Lecture Note 3

Repeated Games, Part I:
Introduction to Relational Contracts

Game theory is rampant in economics. Having long ago invaded industrial organization (the sub-field of economics that studies firms and their product markets), game-theoretic modeling is now commonplace in international, labor, macro, and financial economics, and is gathering steam even in development economics and economic history. Nor is economics alone: accounting, law, marketing, political science, and even sociology are beginning similar experiences.

Why is this? Broadly speaking, two views are possible: fads and fundamentals. While I believe that fads are partly to blame for the current enthusiasm for game theory, I also believe that fundamentals are an important part of the story. Simply put, many economists use game theory because it allows them to study the implications of rationality, self-interest, and equilibrium when the theory of perfect competition does not apply—such as where markets are imperfectly competitive, or where markets are only peripherally relevant, such as in the relationship between a regulator and a firm or a boss and a worker.

By my armchair citation count, repeated games have been applied more broadly than any other game-theoretic model—not only in economics but also in other social sciences and management fields. I believe this is because repeated games capture the following fact of life: when people interact over time, threats and promises concerning future behavior may influence current behavior.

I do not mean to imply that this logic is surprising or rare. To the contrary, I think it is simple and ubiquitous. In this note I present the simplest possible formalization of this logic. I first describe a one-shot interaction between two parties (which works out badly) and then analyze an ongoing relationship in which such interactions occur repeatedly (and work out well because of the parties’ concerns for their reputations). The ongoing-relationship model shows how repeated games allow economists to analyze some aspects of “trust,” “norms,” and “culture” in organizations (but see the concluding section of this

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note for more on these tricky words). More specifically, repeated-game models allow economists to analyze “relational contracts:” informal agreements and unwritten codes of conduct that powerfully affect behavior, both within firms and between.

Firms are riddled with such relational contracts. There are often informal *quid pro quos* between co-workers, as well as unwritten understandings between bosses and subordinates about task-assignment, promotion, and termination decisions. Even ostensibly formal processes such as compensation, transfer pricing, internal auditing, and capital budgeting often cannot be understood without consideration of their associated informal agreements.

Business dealings are also riddled with relational contracts. Supply chains often involve long-run, hand-in-glove supplier relationships through which the parties reach accommodations when unforeseen or uncontracted-for events occur. Similar relationships also exist horizontally, as in the networks of firms in the fashion industry or the diamond trade, and in strategic alliances, joint ventures, and business groups. Whether vertical or horizontal, these relational contracts influence the behaviors of firms in their dealings with other firms.

Both within and between firms, relational contracts help circumvent difficulties in formal contracting (*i.e.*, contracting enforced by a third party, such as a court). For example, a formal contract must be specified *ex ante* in terms that can be verified *ex post* by the third party, whereas a relational contract can be based on outcomes that are observed by only the contracting parties *ex post*, and also on outcomes that are prohibitively costly to specify *ex ante*. A relational contract thus allows the parties to utilize their detailed knowledge of their specific situation and to adapt to new information as it becomes available. For the same reasons, however, relational contracts cannot be enforced by a third party and so must be self-enforcing agreements (similar to the idea of Nash equilibrium familiar from game theory): each party’s reputation must be sufficiently valuable that neither party wishes to renge.

1. A One-Shot Interaction

Suppose that late last night an exciting new project occurred to you. The project would be highly profitable, but is outside your area of expertise, so you would need help in completing it. Furthermore, it would take significant work on your part to explain the project
to someone with the needed expertise. Finally, if you did explain the project to the relevant other, that person could steal your ideas and represent them as primarily his own.

It is not hard to imagine this scenario unfolding in an organization: you work in marketing, and the project is a new product, but you need assistance from someone in engineering, who could later take all (or at least too much of) the credit. To decide whether to pursue the project, it would help to know something about the “trustworthiness” of a particular engineer you could approach. But if you have been buried deep inside marketing, you may not have much information about any of the relevant engineers. In this case, you would be forced to rely either on the average sense of human decency among engineers or on your organization’s culture: “how we do things around here.” If the culture emphasizes teamwork over individual accomplishments, for example, you may have more confidence in approaching an unfamiliar member of the engineering group.

Kreps (1990) captures these issues in the Trust Game shown below. The game begins with a decision node for player 1, who can choose either to Trust or Not Trust player 2. If player 1 chooses Trust then the game reaches a decision node for player 2, who can choose either to Honor or Betray player 1’s Trust. If player 1 chooses Not Trust then the game ends (effectively, 1 terminates the relationship). At the end of each branch of the game tree, player 1’s payoff appears above player 2’s. If player 1 chooses to end the relationship then both players’ payoffs are zero. If 1 chooses to trust 2, however, then both players’ payoffs are one if 2 honors 1’s trust, but player 1 receives -1 and player 2 receives two if player 2 betrays 1’s trust.

We solve the Trust Game by backwards induction—that is, by working backwards through the game tree, one node at a time. If player 2 gets to move (i.e., if player 1 chooses to trust player 2) then 2 can receive either a payoff of one by honoring 1’s trust or a payoff of two by betraying 1’s trust. Since two exceeds one, player 2 will betray 1’s trust if given the move. Knowing this, player 1’s initial choice amounts to either ending the relationship (and so receiving a payoff of zero) or trusting player 2 (and so receiving a payoff of -1, after player 2 betrays 1’s trust). Since zero exceeds -1, player 1 should end the relationship.

The Trust Game has many other interpretations, including the following simple model of a bonus based on subjective performance evaluation. Suppose that an employee is player 1 and the boss player 2. Suppose that if the employee works hard and produces good results then she is supposed to be paid a bonus, but performance is subjective so paying the bonus is left to the discretion of the boss. If the bonus comes out of the boss’s budget, or if the boss must argue with superiors to secure the bonus, then the boss may prefer not to pay
the bonus, even if the employee deserves it. Fearing that the boss will not pay the bonus, the employee may decide not to work hard.

![Figure 1: The Trust Game](image)

2. The Repeated Game

Instead of a one-shot interaction, suppose that you and a particular engineer will play the Trust Game repeatedly, with all previous outcomes observed by both players before the next period’s Trust Game is played. The analysis of this repeated game differs dramatically from the one-shot interaction: the engineer’s actions today may affect your expectation of her actions tomorrow, which may affect your actions tomorrow, which affect her payoffs tomorrow. Thus, actions not in the engineer’s short-run self-interest (as defined by her payoff today) may be consistent with her overall self-interest (as defined by her total payoff over time).

Let me reiterate that I do not think this logic surprising or rare. I nonetheless find it useful to develop a formal model of these ideas in order to isolate some of the key variables that determine when a player should be guided by long- rather than short-run self-interest.

Formally, we will analyze an infinitely repeated game: the game never ends, but both players face an interest rate $r$ per period in discounting their payoffs across periods. (For example, when $r$ is high, a dollar to be received next period is not worth much today—$1/(1+r)$, to be exact.) We can interpret this “infinitely” repeated game somewhat
more realistically by saying that the game ends at a random date. Under this interpretation, the interest rate $r$ reflects not only the time value of money but also the probability that the players will meet again after the current period. (A dollar to be received next period provided that we are still interacting is not worth much if today’s interaction is likely to be our last.) Under either interpretation, the present value of $1$ to be received every period starting tomorrow can be shown to be $1/r$.

Mostly for analytical simplicity (but to some extent for behavioral realism), we will consider the following “trigger” strategies in the infinitely repeated game:

Player 1: In the first period, play Trust. Thereafter, if all moves in all previous periods have been Trust and Honor, play Trust; otherwise, play Not Trust.

Player 2: If Player 1 plays Trust this period, play Honor if all moves in all previous periods have been Trust and Honor; otherwise, play Betray.

These strategies are not forgiving: if cooperation breaks down at any point then it is finished for the rest of the game, replaced by the dictates of short-run self-interest. In most games, reverting to short-run self-interest after a breakdown in cooperation is a middle ground between two plausible alternatives: forgiveness (i.e., an attempt to resuscitate cooperation) and spite (i.e., going against short-run self-interest in order to punish the other player). Both forgiveness and spite deserve analytical attention, but I will focus on the trigger strategies (with their reversion to short-run self-interest after a breakdown of cooperation) as a tractable compromise.\(^2\)

We will analyze whether these trigger strategies are a Nash equilibrium of the infinitely repeated game. That is, given that player 1 is playing her trigger strategy, is it in player 2’s interest to play his? We will see that the trigger strategies are a Nash equilibrium of the infinitely repeated game provided that player 2 is sufficiently patient (i.e., provided that the interest rate $r$ is sufficiently small).

Suppose that player 1 follows his trigger strategy and chooses Trust in the first period. Player 2 then faces a dilemma. As in the one-shot interaction, player 2’s one-period payoff is maximized by choosing to Betray. But in the repeated game, if player 1 is playing the trigger strategy then such a betrayal by player 2 leads player 1 to choose No Trust forever after, producing a payoff of zero for player 2 in each subsequent period. Thus, the key question is how player 2 trades off the short-run temptation (a payoff of 2 instead of 1) with the long-run benefits of cooperation.

\(^2\) In the Trust Game, unlike most, reverting to short-run self-interest is identical to spite: it achieves the harshest possible punishment of player 2.
now) against the long-run cost (a payoff of 0 instead of 1 forever after). The answer depends on the interest rate: if $r$ is sufficiently low then the long-run consideration dominates and player 2 prefers to forego the short-run temptation.

The general point is that cooperation is prone to defection (otherwise we should call cooperation something else—such as a happy alignment of the players’ self-interests), but in some circumstances defection can be met with punishment. A potential defector therefore must weigh the present value of continued cooperation against the short-term gain from defection followed by the long-term loss from punishment. If a player’s payoffs (per period) are $C$ from cooperation, $D$ from defection, and $P$ from punishment (where $D > C > P$) then this decision amounts to evaluating two time-paths of payoffs: $(C, C, C, ...)$ versus $(D, P, P, P, ...)$, as shown in Figure 2.

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\{ 1 + \frac{1}{r} \} C > D + \frac{1}{r} P .
\]

Rearranging the inequality (*) yields $r < (C - P)/(D - C)$. That is, if the players are sufficiently patient (i.e., if $r$ is sufficiently close to zero) then it is optimal to cooperate, foregoing the short-run temptation $(D - C)$ for the long-term gain $(C - P$ forever after). More formally, if the players are sufficiently patient then it is an equilibrium of the infinitely
repeated game for the parties to play the trigger strategies given above, thereby achieving cooperation in every period of the repeated game.

### 3. Richer Models (and Some Lessons)

Even this simplest repeated-game model offers one lesson: grow the value of the relationship (*i.e.*, increase \( C - P \)). That is, cooperation is more likely to continue if the parties do better together (\( C \)) than apart (\( P \)). Additional lessons follow from slightly richer models.

One way to enrich the model is to allow the payoffs to fluctuate over time. As an example, consider a duopoly that is trying to collude to keep prices high. Unfortunately, the duopoly’s industry is subject to cyclical fluctuations in demand. In particular, in a boom the market will bear a price much above cost, whereas in a bust the margins are nearly zero. When will each duopolist be more tempted to break from the collusion by under-cutting price so as to attract the entire market (at least for a time, until the other firm matches the price cut): in a boom or a bust? Clearly in a boom, for there is nothing to gain in a bust. Consequently, collusion may break down (and price wars emerge) in booms, not busts.\(^3\)

To model such fluctuating payoffs, suppose that at the beginning of each period the parties observe the payoffs \( C_t, D_t, \) and \( P_t \) that apply to that period. Suppose also that each period’s payoffs are independently drawn from a given joint probability distribution \( F(C, D, P) \). Then cooperation is optimal in period \( t \) for one player (assuming that the other player is playing a trigger strategy) if the first player’s payoffs satisfy

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C_t + \frac{1}{r} E(C) > D_t + \frac{1}{r} E(P),
\]

where \( E(C) \) is the expected value of \( C \) and likewise for \( E(P) \). This enriched model offers a second lesson: manage the extremes (*i.e.*, the extreme values of \( D_t - C_t \)). That is, the threats to a relationship come when the defection temptation is largest (relative to the cooperation payoff), so these are the events to plan for in advance.

Another way to enrich the basic model is to allow for a permanent shift in the payoffs: rather than a random draw of new payoffs each period, a permanent shift from \( (C, D, P) \) to

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\(^3\) See Rotemberg and Saloner (1986) for the full-blown version of this theory and some supporting evidence.
(C’, D’, P’) occurs at an uncertain date. One interesting possibility is that (C, D, P) satisfy the inequality (*) from the previous section but (C’, D’, P’) do not. The breakdown of several famous “relational contracts” can be understood in these terms. For example, for several decades IBM made a “no layoffs” pledge to its employees. This was not an explicit contract, enforceable by a court, but it was part of “the deal” at IBM: a shared understanding between the firm and its employees about how employment would proceed. As personal computers and work stations reduced the demand for mainframe computers, however, one could imagine that the value of living up to this pledge (namely, C in the model above) fell. Thus, (*) may have held at the original, high value of C but not at the new, low value.

A similar story can be told about bonus payments to investment bankers at First Boston in the early 1990s. Such bonuses are highly subjective (as will be described in more detail in Lecture Note 4). Many bankers left the firm after a second year in which bonuses were unexpectedly low. The new parent company, Credit Suisse, claimed that bonuses were low because performance was low (and First Boston had indeed performed worse than its competitors), but many bankers argued that they should be paid the same as bankers at other firms. There may have been a legitimate misunderstanding between the Swiss (payment for results) and the bankers (match the market). Alternatively, the Swiss may have claimed such a misunderstanding as a way to cover their decision to lower bonuses after the junk-bond market collapsed, because this collapse made bankers specializing in mergers and acquisitions less valuable.

In terms of the model developed in this section, one can again understand the First Boston scenario in terms of an unexpected fall in the value of C. In different scenarios, however, the other variables in (*) may also move unexpectedly. For example, a sudden and dramatic need for cash might increase a player’s defection payoff, D. Similarly, as alternative sources of supply develop, a buyer will have less trouble finding a new supplier should a current supply relationship end; that is, P may increase. Finally, as mentioned above, the interest rate r reflects not only the time value of money but also the probability that the players will meet again after their current interaction. Thus, if exogenous factors (such as illness, promotion, a spouse’s new job, and so on) make it less likely that the parties will meet again then r increases and (*) may fail to hold. That is, there may be “endgame behavior” in which the parties give in to the temptation to defect because the future value from current cooperation is too small.
All these stories about permanent shifts in the payoffs yield a third lesson: *know when to quit (but also when to start)*. For example, in the case where \((C, D, P)\) satisfy \((*)\) but \((C', D', P')\) do not, it may or may not be optimal to cooperate until the payoff shift occurs. The answer depends on how likely the payoff shift is to occur. The analysis behind this answer is much like the argument leading to \((*)\), but now the interest rate \(r\) must reflect not just the time value of money and the probability that the relationship will end but also the probability that the payoffs will shift.

Even if we incorporate sudden shocks into the story, the repeated-game model described here is excessively tidy: cooperation either works perfectly or doesn’t work at all, depending on the interest rate. It is natural to ask what happens when the players are not “sufficiently patient.” In brief, all is not lost, because it may be possible to achieve partial rather than full cooperation.\(^4\) It is also natural to ask why there are never any fights or misunderstandings in the trigger-strategy equilibrium we have analyzed. Green and Porter (1984) developed a richer model in which the players’ actions are only imperfectly observable (*e.g.*, Saudi Arabia cannot tell whether a small country has been selling more oil than its OPEC quota, but everyone can see the price of oil on today’s market). In models with imperfect observability, cooperation breaks down every now and then, but then resumes after a stretch of non-cooperation.

In spite (or perhaps because?) of their excessive tidiness, repeated-game models of such “self-enforcing agreements” (Telser, 1981) have been widely applied in economics, such as in problems of quality assurance in supply relationships (Klein and Leffler, 1981), subjective performance evaluation in incentive compensation (Bull, 1987), and corporate culture (Kreps, 1990). Related ideas can recently be seen in law (“self-enforcing corporate law,” Black and Kraakman, 1996) and in organization theory (“psychological contracts in organizations,” Rousseau, 1995). In Lecture Note 4, “Repeated Games, Part II: Subjective Performance Evaluation,” I consider how relational contracts can be used to provide incentives. In Lecture Note 6, and “Asset Ownership, Part II: Make, Buy, or Cooperate,” I consider how such contracts influence the vertical-integration decision.

In sum, repeated-game models focus on the role of long-run self-interest in overcoming short-term temptation. In everyday parlance, if you understand my long-run

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\(^{4}\) To see how this may work, examine \((*)\). Note that reducing the cooperation payoff from \(C\) to some lower level is no help at all, in and of itself. That is, holding the payoff from defection \((D)\) constant, reducing \(C\) makes it harder to satisfy \((*)\). The trick is that reducing \(C\) may also reduce \(D\): making due with partial cooperation may also limit the players’ opportunities for profitable deviations. If \(D\) falls more than \(C\) does then \((*)\) may hold.
self-interest, you might “trust” me not to yield to certain short-run temptations. Like Williamson (1993), however, I see such “calculative trust” as a contradiction in terms. Instead, I prefer a label such as “assurance” (Yamagishi and Yamagishi, 1994) for repeated-game logics and other arguments in which behavior is determined by self-interested incentives. (For example, if you make me a loan and I know you are a mobster, you may be confident that I will pay you back, but are you trusting me?) I would reserve “trust” as the label for a non-economic phenomenon. I cannot yet model (or even really define) such trust, but I find it too depressing to accept that all trust is merely assurance.

References


