.
\]

The Agent’s optimal actions are therefore \( a_1^* (b) = g_1 b \) and \( a_2^* (b) = g_2 b \), analogous to the special case of the classic agency model where \( c(a) = \frac{1}{2} a^2 \) and so \( a^* (b) = b \). (Note that the salary \( s \) again does not affect the Agent’s optimal actions; the slope of the contract completely determines the Agent’s incentives; the salary is just a transfer of wealth that does not affect incentives.)

To finish analyzing this model we must determine the optimal level of \( b \). It turns out that there is a single value of \( b \) that is efficient. That is, if the parties were to sign a contract with a slope other than this efficient value then they could both be made better off by switching to a contract with the efficient slope (and a new value of \( s \) chosen to make both parties better off after the switch).

To derive the efficient value of \( b \), note that the Principal’s expected payoff from the contract \( w = s + bp \) is

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3 For simplicity, in this model no one ever observes the Agent’s total contribution, even though the Principal eventually receives the payoff \( y - w \). See Lecture Notes 3 and 4 on relational contracts and subjective performance assessment for a more realistic approach to this issue.
where the Agent’s optimal actions in response to the contract have been included in the calculation of the Principal’s expected payoff. Similarly, the Agent’s expected payoff from the contract \( w = s + bp \) is

\[
E(w) - c(a_1, a_2) = s + b[g_1a_1^*(b) + g_2a_2^*(b)] - \frac{1}{2}a_1^*(b)^2 - \frac{1}{2}a_2^*(b)^2.
\]

The sum of these expected payoffs is the expected total surplus

\[
E(y) - c(a_1, a_2) = f_1a_1^*(b) + f_2a_2^*(b) - \frac{1}{2}a_1^*(b)^2 - \frac{1}{2}a_2^*(b)^2.
\]

The efficient value of \( b \) is the value that maximizes this expected total surplus. A little math shows that this efficient value of \( b \) is

\[
b^* = \frac{f_1g_1 + f_2g_2}{g_1^2 + g_2^2}.
\]

This expression for the efficient value of \( b \) may not seem very helpful! Fortunately, it can be restated using Figure 1, which plots both the coefficients \( f_1 \) and \( f_2 \) from the technology of production and the coefficients \( g_1 \) and \( g_2 \) from the technology of performance measurement. (The figure is drawn assuming that \( f_1, f_2, g_1, \) and \( g_2 \) are all positive but this is not necessary.) This figure happens to represent a case in which \( g_1 \) is larger than \( f_1 \) but \( f_2 \) is larger than \( g_2 \). In such a case, paying the Agent on \( p \) will create stronger incentives than the Principal wants for \( a_1 \) but weaker incentives than the Principal wants for \( a_2 \).

There are two important features in Figure 1: scale and alignment. To understand scale, imagine that \( g_1 \) and \( g_2 \) were both much larger than \( f_1 \) and \( f_2 \). Then the Agent can greatly increase \( p \) by choosing high values of \( a_1 \) and \( a_2 \) but these actions will result in a much smaller value of \( y \) (ignoring the realizations of the noise terms for the moment). As a result, the efficient contract should put a small bonus rate on \( p \), as will emerge below. To understand alignment, imagine first that the \( f \) and \( g \) vectors are closely aligned—they lie almost on top of one another (even if one is longer than the other). In this case the incentives created by paying on \( p \) are valuable for increasing \( y \). Alternatively, imagine that the \( f \) and \( g \) vectors are badly aligned—for example, they might be orthogonal to each other (e.g., \( f_1 = 0 \) and \( g_2 = 0 \), so that \( y \) depends on only \( a_2 \) and \( p \) depends on only \( a_1 \)). In this second case the incentives created by paying on \( p \) are useless for increasing \( y \).
Figure 1

It turns out that scale and alignment are hiding in the expression for $b^*$ derived above. With a little more math we can rewrite that efficient slope as

$$b^* = \frac{\sqrt{f_1^2 + f_2^2}}{\sqrt{g_1^2 + g_2^2}} \cos(\theta),$$

where $\theta$ is the angle between the $f$ and $g$ vectors, as shown in Figure 1. Recall (from the Pythagorean Theorem!) that $\sqrt{f_1^2 + f_2^2}$ is the length of the $f$ vector, and correspondingly for $\sqrt{g_1^2 + g_2^2}$, so $\sqrt{f_1^2 + f_2^2} / \sqrt{g_1^2 + g_2^2}$ reflects scaling. For example, if $g$ is much longer than $f$ (as considered above) then the efficient contract should put a small weight on $p$, as shown in this second expression for $b^*$. Recall also that $\cos(0) = 1$ and $\cos(90) = 0$, so $\cos(\theta)$ reflects alignment. For example, if the $f$ and $g$ vectors are closely aligned then $\cos(\theta)$ is nearly 1 so $b^*$ is large, whereas if the $f$ and $g$ vectors are almost orthogonal then $\cos(\theta)$ is nearly 0 so $b^*$ is small.
At one level, the analysis in this section merely formalizes the first two lessons given in the previous section: 1) objective performance measures typically cannot be used to create ideal incentives and 2) efficient bonus rates are consequently often small. At another level, however, we have achieved two things. First, we have understood the two determinants of the efficient bonus rate $b^*$—scale and alignment. Second, we can now dispel a persistent confusion about what makes a good performance measure, as follows.

One might be tempted to say that $p$ is a good performance measure if it is highly correlated with $y$. But what determines the correlation between $p$ and $y$? That is, what would cause $p$ and $y$ to move together if we watched them over time? Given technologies such as $y = f_1 a_1 + f_2 a_2 + \varepsilon$ and $p = g_1 a_1 + g_2 a_2 + \phi$, one important part of the answer involves the two variables we have not discussed thus far—the noise terms $\varepsilon$ and $\phi$. Simply put, $p$ and $y$ will move together over time if the noise terms are highly correlated, regardless of the Agent’s actions. For example, suppose that $p$ is a division’s accounting earnings and $y$ is the firm’s stock price: both are hit by business-cycle variations (noise terms) but earnings reflect only short-run actions while the stock price incorporates both short- and long-term actions. Thus, the earnings and the stock price might be highly correlated because of their noise terms, even though paying on one creates distorted incentives for the other (namely, ignoring long-run actions). Put more abstractly, $y$ and $p$ will be highly correlated if $y = a_1 + \varepsilon$ and $p = a_2 + \varepsilon$, but in this case $p$ is clearly a lousy performance measure. This argument leads to the second important conclusion of this model: $p$ is a valuable performance measure if it induces valuable actions, not if it is highly correlated with $y$. In short, alignment is more important than noise.

References


