Lecture Note 4

Repeated Games, Part II:
Subjective Performance Evaluation

We started this course by studying “formal” or “explicit” incentive contracts—the kind that could be enforced by a court, if necessary. Then we introduced the idea of a “relational” contract that cannot be enforced in court and so relies instead on the parties’ self-interested weighing of the short-term benefits from cheating against the long-run benefits from cooperating. In this note we continue to discuss relational contracts but now focus specifically on relational incentive contracts (such as subjective bonus plans) rather than on the abstract relational contracts analyzed in Lecture Note 3.

For simplicity, in this note we will assume that no formal contracts can be enforced. A more balanced view would be that formal contracts are not completely infeasible but are likely to be imperfect, in which case one should consider how to combine imperfect formal contracts with the relational contracts studied here. We will consider such combinations of relational and formal contracts in our discussion of subjective performance assessments at Brainard, Bennis, and Farrell (a fictional but illustrative law firm); for a more formal treatment, see Baker, Gibbons, and Murphy (1994).

This is the last note in the Incentives part of the course, so we conclude this note by offering some summary comments and beginning a transition from Incentives to the next part of the course, on Structure.

1. The One-Shot Interaction

As we have argued several times in class, for most workers and managers (and even for many executives) it is extremely difficult to measure \( y \)—the dollar value of the agent’s contribution to the firm. More precisely, it is extremely difficult to measure \( y \) in a way that would allow the agent’s pay to be based on \( y \) through a compensation contract that could be enforced by a court, if necessary. We will describe this difficulty by saying that the agent’s contribution to firm value is not objectively measurable. Even if the agent’s contribution to firm value is not objectively measurable, however, it sometimes can be subjectively assessed by superiors who are well placed to observe the subtleties of the agent’s behavior and opportunities. (Another phrase sometimes used to capture this distinction is that the agent’s
contribution to firm value is “observable but not verifiable.” That is, the agent’s contribution is observable by the parties but not verifiable by a court.) Such subjective assessments of an agent’s contribution to firm value may be imperfect, but they may nonetheless complement or improve on the available objective performance measures.

In this section we develop a simplified version of Bull’s (1987) repeated-game model of a bonus based on a subjective assessment of a worker’s total contribution to firm value.¹ In each period, the worker chooses an unobservable action, a, that stochastically determines the worker’s contribution to firm value, y. In particular, y equals either L or H, and the worker’s action, 0 ≤ a ≤ 1, equals the probability that y=H. (That is, higher actions produce higher probabilities of y = H; the action a = 0 guarantees that y = L will occur.) The worker incurs an action cost c(a).

As discussed above, we assume that the worker’s contribution to firm value is too complex and subtle to be verified by a third party, and so cannot be the basis of an enforceable contract. That is, y cannot be objectively measured. On the other hand, we assume that y can be subjectively assessed, as described in the following timing of events within each period. First, the firm offers the worker a compensation package (s, b), where s is a base salary paid when the worker accepts the offer and b is a relational-contract bonus meant to be paid when y=H. Second, the worker either accepts the compensation package or rejects it in favor of an alternative employment opportunity with payoff wₐ. Third, if the worker accepts then the worker chooses an action at cost c(a). The firm does not observe the worker’s action. Fourth, the firm and the worker observe the realization of the worker’s contribution to firm value, y. Finally, if y = H then the firm chooses whether to pay the worker the bonus b specified in the relational contract.² The firm’s payoff when the worker’s contribution is y and total compensation is w is the profit y - w. The worker’s payoff from choosing an action with cost c(a) and receiving total compensation w is w - c(a), and the worker is risk-neutral.

In a single-period employment relationship with this timing (or in the final period of a multi-period relationship with a known, finite duration), the firm would choose not to pay a bonus, so the worker (anticipating the firm’s decision) would choose not to supply effort, so the firm (anticipating the worker’s choice) would not pay a salary greater than L. Whether the worker is employed at this firm would then depend on whether wₐ is greater or

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¹ The single worker we consider could just as well be a sequence of workers, each of whom works for a known finite number of periods, provided that each worker learns the history of play before beginning employment.

² One could construct an analogous model in which the firm rewards the worker with a promotion rather than with a bonus. Indeed, relational contracts concerning promotions may be more prevalent than the pay-for-performance relational contracts we discuss here.
less than \( L \). If \( w_a < L \) then the worker will be employed at the firm, but will not supply any effort. We will assume hereafter that \( w_a > L \), in which case the worker would not be employed at this firm in the single-period model.

2. The Repeated Game

In many settings, there is some prospect of an ongoing relationship, which may cause the firm to value its reputation for honoring its relational contracts. To capture this prospect, we analyze what is formally described as an infinitely repeated game but is better interpreted as a repeated game that ends randomly, as described in Lecture Note 3. In analyzing this repeated game, we focus on trigger strategies: roughly speaking, the parties begin by cooperating and then continue to cooperate unless one side defects, in which case they refuse to cooperate forever after. We solve for the trigger-strategy equilibrium that maximizes the firm’s expected profit, subject to making the terms of employment sufficiently attractive that the worker chooses to work at this firm. The key issue is how large a bonus the worker can trust the firm to pay. The answer depends on the firm’s interest rate, \( r \), and on the firm’s expected profit per period, which we now derive.

If the worker believes the firm will honor the relational contract (i.e., that the firm will pay the bonus \( b \) after observing performance \( y = H \)), then the worker’s optimal action solves

\[
\max_a \quad s + a \cdot b - c(a) .
\]

Under assumptions similar to Lecture Notes 1 and 2 (namely, that \( c(a) \) is convex, and now also that \( c'(a) \) approaches infinity as the worker’s action approaches one), the solution satisfies \( c'(a) = b \). For an arbitrary \( b \), we denote the worker’s optimal action by \( a^*(b) \), in which case the firm’s expected profit per period is

\[
L + a^*(b) \cdot [H - L] - [s + a^*(b) \cdot b] .
\]

The worker will work for the firm if her expected payoff exceeds her alternative payoff:

\[
s + a^*(b) \cdot b - c[a^*(b)] \geq w_a .
\]

Assuming that the alternative payoff is not too high (i.e., not so high as to prevent the firm from both attracting the worker and making a profit), the optimal base salary for the firm to offer is the lowest salary sufficient to induce the worker to join the firm. Given this salary, the firm’s expected profit per period when the relational-contract bonus is \( b \) is
\[ E\pi(b) \equiv L + a^*(b) \cdot [H - L] - c[a^*(b)] - w_a. \]

This expression for the firm’s expected profit per period allows us to determine how large a bonus the worker can trust the firm to pay.

Given the worker’s trigger strategy, if the firm does not pay the bonus \( b \) when the worker’s contribution is \( y = H \) then the firm’s payoff is \( H - s \) this period but zero thereafter (because \( w_a > L \), so the worker will not be employed by this firm if trust collapses), whereas if the firm does pay the bonus then its payoff is \( H - s - b \) this period but \( E\pi(b) \) thereafter. Thus, the firm should pay the bonus if and only if

\[
(H - s - b) + \frac{E\pi(b)}{r} \geq (H - s - 0) + \frac{0}{r}, \quad \text{or} \quad E\pi(b) \geq rb,
\]

where \( 1/r \) is the present value of $1 received next period and every period thereafter.

The efficient relational contract sets \( b \) to maximize expected profit per period, \( E\pi(b) \), subject to the firm’s reneging constraint \( E\pi(b) \geq rb \), as shown in Figure 1. For high enough values of \( r \), such as \( r_H \), no value of \( b \) generates enough expected profit to dissuade the firm from reneging—that is, no value of \( b \) satisfies \( E\pi(b) \geq rb \). For small enough values of \( r \), such as \( r_L \), first-best incentives can be provided through a relational contract with bonus \( b_{FB} \). Finally, for intermediate values of \( r \), such as \( r_M \), the efficient relational-contract bonus \( (b^*) \) is the largest value of \( b \) that satisfies the reneging constraint. In this case, a larger value of \( b \) (but still less than \( b_{FB} \)) would improve incentives (if the worker believed that such a bonus would be paid) but is not credible (because \( E\pi(b) < rb \) for all such larger values of \( b \)).

\[ \text{Figure 1} \]
For interest rates in the intermediate range, the efficient bonus falls as \( r \) increases, because the higher value of \( r \) makes future profits less valuable, so the firm is more tempted to renege. (To see this, increase \( r_M \) from \( r_L \) to \( r_H \) in Figure 1 and consider the movement in \( b^* \).) Similarly, as \( w_a \) increases, the firm’s expected profit falls, so the largest feasible relational-contract bonus falls. (To see this, consider lowering the \( E\pi(b) \) curve straight down in Figure 1, by the amount of the increase in \( w_a \), and consider the movement in \( b^* \) for a fixed interest rate, such as \( r_M \).)

3. Interpretation and Extensions

In the model above a relational contract is a repeated-game equilibrium. Designing and managing such a contract involves determining a value of \( b \), communicating to the Agent that \( b \) will be paid if \( y = H \), assessing whether \( y = H \), and deciding whether to pay \( b \) if \( y = H \). In the model, these tasks are trivial; in reality, each requires judgment and knowledge of the specifics of complex situations. Furthermore, hard as these tasks are in practice, much evidence suggests that changing a relational contract is harder still: managers must end one relational contract but preserve enough credibility to begin another, and the new contract they seek to begin often looks suspiciously like reneging on the old contract they seek to end!

Many illustrious relational contracts have come under substantial stress (and sometimes failed) when the world has changed important parameters (here, \( r \) and \( w_a \)) in unfortunate directions. The lifetime employment policy at IBM, for example, came under great stress when the firm’s profit collapsed in the mainframe market; eventually, IBM abandoned the policy. Similarly, Lincoln Electric’s bonus payments to US workers were threatened when overseas acquisitions incurred large losses. In the model above, the firm’s profit falls when \( w_a \) rises (that is, when alternative employment options become more attractive), but it is straightforward to enrich the model so that a reduction in the firm’s profit from any source threatens the viability of the relational contract.

Some firms use formal and relational incentive contracts in combination. For example, recall that Lincoln Electric uses both piece rates and subjective bonuses. Baker, Gibbons, and Murphy (1994) explore the simultaneous use of formal contracts based on objective performance measures (\( p \)) and relational contracts based on subjective assessments of total contribution (\( y \)). They analyze how the relational contract can reduce the distorted incentives that would be created by the formal contract on its own, while the formal contract can reduce the size of the reneging temptation that the firm would face if it used only a relational contract. Thus, the combination of formal and relational contracts can
reduce distortion in the worker’s incentives and reduce the firm’s temptation to renege on a promised bonus.

4. Summary Comments on Incentives

We have now seen many examples of formulas run amuck and (essentially) no examples of employees with successful incentives created solely by a formula. But even after these discussions it is easy to forget exactly what we mean by a “formula,” so let me give a definition: a formula has objective weights and objective measures. In the \( w = s + bp + B(y) \) framework, the objective weight is \( b \) and the objective measure is \( p \). Notice that this definition of a formula implies that many alternatives are hiding in \( B(y) \), including the following.

Objective measures but subjective weights: As an example, imagine that a law firm announces that pay will be based on origination credits and billable hours but does not announce what weights these measures will have.

Objective weights but subjective measures: It is common for division managers to be paid (say) 50\% on individual performance, 25\% on division performance, and 25\% on corporate performance (objective weights), but it is often the case that one or more of these performance measures is not completely objective (e.g., individual performance may be based on a 360-degree evaluation, or corporate performance may be subjectively determined by the board, relative to a “par” year).

Total subjectivity: A senior professor at Cornell used to tell assistant professors that the way to get tenure was to “find an oar” (i.e., find a way to help move the school forward). He refused to say more than this, arguing that (a) he wanted junior people for their new ideas, not to implement the ideas of senior people, and (b) even if he did know how to move the school forward, he was in part testing to see if the junior people could figure it out.

5. From Incentives to Structure

The previous section shows how narrow a true formula is and hence how pervasive subjectivity is. But our only formal model of a subjective bonus remains the model in Section 2 above, and in this model the only difficulty with a subjective bonus was the possibility of reneging. In practice, however, there is another difficulty with subjective bonuses: the notation “\( B(y) \)” suggests that the Principal and the Agent can easily observe...
the Agent’s total contribution to firm value, \( y \), but it is often the case that the parties must make do with inferior subjective performance assessments, as follows.

One possibility is that the Principal and the Agent can subjectively assess team performance but not individual performance. As a simple model of team performance, suppose that there are two team members \((i = 1, 2)\), member \( i \) chooses action \( a_i \), and team performance is

\[
Y = a_1 + a_2 + ka_1a_2,
\]

where \( k > 0 \) is a constant that reflects the importance of coordinating the actions of the team’s members. If the Principal and Agents can subjectively assess team performance then one approach would be to pay each team member the same subjective bonus, \( B(Y) \).

But it would be natural to consider paying different team members different bonuses, depending on perceived differences in their contributions. To model this, suppose that the Principal and the Agents can also subjectively assess the individual contribution \( q_i = a_i \) for team member \( i \). For example, \( q_i \) might subjectively reflect Agent \( i \)’s contributions in his or her function but ignore Agent \( i \)’s cross-functional contributions to total firm value \( (Y) \). If the Principal and the Agents can subjectively assess not only \( Y \) but also \( q_i \) then one could consider a subjective bonus plan such as

\[
B_i(q_i, Y) = mq_i + nY,
\]

and one might expect something like the optimal weight on \( Y \) would increase and the optimal weight on \( q_i \) would decrease as the importance of coordinating team actions increases (i.e., \( n^* \) increases and \( m^* \) decreases as \( k \) increases).

This brief note is not the place for a detailed analysis of a specific subjective bonus plan in a specific model of team and individual subjective performance assessment. Instead, the simple points of this section are that (1) it is typically not possible to observe an individual’s total contribution to firm value \( (y) \), (2) the alternatives are often either more aggregated \( (Y) \) or less complete \( (q_i) \), and (3) optimal subjective bonus plans must balance the pros and cons of the latter two kinds of subjective assessments.

But one last point is perhaps more important than any of these three: the available (objective and subjective) performance measures often depend on how the work is structured (e.g., by function, product, or geography, and perhaps with cross-cutting teams on top). For example, suppose there are four agents working on four tasks: two tasks are in engineering and two are in manufacturing, but one of the engineering tasks and one of the manufacturing tasks pertain to one product while the others pertain to another product. Suppose also that there are coordination externalities (i.e., total contributions of the form \( a_i \)
+ a_2 + k a_1 a_2) both within each function and within each product. Then organizing by either function or product will cause performance measures to omit a coordination externality, and even the addition of cross-cutting teams (such as product teams overlaid on a functional organization) may not provide the right performance measures, not to mention the right incentives.

We turn next to such structural issues, and to their connection with the incentives issues studied thus far.

References
