3.1 Data

In chapter 1, we evaluated the following sentences:

(77) Ray and Tess wrote poems.
(78) Ray and the boys wrote poems.
(79) Ray and Tess and Jess wrote poems.

and concluded that our two theories part ways when it comes to the interpretation of a conjoined noun phrase one of whose conjuncts is itself plural (formed by conjunction or common noun pluralization), as in (78) and (79). As a result of the fact that set formation is a non-associative operation\(^{10}\), on the sets approach syntactic complexity is mapped into semantic complexity, while such is not the case on the union approach and this difference starts to have effects when a conjunct is plural. The sets approach requires many more types (in the logical sense) of entities in the domain of discourse than does the union approach. This difference is recorded in the relationship between $D$ and $D^*$ on the two approaches:

(80) Union theory: $D^*$ is the set of all non-empty subsets of $D$.

\(^{10}\) $O$ is an associative operation iff for any $a,b,c$ in the domain of $O$: $(a \circ b) \circ c = a \circ (b \circ c)$. By set-formation we mean the operation $\ast$ such that $a^\ast b = \{a,b\}$. $(a^\ast b)^\ast c \neq a^\ast(b^\ast c)$.
Sets theory\textsuperscript{11}:

\[
D_0 = D
\]

\[
D_{n+1} = D_n \cup \text{POW}_{\geq 2}(D_n)
\]

\[
D^* = \bigcup_{n<\omega} D_n
\]

If it can be shown that the "extra" entities of the sets approach are required of a model that interprets plural expressions, then the sets theory is to be preferred over the union theory. The purpose of chapters 4-9 is to examine various arguments supporting the need for the sets domain. This chapter is meant as a preview of the different kinds of arguments to be discussed.

Assuming a sets model and a union model that agree on the domain of singularities, it is possible to find a pair of noun phrases that have identical denotations on a union interpretation but different denotations under a sets interpretation. For example, if \(D\), the domain of singularities, contains exactly one boy and exactly three girls, then the noun phrases \emph{the boy and the girls} and the noun phrase \emph{the children} will (assuming they are felicitous) have the same denotation for the union theory but not for the sets theory (cf. chapter 1, page 7). One line of attack on the union theory has been to identify predicates that are sensitive to these differences. I will argue that predicates of the required type are not in fact found in English. In chapter 4, I will give a general account of the obstacles to any argument in favor of the sets theory which capitalizes on differences in denotation between plural noun phrases that emerge on that theory but not on the union theory.

In chapter 5, I will take up a specific subcase of the type of argument discussed in chapter 4. This subcase involves examples of sentences which, on their distributive readings, argue for the sets theory. For example, consider a context in which the following statements are true:

\[
(82) \quad \text{Every woman is either an author or an athlete and all authors and athletes are women.}
\]

\[
(83) \quad \begin{align*}
\text{a. The authors are outnumbered by the men.} \\
\text{b. The athletes are outnumbered by the men.} \\
\text{c. But, the women, altogether, outnumber the men.}
\end{align*}
\]

\textsuperscript{11} \text{POW}_{\geq 2}(X)\text{ is the set of all the non-empty non-singleton subsets of } X. \text{ This inductive definition is taken from Hoeksema (1983:81), where it is credited to Johan van Benthem.}
There are those who would say that in such a context (84) below is ambiguous, having one true reading. That true reading would be described as one in which there is distributivity on the subject argument. It is captured in the unambiguous (85).

(84) The authors and the athletes are outnumbered by the men.
(85) The authors are outnumbered by the men and the athletes are outnumbered by the men.

Now, on the union theory alone, given (82), the subject noun phrase of (84) has the same denotation as the subject noun phrase of (86):

(86) The women are outnumbered by the men.

hence presumably (84) and (86) should have the same truth value in this context. However, given (83c), (86) is false. This is a specific example of the type of anti-union argument identified above. A predicate is isolated which distinguishes noun phrases that are co-referent on the union theory. One of the reasons for treating it separately is that distributivity holds a unique place in the literature on plurals. Moreover, the anti-union arguments of chapter 4 rely on finding specific predicates of English that are sensitive to semantic differences that emerge only on the sets theory. In contrast to this, all predicates can have distributive readings.

In chapter 5, I will take issue with specific assumptions underlying analyses of distributivity. I will argue that even without the results of chapter 4 it is not clear that distributivity offers any hope for distinguishing our two theories. In particular note that the argument outlined with the help of examples (82)-(86) crucially relies on having a truly ambiguous VP in (84). By this I mean, that the VP in (84) is said to have two distinct denotations. It is the "distributive denotation" of that VP which is being used here to argue against the union theory. I will argue against some of the assumptions underlying this analysis, in particular its failure to recognize the context-dependent aspects of distributivity. The anti-union argument outlined here will not survive the analysis of distributivity that I will propose.

As Lønning (1989) has argued, probably the most convincing arguments for the sets theory have been made using predicates with reciprocals such as infuriate each other. So in the first part of chapter 6, I discuss reciprocals focussing on what they have to tell us about the union-sets debate. In the second half of the chapter, I give an analysis of reciprocals drawing on the context dependent analysis of distributivity from chapter 5 and the work of Heim, Lasnik and May (1991a,b) and Sauerland
Chapter 7 is devoted to remarks concerning floated quantifiers. Floated quantifiers have been used as a starting point in the development of theories of distributivity including the one in chapter 5. In chapter 7, we return to compare the resulting theory of distributivity with the behavior of floated quantifiers. Another issue briefly discussed is the relation between floated quantifiers and their non-floated counterparts. In the second part of the chapter, attention is turned to the floated quantifier both, said to involve a notion of duality. A speaker who uses the expression both camels presupposes that there are two and only two camels under discussion. Peter Lasersohn has used this duality presupposition of both to probe the semantics of plural noun phrases using the example in (87):

(87) both Awbery and Jones and Thomas.

He concludes that the sets theory is correct on the basis of this evidence. In chapter 7, I respond to his argument claiming that in examples like (87), both is part of a complex conjunction and in fact has no semantic duality requirement.

At the heart of chapter 8 is a potential argument in favor of the sets theory based on the richness of the domain of discourse, \( D^* \), on that theory as compared with \( D^* \) for the union theory, again assuming the two start off with the same domain of singularities, \( D \). Recall from (81) above, the elements of \( D^* \) in the sets theory come in an infinite variety of (logical) types. If this theory is correct, you might expect the richness of the domain of discourse to be exploited by the language. One place predicates of English, denoting subsets of \( D^* \), might very well be organized type-theoretically. Particular predicates might be defined only for pluralities of a certain order. Imagine that you had a verb like that, say \( V_2 \) of type 2. If that were the case then the truth or well-formedness of sentences of the form NP \( V_2 \) would depend in part on the syntactic complexity of the subject NP. For example, the children \( V_2 \) could but the boys and the girls \( V_2 \) couldn't be true. Such verbs would constitute strong evidence in favor of the sets theory. On the other hand, if it turns out that there are no such verbs, then all else being equal we should opt for the simpler union theory.

Chapter 9 is devoted to the semantics of collective nouns. Collective nouns are characterized in Jespersen (§4.8) as follows:

A collective noun is defined in the NED [New English Dictionary by Murray, Bradly, Craigie. Oxf. 1884] as 'a substantive which (in the singular) denotes a collection or number of individuals.'
may accept this definition (though it does not always agree with practice followed in that dictionary), and give as examples a library = ‘collection of books’, a train (railway-carriages), a forest (trees), a nation (men and women), an army (soldiers). All of these may be used with such words as one (one library) or that; and we may use them in the plural: libraries, trains, etc.

According as the idea of plurality is more or less prominent in the mind of the speaker, there is in all languages and at all times a tendency to forget the fact that collectives are grammatically singular, and we often find plural constructions, partial or total. ... It should, however, be noticed that it is only with collectives denoting living beings that the plural construction is found: words like library or train never take the plural.

This view of collective nouns suggests that rule [7] from chapter 1:

\[ [7] \text{If } \alpha \text{ is a singular common noun, then } \| \alpha \|_M = V(\alpha) \text{ and } V(\alpha) \subseteq D. \]

is not fully general. Rather, it appears that collective nouns, at least the animate ones, denote subsets of \( D \). In more recent times collective nouns have been treated this way in formal accounts of the semantics of English. These accounts give formal expression to the synonymy of pairs such as the following:

(88) a. The committee voted.
    b. The members of the committee voted.

(89) a. The members of the group gathered in the park.
    b. The group gathered in the park.

To see why this is relevant to the choice of the sets or the union theory we need to consider examples in which a collective noun forms part of a (syntactically) plural noun phrase, for recall, the two theories part ways once pluralization is iterated. If one maintains that in the extension of committee there are (non-singleton) sets, pluralities, then given a domain in which the noun phrase the committees is felicitous, it will denote (assuming rules [6] and [8] of chapter 1) the set of all committees, each of which is a set, hence a set of sets. This is troublesome for the union theorist because

---

12In the formal semantics tradition, this practice goes back at least to R. Montague (cf. Bartsch 1973:79).
his domain of discourse has to be enlarged to accommodate these entities. He must now adopt a domain more like that we've come to associate with the sets theory. Moreover, having accepted this analysis for collective nouns, the attack on the union theory can be mounted even with respect to the interpretation of conjunction as follows. A context is chosen in which there are exactly two committees, A and B, and then the referents of the subjects of the following pair are identified:

(90) The committees voted.
(91) The members of committee A and the members of committee B voted.

This identification forces us into a denotation for the subject of (91) that is predicted by the union theory to be impossible (assuming member of committee A denotes a subset of D).

This constitutes a serious blow to the union theory, if the position outlined so far is tenable. However, we should not be too quick to identify the denotation of a noun phrase such as the committee with the denotation of the members of the committee, since there are predicates that distinguish them. As an illustration consider that if a normally seven-membered committee loses five of its members and the remaining members are both seven feet tall then:

(92) The committee is small.

but:

(93) The members of the committee are not small.

Furthermore, while we can say:

(94) The members of the committee are tall.

we cannot, as Bennett (1974:223) pointed out, say:

(95) "The committee is tall.

In fact then, pairs of noun phrases such as the committee / the members of the committee or the team / the players seem not to be extensionally equivalent. In chapter 9, we will consider the conflicting data of (88)-(91) and (92)-(95). Further, we will follow the lead of Jespersen and Dougherty (1970) in devising plausible tests for semantic plurality in an attempt to
decide when and whether collective noun phrases denote pluralities.

3.2 Methodology

I would like now to end with a somewhat lengthy methodological note. Much of the argumentation in this book turns on a comparison of pairs of noun phrases in contexts in which they are assigned the same denotation in one theory and different denotations in the other. For example the boy and the girls and the children, if felicitous, are co-referent according to the union theory, but not according to the sets theory. Let us say that two or more noun phrases share a predicate if applying the predicate to both of them yields the same truth value. Generally, arguments for or against coreference of noun phrases turn on predicate sharing. For example, the sets theorist might argue for the non-coreference of the noun phrases the boy and the girls and the children by isolating predicates that are not shared by the two. This method of argumentation is fairly straightforward so long as we stick to the two extremes: either the noun phrases share all predicates of the language or they don’t share any. Consider first two noun phrases that share all (appropriate) predicates of the language. In this case, coreference is plausible even if not inevitable. A diehard multiplier of entities could still maintain that the two noun phrases denote distinct entities differing with respect to properties that are not expressible in the language. Nonetheless, in this case one normally chooses the more intuitive option of inferring coreference from the sharing of all predicates of the language. Assigning the two noun phrases the same denotation, that is treating them as coreferent, is in a sense a way of capturing in our semantics the fact that all predicates are shared by the noun phrases in question. At the other extreme we have noun phrases that do not share any predicates. In this case, it is hard to see any basis for assuming coreference. In sum then, the sharing of all predicates is associated with coreference while the sharing of no predicates is associated

13This connection between predicate sharing and coreference is similar to Leibniz’s principle of the Identity of Indiscernibles, though it differs in its reference to language. We connect predicate sharing and noun phrase coreference whereas Leibniz’s principle is about property sharing and identity of objects. An earlier example of a linguistic interpretation of Leibniz’s principle is found in Wilson (1953) who writes "The principle of the identity of indiscernibles may be taken to mean that if two objects O₁ and O₂ are numerically different then they are qualitatively different, they differ in some mentionable respect."
with non-coreference. Expressing approximately this idea, Link (1983: 304) writes:

Our guide in ontological matters has to be language itself, it seems to me. So if we have, for instance, two expressions \(a\) and \(b\) that refer to entities occupying the same place at the same time but have different sets of predicates applying to them, then the entities referred to are simply not the same.

Unfortunately, neither of the extremes examined so far is representative. Rarely, if ever, do two noun phrases share all predicates. The unintuitive conclusion drawn in (96) based on Link’s dictum illustrates this:

\[
\begin{align*}
(96) & \quad a = 'George and Mike' \\
& \quad b = 'Mike and George' \\
& \quad P = 'are running with Dan and Lloyd respectively.' \\
& \quad P(a) \land \neg P(b).
\end{align*}
\]

This means that "total predicate sharing" is hopeless as a criterion for coreference. Total non-sharing of predicates fares even worse as a criterion for non-coreference. Quite often, two intuitively non-coreferent noun phrases share some predicates. Think of all the predicates that hold both of the number two and of the number four or of Joe and Joe’s brain. Thus, the observation that a pair of noun phrases share some predicates is not sufficient grounds to identify their referents. The key then in making sense of the connection between coreference and predicate sharing is to develop theories rich enough to allow the definition of a subset of predicates all of which must be shared by coreferent noun phrases. The most well-known example of this is the distinction between extensional and intensional contexts. Noun phrases that are found to share all predicates classified as extensional can safely be counted as coreferent even though they may differ in the sense they express and hence not share some intensional predicates. Theories concerned with discourse afford another distinction which can be appealed to here. These theories make reference to the potential of an expression to change the discourse context. A noun phrase’s context change potential is a function of but not identical to its reference. Two noun phrases may have different context change potentials while remaining coreferent. Predicates whose interpretation makes crucial use of discourse properties of noun phrases will not be counted in deciding coreference. An example of such a predicate, I would argue, is the verb phrase in (96) above.
containing the adverb *respectively*. This adverb relies for its interpretation on the linear order in which discourse referents are introduced. Some have used this property of *respectively* in determining the kind of things noun phrase conjunctions denote (cf. section 2.10 of Link to-appear; Lasersohn 1988:139). However, I would argue that linear order is a property of the discourse itself and not of the entities referred to. Evidence for this claim derives from examples in which this ordering information is not contributed by a conjoined noun phrase subject but rather enters the discourse from outside the sentence containing *respectively*. Here are two such examples; the first is from the text of Jackendoff (1972:325, italics are mine):

(97) ... there are the ambiguous readings of (8.22) and (8.23).
(8.22) They’re fighting about nothing.
(8.23) I will force you to marry no one.
One reading of these sentences is synonymous with (8.24) and (8.25), respectively.
(8.24) It is not so that they’re fighting about anything.
(8.25) It is not so that I will force you to marry anyone.

(98) The first book is 2,000 pages long and it barely fits in the book bag. The second one is only 20 pages long, you can put it in your pocket. I refer to these books as the fat book and the skinny book, respectively.

Other expressions that behave like *respectively* are *correspondingly*, *analogously*, *equivalently* and *in that order* (for examples cf. Dougherty 1970:896). Comparisons in general often depend on linear order for their interpretation.

Verb phrases containing pronouns provide another clear class of predicates whose failure to justify non-coreference claims is explained with the help of a theory of discourse. One such example occurs in (99) and (100) below:

(99) Ray and the women relate that by late August each person was aware that he had only six months to live.

(100) The authors relate that by late August each person was aware that he had only six months to live.

Assume that both (99) and (100) are initial utterances in a discourse between two speakers who are aware of who Ray, the women and the authors are and that Ray and the women are the authors. (99) has a reading in which
be is interpreted as referring to Ray. (100) lacks this reading. One might argue that this difference arises from the fact that the subjects of (99) and (100) are non-coreferent. This would constitute an argument against the union theory and for the sets theory which predicts non-coreference here. However, I would argue that an adequate theory of discourse would assign these noun phrases different context change potentials, though not necessarily different referents. Note that the verb phrase in question distinguishes as well between the noun phrases Ray and Tess and the authors which, assuming now that the authors were just Ray and Tess, even the sets theory counts as coreferent.

A debate about the coreference of definite noun phrases is a debate about predicate sharing but in a trickier way than at first envisioned. It is a two stepped affair. First one needs to determine whether predicates are shared or not. If certain predicates are found that are not shared then we must wonder whether there is some explanation for this other than the referential properties of the noun phrases. Clearly, what drives the discussion to begin with is some initial intuition that the noun phrases in question are coreferent. These intuitions are very much in the background of the debate in this book. They have cropped up already in the form of identity statements such as:

(101)  a. The children are just the boy and the girls.
       b. The players are the team.

In the following chapters we try to determine whether the relevant predicate sharing is found to accord with these intuitions.
Chapter 4
General Arguments from VP Denotations

In this section, our focus will be on arguments in favor of the sets theory based on possible VP denotations. The discussion will run as follows. First I will make some preliminary adjustments to our theories in order to accommodate the kinds of examples we need to consider. Next I will present a number of examples from work by Hoeksema, Link and Landman which argue for the sets approach. I will temporarily adopt this theory. Then I will argue for the introduction of two shifting operations which can apply to predicates to allow them to apply to entities of different types. Next, I will show that once these operations are in place, the motivation for the sets approach is eroded. I end this section by returning to the simpler union approach.

Let us assume for the remainder of our discussion that we have some male and female cows and some male and female pigs, that the cows and the pigs comprise all the animals there are, and that the males are young and old and so are the females. In order to talk about these various cows and pigs we add to our theories the category IADJ (intersective adjective) whose members include young, old, male and female. We also add the following rules to both theories:

[9] If α is an IADJ then \( \| \alpha \|_M = V(\alpha) \) and \( V(\alpha) \subseteq D \).
[10] If α is an IADJ and β is a CN then \( \alpha \beta \) is a CN and \( \| \alpha \beta \| = \| \alpha \| \cap \| \beta \| \).

and we assume for a noun phrase like the young pigs that young pigs is the plural of young pig.

The noun phrase the cows denotes the set of all the cows. The noun phrase the pigs likewise denotes the set of all the pigs and the noun phrase the animals will denote the set of all the animals. So far we have three distinct noun phrases and three distinct entities. Now we come to the noun phrase the cows and the pigs. On the union approach we get the
union of the cows and the pigs which is the set of all the cows and pigs which is just the set of all the animals. So the cows and the pigs are just the animals on this approach and we still have only three entities. On the sets approach, the noun phrase the cows and the pigs denotes a set of two sets, a cow set and a pig set. This is different from the noun phrase the animals which denotes a set of individuals, not a set of sets. So now, on the sets approach, we have four entities: a purely bovine plurality, a purely porcine plurality, an animal plurality, and finally a plurality composed of two pluralities. Of course, this last one is not the only new animal entity that the sets approach has but that the union approach lacks. For there is also the denotation of the young animals and the old animals, another set of two sets and there is the male animals and the female animals, yet another distinct entity. And so on. All we want now is some linguistic evidence to show that we need these extra entities.

The list in (102)-(104) below, inspired by examples in Link (1984) and Landman (1989a), contains the evidence we need for the extra higher order entities of the sets approach. Let me note that I will be ignoring distributive readings throughout. The issue of distributivity will be taken up in chapter 5.

(102) a. The cows and the pigs were separated.
   b. The young animals and the old animals were separated.

(103) a. The cows and the pigs talked to each other.
   b. The young animals and the old animals talked to each other.

(104) a. The cows and the pigs were given different foods.
   b. The young animals and the old animals were given different foods.

Each example consists of an a. and b. pair which seem, in the context we’re assuming here, to be independent in the sense that one could be true while the other is false. Consider the pair in (102). It has been claimed that we do not want it to follow necessarily from the fact that the cows and the pigs were separated that the young animals and the old animals were separated, even with our assumption that the animals are just the cows and the pigs. On the union approach the noun phrase subjects of the a. and b. sentences would have the same denotation, namely something corresponding to the set of all the animals. This would mean that if any of the a. sentences was true the corresponding b. sentence would also have to be true. In order to avoid this undesirable consequence, we adopt the sets theory, under which the noun phrase the cows and the pigs and the noun phrase the young animals and the old animals have different denotations, and hence the a. and b. sentences remain independent.
We have just considered examples the main predicates of which appear to distinguish sets of higher than first order. Since such sets are not found in the domain of the union approach, we had cause to adopt the sets approach. But, of course, not all predicates are like the ones in (102)-(104). Some predicates, such as fill the room and are asleep, intuitively appear to hold of first order sets. This fact, by itself, is not a problem for the sets approach whose domain includes all the elements in the domain of the union approach, including first order sets. However, these intuitively first-order predicates have a property that is somewhat surprising, given the assumptions of the sets approach. Consider the following:

(105) a. The animals filled the barn to capacity.  
     b. The cows and the pigs filled the barn to capacity.  
     c. The young animals and the old animals filled the barn to capacity.

(106) a. The animals were sleeping in the barn.  
     b. The cows and the pigs were sleeping in the barn.  
     c. The young animals and the old animals were sleeping in the barn.

If (105a) is true then in the context we are assuming, (105b) and (105c) will follow. On the sets approach which we have just adopted, nothing guarantees this since the subject noun phrases in (105a)-(105c) are not coreferent. The inference from (105a) to (105b) and (105c) is an example of what I call the Upward Closure Phenomenon, whereby

(107) **Upward Closure Phenomenon**  
An English predicate that is true of a first order plurality \( G \) (non-singleton set of individuals), is true as well of all higher order pluralities formed using all the members of \( G \).

We need to add something to our sets theory now which will guarantee that the Upward Closure Phenomenon holds for the sentences generated and interpreted by this theory. We do this by constraining the interpretation function, \( \cdot \cdot \cdot \), so as to eliminate any predicates that do not have the Upward Closure property:

(108) **LIFT constraint on \( \cdot \cdot \cdot \).**  
For any predicate of English, \( \delta \), and \( Y \in D^* \):

\[
\text{if} \{ x \in D \mid x \in^* Y \} \in \cdot \cdot \cdot \delta \cdot \cdot \cdot \text{ then } Y \in \cdot \cdot \cdot \delta \cdot \cdot \cdot
\]

\( \in^* \) is meant to indicate the transitive closure of \( \in \) defined as follows:
\( \forall x, z [x \in^* z \leftrightarrow (x \in z \lor \exists y [x \in^* y \land y \in z \land y \models z])] \)

According to (108), if \( \delta \) is true of some first order plurality \( A \), \( \delta \) will be true of a plurality of any order if the individuals involved are just the members of \( A \). For example:

\[
\{a,b,c,d\} \in \parallel \delta \parallel \rightarrow \{\{a,b\},\{c,d\}\} \in \parallel \delta \parallel
\]

To see how this works, reconsider the example in (105). Assume that (105a) is true. If that is the case, then the plurality composed of all the individual animals is in \( \parallel \text{fill the room to capacity} \parallel \). The LIFT constraint now guarantees that any set of any order whose urelements are the animals will be in \( \parallel \text{fill the room to capacity} \parallel \). The subject noun phrases of (105b) and (105c) both denote such sets, hence these sentences would also come out true.\(^{14}\)

It is worth noting here that the problem surrounding (105)-(106) would not have arisen had we chosen the union theory instead of the sets theory. This is so because according to that theory all the subject NPs in (105)-(106) have the same denotation provided the domain of discourse is as assumed here where the animals are just young and old cows and pigs.

\(^{14}\)This LIFT constraint is written in such a way that it will apply to all predicates, not just verbal predicates. This generality is desirable since the Upward Closure phenomenon shows up not only with verbs, but also for example with nouns:

i. The cows are mammals.

entails that:

ii. The young cows and the old cows are mammals.

However, this causes a problem for our interpretation of definite plural noun phrases. A plural noun can no longer be taken to denote the power set of the denotation of its singular counterpart minus the empty set, but must now denote the "lifted" version of this set, that is one that conforms to the LIFT constraint. Furthermore, we can no longer be content with saying that the denotation of the definite article is a function that takes a set of sets and returns the largest one. Rather, we must say that it returns the largest first order set. This will still work for the singular case, since singletons are first order.
One might be tempted therefore to suggest that the conjunction *and* is ambiguous with a union interpretation associated with the examples in (105)-(106) and a sets interpretation associated with the noun phrases in (102)-(104). This will not work however, because, as pointed out in Landman (1989a:§2.4), there are examples such as (109) in which predicates of the (105)-(106) type are conjoined with predicates of the (102)-(104) type:

(109) The Cows and the Pigs account for more than half the population of New Blinks but hate each other intensely.

(110) a. The Cows and the Pigs account for more than half the population of New Blinks.
b. The Cows and the Pigs hate each other intensely.

To see the problem here, let us assume for the moment that *and* is ambiguous between a union and a sets interpretation and that we do not have something like the LIFT constraint. The first VP conjunct in (109) is one that would be true in virtue of some fact about a first order set of individuals; it is like *fill to capacity*. The denotation of the subject NP could be in the extension of such a VP if we interpreted it using the union interpretation of *and*. The second VP is of the type that was used by Link (1984) to argue for a higher order theory like our sets theory (cf. (103) above). The denotation of the subject NP could be in the extension of such a VP if we interpreted it using the sets interpretation of *and* (assuming a reading in which Cows love Cows and Pigs love Pigs but Cows hate Pigs and vice versa). We assume that VP conjunction with *but* has the same truth conditions as with *and*, and so it is interpreted as set intersection.

The problem is that nothing guarantees that the intersection of the denotations of these two VPs will contain either type of NP denotation even if the sentences in (110) were true and this is counterintuitive. The reader may recognize this argument as a higher order version of the one given at the end of section 1.3 against an ambiguous interpretation for *and*.

Because of examples like (109), we stick to the sets theory and introduce the LIFT constraint. This then guarantees that we can safely use the sets interpretation of *and* in (109), and assuming the sentences in (110) are true, (109) will be true as well. This way of doing things is essentially an adaptation of the type-shifting operations introduced in Landman (1989a:§2.4) or the meaning postulates of Hoeksema (1987a:28-29).

Reviewing briefly, because of the examples in (102)-(104) repeated below:
we adopted the sets theory. In order to prevent the inference from the truth of an a. sentence to the corresponding b. sentence, we interpret conjunction as set-formation and this insures that the noun phrases *the cows and the pigs* and *the young animals and the old animals* will have different denotations. Next, we saw that certain predicates which are true of first order pluralities seem to be true as well of all higher order pluralities formed from the same individuals, thus blurring the distinctions introduced with the adoption of the sets theory. To handle this Upward Closure Phenomenon, we constrain \( \| \cdot \| \) in such a way that a predicate of English will be true of a higher order plurality in \( D^* \) if it is true of the set of individuals which are the urelements of that higher order plurality.

The LIFT constraint is worded in such a way that it applies to any predicate of English. As far as intuitively first order predicates like the ones of (105)-(106) are concerned, I think this is correct. As far as non-first order predicates like those in (102)-(104) are concerned, the constraint would appear to be irrelevant, since its application is limited to predicates having first order sets in their extension to begin with. But this last statement, relies on an assumption which I would like now to challenge. I would like to claim that not only is there an upward closure phenomenon in English but there is a downward closure phenomenon as well. I claim that:

\[(111) \text{ Downward Closure Phenomenon} \]

1° There are no predicates of English that have higher order pluralities in their extension but that cannot also have first order pluralities in their extension.

2° If a predicate of English is true of a plurality \( G \) of any order, it will also be true of that first order plurality \( G' \) which is composed of the individuals used to generate \( G \).

The first part says that there are no predicates of English that have exclusively higher order pluralities in their extension. In other words, there are no predicates that are strictly typed for higher order groups. Support for this claim comes from the fact that the predicates used in (102)-(104) to
argue for adopting the sets theory can be applied to noun phrases denoting first order sets. Examples of this appear in (112).

(112) a. The boys were separated.
   b. The boys talked to each other.
   c. The boys were given different foods.

We return to this conjecture in chapter 8.

The second part of (111) says that if a predicate of English is true of a plurality G of any order, it will also be true of that first order plurality G' which is composed of the individuals used to generate G. This is, in a sense, a stronger version of the first part. It speaks not about the kinds of things that can be in a predicate's extension, but about specific entities that we find there. Evidence for this claim follows in examples (113)-(115). Recall that distributive readings are ignored here and that we are assuming that the animals are just the cows and the pigs. I claim that in each example the b. sentence follows from the a. sentence.

(113) a. The cows and the pigs were separated.
   b. The animals were separated.

(114) a. The cows and the pigs talked to each other.
   b. The animals talked to each other.

(115) a. The cows and the pigs were given different foods.
   b. The animals were given different foods.

An appropriate context for these sentences might be one where a speaker says a. and his hearer replies with b. adding that he is not interested in how it was done, just that it was done. Another context for these examples might be one in which the a. sentence is true, but the speaker didn't have enough information to say that, either because he didn't realize that there were only cows and pigs or he simply could not distinguish a cow from a pig.

So far, there is nothing in the system we are working with that will guarantee the types of inferences exemplified in (113)-(115) since the sets theory assigns the noun phrase the animals a different denotation from that assigned to the noun phrase the cows and the pigs. We need to add something to our sets theory which will guarantee that the Downward Closure Phenomenon holds for the sentences generated and interpreted by that theory. Again we do this by constraining the interpretation function, \[ \| \cdot \| \], so as to eliminate any predicates that would falsify the generalization:
LOWER constraint on $|| \cdot ||$.

For any predicate of English, $\delta$, and $K \in D^*$:

if $K \in || \delta ||$ then $\{ x \in D \mid x \in^* K \} \in || \delta ||$

[$\in^*$ is the transitive closure of $\in$]

According to (116), if $\delta$ is true of a plurality $G$ of any order, then $\delta$ will be true of that first order plurality $G'$ which is composed of the individuals used to form $G$, for example:

$$\{\{a,b\},\{c,d\}\} \in || \delta || \rightarrow \{a,b,c,d\} \in || \delta ||$$

To see how this works, reconsider the example in (113). Assume that (113a) is true. If that is the case, then $||$ were separated $||$ will contain, among other things, a two membered set consisting of the set of all the cows and the set of all the pigs. Since $||$ were separated $||$ contains this set, it will also have to contain the first order set containing all the cows and pigs; that is what the LOWER constraint requires. Since the animals are just the cows and the pigs, it follows then that (113b) is true.

Reviewing again, because of the examples in (102)-(104) we assume the domain of discourse $D^*$ given by the sets theory and we interpret conjunction as set-formation. This means that the noun phrases the animals and the cows and the pigs cannot be coreferent. But then we come to find out that English does not want to cooperate. Predicates of English are just not fine-grained enough. So the next thing we need to do is introduce two constraints on the interpretation function that serve to blur the distinctions. At this point I want to give some thought to the interaction of these two constraints.

For our first example, I want to return to (105) repeated below:

(105)

a. The animals filled the barn to capacity.
b. The cows and the pigs filled the barn to capacity.
c. The young animals and the old animals filled the barn to capacity.

I claimed above that (105a) implies (105b) and (105c). That intuition was incorporated into our theory in the form of the LIFT constraint. In fact, the entailment goes the other way as well. Namely, (105b) and (105c) (on their non-distributive readings) entail (105a). This entailment is not covered by the LIFT constraint. However, it is covered by the LOWER constraint. In effect then, the combination of the LIFT and the LOWER constraints guarantee that as long as the animals are just the cows and the pigs, (105a)-(105c) are truth conditionally equivalent.
General Arguments from VP Denotations

Now we go the other way, reexamining the examples that motivated LOWER to see what effect LIFT has on them. I will do this by way of a piece of reasoning given in (117) which is explained by appeal to these constraints.

(117) Given:
   a. The young animals and the old animals are just the cows and the pigs.

Assume:
   b. The young animals and the old animals were separated.

Then:
   c. The animals were separated. (by LOWER)
   d. The animals were separated by age.
   e. The cows and the pigs were separated by age. (LIFT)
   f. The cows and the pigs were separated.

The intuition that c. follows from b. is explained by the LOWER constraint. (117d) seems to follow from (117b) and (117c) and the meaning of by age. Now from (117d) I think we can conclude (117e). This step is more evidence for the LIFT constraint. If || separated-by-age || contains the set of all animals, then it must, by the LIFT constraint, contain all sets whose urelements are the animals. In particular, it must contain the two membered set denoted by the subject of e. Finally, the step from e. to f. comes about because by age is a standard modifier, that is, it obeys the following schema:

(118) NP VP by age ---> NP VP. (NP is monotonically increasing)

If some things are separated by age then they are separated. I take the inferences traced out in (117) to be intuitively correct, and yet they have allowed us to go from (117b) to (117f). Now I do not deny that (117f) is a misleading thing to say, if you could have said (117b). It is misleading, but not false. (117f) follows from (117b) because English does not respect the distinctions that the sets theory makes. Here is another example of reasoning in which LOWER and LIFT appear to interact:
(19) Scene: four lawyers: Robert (defense), John (defense), Marcia (prosecution), Hank (prosecution).

Given:

a. The defense lawyers and the prosecution lawyers used to fight each other in court every day.

Then:

b. The lawyers used to fight each other in court every day. (LOWER)
c. That woman and those three men used to fight each other in court every day. (LIFT)

Here again the steps from premise to conclusion seem intuitively correct. And again the subject NP of our premise and the subject NP of the conclusion have in common that they denote sets built from the same urelements. A tendentious way to put this would be that what they have in common is that they are assigned the same denotation by the union theory. We can summarize this pattern by combining our two constraints as follows:

(120) LIFT and LOWER constraints combined.

if \( K \subseteq P \) then \( \{ x \in D \mid x \in^* K \} \subseteq P \)

if \( \{ x \in D \mid x \in^* Y \} \subseteq P \) then \( Y \subseteq P \)

\( (K,Y \) are variables over elements of \( D^* \)\)

It follows from (120) that:

(121) if \( K \subseteq P \)

and \( \{ x \in D \mid x \in^* K \} = \{ x \in D \mid x \in^* Y \} \)

What (121) says is that we can go directly from (122) to (123):

(122) The cows and the pigs were separated.
(123) The young animals and the old animals were separated.

But now if LOWER and LIFT can conspire like this our original motivation for moving from the sparse universe of the union theory to the more complicated world of the sets theory is undercut.

Recall, our original motivation was to achieve a logic that would allow a sentence like (122) to be true, without it following, in the relevant context, that (123) was true. Having accommodated the Upward Closure Phenomenon and the Downward Closure Phenomenon, we now have a
system that precisely allows us to go from (122) to (123).

I elaborate. (122) and (123) differ in meaning. No question about that. The issue here is that the argument based on these examples in favor of the sets theory rests on the belief that the difference between (122) and (123) can be captured in a semantics which takes predicates to denote sets of objects which are themselves distinguished by a set-theoretic principle of extensionality. In particular, given two different sets formed from the same urelements, for example \{a,b\}, c and \{a\{b,c\}\}, this account rests on the possibility of having a predicate of English whose denotation includes one of these sets but not the other. It is this belief that I am challenging here.

In order to argue for a sets theory we need to have predicates that distinguish plural entities which differ only by the way they are grouped. We thought we had this. However, English has two properties that conspire against us. The Downward Closure Phenomenon guarantees that a predicate of English that is true of a particular grouping of a set of singularities will be true of that set itself. On the other hand, the Upward Closure Phenomenon guarantees that predicates of English that are true of a set of singularities to be true of those individuals on any grouping. Putting these two together, it turns out that a predicate of English that is true of a set of singularities on some complicated grouping will be true of the set itself, and hence true of those singularities on any grouping, so groupings cannot matter.

Of course, (122) and (123) differ in meaning. Both say more than (124).

(124) The animals were separated.

In (122) we understand the animals to be separated by species, in (123) by age. I also grant that this added information is coming from the NP subjects. So I do not dispute that (122) and (123) differ and that that difference has to do with the meaning of the subject NPs. What I do dispute is that the difference can be handled by an account that rests on having predicate denotations be sets of entities that are as fine grained as the sets theorist needs to support his theory. Extensions of predicates of English do not have this kind of structure.

The differences here do have to do with NP meanings, but not with the objects that NPs refer to. In the terminology introduced in section 3.2 we may say that those predicates not shared by our putatively non-coreferent noun phrases are not relevant here.
For completeness I would like to turn now to a slightly different version of the argument made here for the sets theory, due to J. Hoeksema. Consider (125) from Hoeksema (1983):

(125)  [[Blücher and Wellington] and Napoleon] fought against each other near Waterloo. ≡ [Blücher and [Wellington and Napoleon]] fought against each other near Waterloo.

Hoeksema argues on the basis of the example in (125) that the interpretation of and must not be associative otherwise the bracketing on the conjunction would not affect the denotation of the noun phrase subject. The problem with this argument is that, as with the examples used above, it relies on having predicates that are sensitive to these groupings. In this particular example, the extension of the predicate fight against each other would have to encode information not only about who was fighting but also about who was allied with whom in the battle. However as I have argued up to now, English predicates do not seem to be as fine grained as the sets theorist requires. As above, the first thing to note is that the Downward Closure Phenomenon crops up with the predicate in this example. Were I embarrassed about the fact that I couldn’t pronounce Blücher correctly, I might report the events of (125) as:

(126)  Those famous European generals fought each other at Waterloo.

(126) follows from (125). If a higher order plurality built from those three generals is in || fought each other at Waterloo ||, then so is the first order plurality having each of them as a member.

The example in (127) makes the case even stronger:

(127)  Despite their current membership in a common market, only 50 years ago, Germany, England, France and Italy were battling each other in one of the worst wars in history.

I take (127) to be true. If Hoeksema is correct and the alignment of the forces is encoded in the extension of the predicate were battling each other then (127) should only be true if the subject noun phrase denotes a set of two sets, one containing Germany and Italy and the other France and England. But even the sets theory doesn’t assign this denotation to the subject of (127). (128) below is yet another example of this type:

(128)  J. and K. played match, but L. and M. played with each other.

A non-associative interpretation of the conjunction J. and K. played match, but L. and M. played with each other takes here (130) as the sister.

(129)  a. J. and K. played match.

Consider the way in which the world is organized: J. and K. are members of the team that is reminiscent of (103)

(103)  a. The emergency committee consists of eight people.

In (103a) the phrase a committee of eight people is a group of individuals, while the group of eight people is a different group of individuals. As shown in (103) by using different phrases. (103b) by using the same phrases. Where as (103a) denotes a noun phrase, the same two sets are arrived at. (103b) is a further test.

(130)  The American and the English fought each other.

The data that I have reviewed so far shows the Downward Closure Phenomenon in examples that have a subject N. and N. phrase.
John and Mary and Bill and Sue played tennis with each other. In the first match, the men played the women, and in the second match John and Mary played Bill and Sue.

A non-associative and might seem attractive on the basis of the second match, but must be abandoned on the basis of the first match.

Finally one bit of circumstantial evidence in favor of the position taken here comes from pairs of the following sort:

129. a. The women from the two communities hated each other.
   b. The women who belonged to the two organizations hated each other.

Consider a context in which the women from the two communities are just the women from the two organizations, the memberships of the two organizations are totally non-overlapping and each organization has members from both communities. The examples in (129) differ in way that is reminiscent of our earlier Linkian examples, repeated here:

103. a. The cows and the pigs talked to each other.
   b. The young animals and the old animals talked to each other.

In (103a) and (103b) the individuals doing the talking are the same while the grouping of the conversationalists differ. In (129a) and (129b), the individuals doing the hating are the same; the battle lines are drawn differently. The sets theorist attempts to account for the differences in (103) by incorporating the groupings in the denotations of the noun phrases. This is achieved through a judicious choice of interpretation for and. What (129) purports to show is that the phenomenon is not limited to noun phrases involving conjunction. To convince us that he is correct, the sets theorist will have to show how the correctly "grouped" denotations are arrived at for the subjects of (129). A similar point is made with (130) below, reminiscent of (102):

130. The delegates from each of the countries were separated.
102. The cows and the pigs were separated.

The verdict based on this section is in favor the union theory. The data that I used to demonstrate the Upward Closure Phenomenon and the Downward Closure Phenomenon now have a simpler explanation. The examples in (131) below all have the same truth conditions because all the subject NP's have the same denotation in the context we are assuming.
(131)  a. The animals filled the barn to capacity.
b. The cows and the pigs filled the barn to capacity.
c. The young animals and the old animals filled the barn to capacity.

If the sentence in (122) is true,

(122) The cows and the pigs were separated.

then (124),

(124) The animals were separated.

can be uttered truthfully in the context we’ve assumed up to now, since the subjects of (122) and (124) are assigned the same denotation in the union theory and the predicates are identical.

While a case has been made here for the union approach, there is something that still remains unexplained. On the sets approach, (122) and (124) are truth-conditionally distinct and furthermore that difference depends on a difference in the reference of the subjects of those sentences. I have just argued against these two claims. Nevertheless, the fact remains that (122) and (124) do differ in some way and one would like to know why that is so according to the union theorist. In the following chapter, five, we will take up a different sort of argument for the sets approach, this will involve the phenomenon of distributivity, ignored to this point. The approach we take there to distributivity will, I believe, shed some light on the data examined in this section, so in chapter 6 we return to the questions left open here.

5.1 A CONCEPT OF DISTRIBUTIVITY

A.

(84) The cows and the pigs were separated.

have been mentioned. Landman and others have shown distributivity to be a widespread phenomenon even in a wide variety of natural languages.

It is argued that the phenomenon of distributivity in a clause of the form (84) is due to the fact that the plural denotation of the subject has a plural meaning in a way that the singular denotation of the same subject does not. However, if a plural subject has a single denotation, it will not lead to distributivity, it will require a difference in meaning.