Chapter 7
Notes on Floated Quantifiers

7.0 Introduction

Traditionally, the occurrence of each in (327a), was thought to be derived transformationally, ‘floated’ rightward, from a position preceding the subject NP as in (327b)

\[
\begin{align*}
(327) & & a. & & \text{The frogs each leapt off a lily pad.} \\
& & b. & & \text{Each of the frogs leapt off a lily pad.}
\end{align*}
\]

In addition to each, both and all are common examples of floated quantifiers. This chapter is meant to be a small collection of notes connected with these quantifiers. In section 7.1, Part operators are compared with the floated quantifiers, on which they were modelled. In section 7.2, we turn our attention to both in its non-floated incarnations. That section is a response to Peter Lasersohn’s claim that the semantics of both demands that the sets theory be preferred over the union theory. The results of that section will be used in section 7.3 to explore the claim that floated and non-floated both are syntactically related.

7.1 Part Operators versus Floated Quantifiers

The theory of distributivity outlined in chapter 5 was based on a generalization of Dowty and Brodie’s (1984) account of floated quantifiers as verb phrase modifiers. This history is likely to lead to the assumption expressed in chapter 5 that covert distributivity operators are simply covert instances of the overt operators. The purpose of this section is to cast doubt on that assumption. It is entirely possible that floated quantifiers are not VP modifiers at all (cf. Sportiche 1988, discussed below) in which case the Part operators might be the only operators of this kind in the language. If this is case, one might want to reanalyze them as interpretations of the plural marking itself. In any case, the analysis of distributivity and the grammar of floated quantifiers are separate questions, theoretically, even if
not historically.

The need to distinguish Part operators from floated quantifiers is nowhere more apparent than in the analysis of the reciprocals. Recall in chapter 6 we said that a reciprocal is uninterpretable unless bound by a Part operator, yielding intermediate representations of the following kind:

(328) They\textsubscript{j} Part\textsubscript{j}[saw each other\textsubscript{j}].

Taking floated each to be an overt incarnation of the Part operator, leads to the false prediction that (329) below should be grammatical.

(329) *They\textsubscript{j} each\textsubscript{j} saw each other\textsubscript{j}.

The ungrammaticality of (329) shows that as long as we preserve our analysis of reciprocals, we need to adopt a different analysis for each than was presented when the Part operators were first motivated. We will leave this as an open problem. The interested reader is referred to the remarks and replies of Williams (1991) and Heim, Lasnik and May (1991b) in which the ungrammaticality of (329) is explained partially on syntactic grounds.

Turning to the semantics of the floated quantifiers, we note differences between them and the Part operators with respect to both quantificational force and domain of quantification. Part operators always have universal force and they quantify over singularities and pluralities, depending on the context. While the force of each, all and both is also universal, in some dialects there are floated quantifiers with other than universal force. Quirk et. al (1985:1399) cite the following examples:

(330) They are none of them very enthusiastic. \textit{<Informal>}
My sisters don't either of them eat enough. \textit{<Informal>}

The example in (331) below is taken from the OED (OED, \textit{many} §A,3,a.: from 1567 John Maplet) while the one in (332) is from a story by Graham Greene (Greene 1973:395):

(331) We be many of us cut off before we come to olde age.

(332) His ambition had been to be a playwright and now that the London theatres were most of them closed, he was no longer taunted by the sight of other men's success.

One might argue that these are not true floated quantifiers because of the pronominal complements, but it should be pointed out that the 'true'
Notes on Floated Quantifiers

floated quantifiers all have uses in which they behave as if they had a missing pronoun complement or were themselves pronominal. *each* is used that way in (333):

(333) The frogs leapt off the lily pad. *Each* left a ring of wavelets on the surface of the lake.

With respect to domain of quantification, even the universal floated quantifiers differ from Part operators. The domain of quantification for *each* includes only singularities, as noted in chapter 5, while *all* can quantify over subparts of singularities, as in the following example:

(334) The room was all cleaned up.

In chapter 9, we return to some of these uses of *all*. *Both* is peculiar in that it's domain is restricted in size. The subject of a verb phrase commencing with *both* is presupposed to refer to a two-membered plurality. *The frogs both leapt into the lake* is not felicitous in a conversation about several frogs. We will refer to this property as the "duality presupposition" of *both*.

7.2 *Both* in the Sets and the Union Theories

Floated *both* shares the duality presupposition with at least some of its non-floated counterparts. It is this presupposition that forms the basis of the following argument for the sets theory found in Lasersohn (1988:132, 1995:§9.4). First, consider the evidence for the presence of the duality presupposition:

(335) a. Both children are asleep.
b. Both the children are asleep.
c. Both John and Mary are asleep.

a. and b. are infelicitous in a context in which it is presupposed that there are more or less than two children. The duality presupposition is never violated in example (335c), in which the noun phrase that *both* is combined with denotes a plurality of two. Next, consider the following example, which occurred in a discussion of two grammars of Welsh, one written by Jones and Thomas and the other written by Awbery:

(336) In contrast, both Awbery and Jones and Thomas need extra statements in their grammars to make the distinction.
this example should be infelicitous since the phrase *Awbrey and Jones and Thomas* denotes a plurality of cardinality greater than two. But (336) is not infelicitous and the reason, according to Lasersohn, is that the noun phrase *Awbrey and Jones and Thomas* in fact does denote a plurality of cardinality two, one of whose members happens to be a plurality itself. This may be impossible according to the union theory, but not according to the set-theory, hence we have an argument in favor of the latter.

The basis of this account of the felicity of (336) lies in the semantics of the complement of *both*, not in its syntax. This means that any definite noun phrase that denotes a set of two pluralities should be able to serve as the complement of *both*. In chapter 4, we discussed some examples that didn’t involve conjunctions of plural noun phrases but that looked like they would have to denote higher order pluralities on the sets theorist’s account of things. The subject of (337b) is an example of this kind:

(337) a. The children from Mexico and the children from Venezuela attended different summer camps.

b. The children from Mexico and from Venezuela attended different summer camps.

In the absence of any other context, the most salient interpretation of (337a) is one in which the children from Mexico attended different camps than did the Venezuelans. On the sets view, this interpretation arises because the subject of (337a) denotes a set of two pluralities. This division is ‘encoded’ in the verb phrase extension. (337b) has the same interpretation as (337a) hence its subject must also denote a two-membered set. In chapter 4, no rules of interpretation leading to this result were proposed, rather these examples were simply left as a challenge to the sets theorist. But no matter what solution is arrived at, unless there is a reanalysis of the verb phrase of (337a), it would seem to me that the subjects of (337a-b) are co-denoting. Given this conclusion, we now predict, contrary to intuition, that the subject of (337b) is a potential complement of *both*:

(338) ?Both (of) the children from Mexico and from Venezuela attended the summer camp.

By similar reasoning based on (339a) below, (339b) is counterfactually predicted to be good (assume that there are several children from each of the communities):

(339) a. The children from Mexico and from Venezuela are fed.

b. The children from Mexico and from Venezuela are fed.

The example above does not appear to me to be such a case. I don’t think anyone should rule out cases of the sort described below:

(340) a. The children from Mexico and from Venezuela are fed.

b. The children from Mexico and from Venezuela are fed.

The example above does not appear to me to be such a case. I don’t think anyone should rule out cases of the sort described below:
(339)  a. The children from the two communities attended different summer camps.
   b. #Both (of) the children from the two communities attended the summer camp.

In (340) below, we have another kind example of a noun phrase that would denote a higher order plurality on the sets account but which doesn't involve a conjunction of plurals:

(340)  The women and the men disagreed about the morning’s activities. That was why the guide put them in separate busses.

The pronominal object of put has a higher order antecedent, hence it denotes a higher order plurality. This is how the second sentence comes to mean that the women’s bus was separate from the men’s. Given that such pronouns denote higher order pluralities, again it is expected that they should function as complements of both. But this prediction is not borne out either:

(341)  #The women and the men disagreed about the morning’s activities. To avoid any conflicts in the afternoon, the guide put both of them on a bus to the city center for a free shopping day.

The examples in (338), (339b) and (341) suggest that the sets-based explanation for why (336) is felicitous is mistaken. However, if we reject Lasersohn’s account of (336), then we are left with no account of why it is felicitous, not according to the sets theorist nor the union theorist.

The solution to this dilemma seems to me to lie not in the semantics of plurals, but rather in the grammar of both. The basis for examining the subject of (336) in connection with those of (335) repeated below:

(335)  a. Both children are asleep.
   b. Both the children are asleep.
   c. Both John and Mary are asleep.

is that they all appear to be noun phrases formed by combining both with a nominal complement. But there is another possible source for the occurrence of both in (335c) and in (336). Both introduces conjunctions of categories other than nominal ones, as in the following examples:
(342) a. John both opened the door and turned up the music.
   b. The chicken is both cold and sour.
   c. He invented both a talking cup and a singing saucer.

One reason to recognize the occurrences of both in (342) as syntactically
distinct from those in (335a-b) is the fact that in the latter cases both is a
deterner and the uses in (342) are not standard cross-categorial
generalizations of determiners. Even both’s near relative all doesn’t
generalize this way:

(343) a. * all John and Mary and Bill were asleep.
   b. * John all walked and talked and sang.

Moreover, if the occurrences of both in (342) were the result of a
generalization of determiner both then we would have to say that the
complements of both denote two-membered pluralities. This by itself is not
impossible and in fact Lasersohn (1995) provides just such an analysis for
conjunctions in non-nominal categories. Strong evidence in favor of this
view of both would come from combining it with non-nominal expressions
other than conjunctions that are otherwise thought to denote two
membered pluralities. As far as I know, such evidence is not available.
that are supposedly used to refer to groups of events. None of them licence
both: \(^{39}\)

(344) a. John both kept (on) swimming.
   b. John both swam and swam.
   c. John both swam again and again.

The inability of both to combine with non-nominal non-
conjunctions casts doubt on the hypothesis that both in (342) is of a kind
with both in (335a,b). This would suggest that there are at least two boths
in English. The first, both\(_{det}\), is a determiner which requires that its

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\(^{39}\) The discussions of the semantics of alternately in Lasersohn (1992: §6,
1995: §14) suggest that ”murder one’s parent’s” might denote a group of two
events. One could imagine a use of both here to mean something like ”he
murdered both of his parents.” But this is impossible:

i. a. *He both murdered each of his parents.
   b. *He both murdered his parents / a parent.

On the other hand, if we consider (346c), we can see that the
definition and use of both given above leaves out how both can

(347)

and “both” are used in the following sentences:

(348)
complement denote a plurality with two singular parts. The second, \(_\text{both}^{\_\text{Conj}}\), introduces conjunctions cross-categorially. The comparison with \textit{all} in (343) would suggest that \textit{both} in (335c) and in (336) is not \textit{both}^{\_\text{det}} but rather \textit{both}^{\_\text{Conj}}. This ambiguity hypothesis is by no means new. One finds the distinction made in early work in generative grammar. In Stockwell, Schachter and Partee (1973), for example, \textit{both} is included in the category QUANTifier in the section on determiners, and in the section on conjunction there is a transformation called \textit{Both} Insertion which produces among other things \textit{both John and Mary}.

If one accepts the ambiguity hypothesis, then the problem of (336) is resolved. The subject is formed with \textit{both}^{\_\text{Conj}} which has no semantic duality requirement. The semantics of the expression \textit{Awbery and Jones and Thomas} is not an issue here, hence the choice between the sets and the union theories is not at stake.

Further evidence for the ambiguity hypothesis comes from the syntactic difference between the two \textit{both}'s brought out by the following paradigm:

\begin{align*}
(345) & \quad \text{a. both the boys} \\
& \quad \text{b. both of the boys.} \\
& \quad \text{c. both John and Mary.} \\
& \quad \text{d. *both of John and Mary.}
\end{align*}

\begin{align*}
(346) & \quad \text{a. all the boys} \\
& \quad \text{b. all of the boys.} \\
& \quad \text{c. *all John and Mary and Sue.} \\
& \quad \text{d. *all of John and Mary and Sue}
\end{align*}

On the assumption that \textit{all} and \textit{both}^{\_\text{det}} are members of the same category we can explain (345). \textit{both}^{\_\text{det}} is impossible in (345c,d) just as \textit{all} is in (346c,d). (345c) is grammatical because \textit{both}^{\_\text{Conj}} is used. But, by definition, \textit{both}^{\_\text{Conj}} cannot combine with a non-conjunction, so (345d) is out because of the intervening \textit{of}. Lists apparently do not count as conjunctions either, as the following shows:

\begin{align*}
(347) & \quad \text{*The chicken was both cold, sour and expensive.}
\end{align*}

and this would explain the anomaly of Lasersohn's (1988,1995) example below:

\begin{align*}
(348) & \quad \text{*both John, Mary and Bill are asleep.}
\end{align*}
Summarizing now, according to the ambiguity hypothesis both in (349a) is not the same word as both in (349b), despite superficial appearances.

(349) a. both John and Mary  
b. both (of) the boys

In (349a), we have an item that introduces conjunctions. In (349b), we have a determiner similar in character to all but with a duality presupposition. Since John and Mary happens to denote a two-membered plurality, one is lead to misanalyze both in (349a) as an instance of both in (349b). Circumstantial evidence for the ambiguity proposal comes from a comparison with either, the counterpart of both with respect to disjunction. Consider the paradigm below:

(350) a. either boy  
b. either of the boys.  
c. either sour or cold.  
d. either John or Mary.

In (350a,b), either is a determiner. In a context where it is known that there are more than two boys, both (350a) and (350b) are felicitous indicating that either introduces a duality presupposition like that of both. Comparing (350a,b) to (350c), we are lead to posit a second either which can introduce disjunctions. So far this story is parallel to what was said above concerning both. The difference comes with (350d). Here there is no chance for misanalysis, since the disjunction, John or Mary, does not denote a two-membered plurality, hence the presupposition associated with either in (350a,b) could not be met here, hence this is clearly a case of the second, cross-categorical, pre-disjunction, either.

Returning again to Lasersohn’s (336):

(336) In contrast, both Awbery and Jones and Thomas need extra statements in their grammars to make the distinction.

On the present theory, both in this example carries no presupposition of duality, hence this example doesn’t count as evidence against the union theory. On the other hand, the problems raised for the chapter 4 proponents of the sets-theory, in connection with (339b) and (341):

(339) b. ?Both (of) the children from the two communities attended the summer camp.

(351)
(341) #The women and the men disagreed about the morning’s activities. 
To avoid any conflicts in the afternoon, the guide put both of them on a bus to the city center for a free shopping day.

still remain, since they relied on a comparison of the above examples with example (335b):

(335) b. Both the children are asleep.

All of these examples, on the proposed account, involve the duality presupposing variant, both$_{det}$. According to the union theory, this presupposition is not met in (339b) or (341) which is why they are infelicitous as compared with (335b).

7.3 Floated both: A Sometime Distributivity Marker

The proposal concerning the ambiguity of pre-nominal both turns out to have consequences for syntactic analyses of the floated quantifier. Floated occurrences of both have often been viewed as related to their non-floated counterparts via a syntactic operation, unlike in the Dowty and Brodie (1984) analysis mentioned above. Traditionally, both was thought to float out of an NP into a verb phrase. Somewhat more recently, Sportiche (1988) has pursued that idea ‘in reverse’. According to his analysis of floated quantifiers, the both phrase would be base generated as the sister of VP and the NP complement of both would be moved leftward stranding the quantifier. An examination of data below in terms of our ambiguity hypothesis will lead to the rejection of this analysis, at least for some speakers. Before turning to the data, I would like to note that the degree of disagreement that I encountered in speaker intuitions has led me to suspect that the movement analysis might be correct for a subset of speakers and perhaps is accurate as an account of the history of floated both. I should also note that Sportiche’s analysis was for floated quantifiers in general so it is possible that the analysis is wrong for both but not for the others.

Turning to the data, if one assumes that the movement posited by Sportiche occurs regardless of which both is involved, then the grammaticality judgements for the examples in (351) entails those for the examples in (352):

(351) a. Both$_{det}$ the children are asleep.  
b. Both$_{Conj}$ [John and Mary] are asleep.  
c. Both$_{Conj}$ [Awbery and [Jones and Thomas]] need extra
statements.
d. *Both Awbery, Jones and Thomas need extra statements.

(352) a. The children are both asleep.
b. John and Mary are both asleep.
c. Awbery and Jones and Thomas both need extra statements.
d. *Awbery, Jones and Thomas both need extra statements.

Some speakers do not accept (352c). At first sight, it looks as if this intuition could be explained by assuming that for these speakers there is no movement away from \( both_{\text{conj}} \). The problem is that they do accept (352b), which would be derived through movement away from \( both_{\text{conj}} \) (compare (351b)). For these speakers then, a simple movement analysis is incompatible with the duality hypothesis argued for above.

We turn now to those speakers who do accept (352c). Among these speakers there are some for whom the examples below are acceptable despite the predictions of the movement analysis. In (353-355) below, the NP that has moved away from \( both \) into a VP external position isn’t formed with a conjunction. This means that it was the complement of \( both_{\text{det}} \) and hence it should, contrary to fact, denote a pair of singularities in order to satisfy the duality presupposition.

(353) a. The lawyers and the physicians disagreed about the morning’s activities. But in the afternoon, they will both go downtown to the museum.

b. Initially, neither the bankers nor the city councilors showed any interest in the plan. But now that the water rights have been clarified, they’ve suddenly both claimed it as their own.

(354) After the Civil War, the draft age was changed when it was discovered that those men who were either too young to fight or were too old to be drafted had both far outnumbered the fighting men.

(355) The administration thinks that the physics instructors and the math instructors cover five courses. In fact, \( those_{\text{instructors}} both_{\text{cover}} only two courses \). Only the English teachers cover five courses.

In the examples in (353), the putatively moved NP is a pronoun, which on its salient interpretation might be said to denote a pair according to the sets theory. This would constitute an argument for the sets theory, were it not
for the fact that the unfloated variants don't sound good:

(356)  ??Initially, neither the bankers nor the city councilors showed any interest in the plan. But now that the water rights have been clarified, both of them have suddenly claimed it as their own.

Given this discrepancy, the examples in (353) along with those in (354-355) are unpredicted by the movement analysis, on both the sets theory and the union theory.

To this point, we have isolated at least two groups of speakers for whom the movement analysis makes wrong predictions. For these speakers, we have excluded the possibility of deriving floated *both from both_{det} or from both_{[Conj]}. Before returning to the alternatives to movement mentioned earlier, I would like to address a possible response to the conclusions drawn so far.

The arguments provided above are arguments against an analysis in which there is truly movement of an NP out of a construction commencing with both. Arguments similar to the one mentioned above concerning (351-352) have been made with other quantifiers. Compare the grammaticality of (357a) below with the ungrammaticality of (357b), its putative source on the movement theory:

(357)  a. A, B and C all have knotted ends.
     b. * All of A, B and C have knotted ends

Sportiche (1988:440-441) claims that his theory is immune to this type of argument, since strictly speaking he doesn't require actual movement of the subject in (357a), allowing in fact for base generation of a structure like the one in (358) below, where [e]_{NP} is an NP-trace which is anaphoric to the subject NP:

(358)  [A, B and C]_{NP} ... [ [all [e]_{NP} [have knotted ends]].

Given this structure for (357a), one can say about (357b) that conjoined NPs cannot be the object of a partitive quantifier, and (357a) is grammatical because the object of the quantifier, [e]_{NP}, is not a conjoined NP. This type of reply applies equally well to our discussion above of (351-352). But Sportiche's theory is not similarly immune to the second argument made here. Compare the following two examples, repeated from above embellished with markers to indicate the position of trace in Sportiche's analysis:
Initially, neither the bankers nor the city councilors showed any interest in the plan. But now that the water rights have been clarified,

a. they have suddenly [ [both [ e_i ]_NP] [claimed it as their own]].

b. both of them_i have suddenly claimed it as their own.

On the theory proposed in the previous section, (359b) is ungrammatical because of a property of the referent of the pronoun, namely it is a plurality with more than two members. The referent of the subject NP of (359a) shares this property. But if the referent of the subject NP of (359a) has this property then so does the referent of the trace which is anaphoric to that NP. So even if there isn’t actual movement, there is coreference even on Sportiche’s theory and so the second argument stands.

It appears then that for the speakers whose judgements are reflected in (353-355), we must posit a third both, this one a VP modifier. Given the apparent similarity with the Part operators, one is tempted to view floated both simply as a Part operator with a special proviso that there are exactly two cells in the cover containing elements of the referent of the subject of the both VP and these cells contain all and only elements of that entity. Adopting the notation of Beaver (1992) to handle the presupposition, this idea could be spelled out as follows:

(360) \[
\text{both}_{\text{float}} \approx \approx > \lambda x \lambda y [\forall x ((\text{Cov}(x) \land (x \subseteq y)) \rightarrow P(x)] \land \\
\text{Presup}(|\text{Cov}/y| = 2)]
\]

Definition: For any X, a set of sets of individuals, and any y, a set of individuals, X/y is the largest subset of X that covers y, if there is one, otherwise it is undefined.

This analysis treats both as a special kind of Part operator. A possible argument for this analysis is that unlike floated each, both can introduce a reciprocal verb phrase (compare (329) above):

(361) They both saw each other.

Unfortunately, this analysis is precluded by other differences between floated both and the Part operators. Even though floated both is often used to indicate a distributive reading, it does not force one, as the following shows:

(362) John made the soup, I made the eggplant and we both made the
Notes on Floated Quantifiers

pot roast.

As it happens, floated both shares this semantic property with its non-floated counterparts. The following quote shows that both\textsubscript{det}, while often associated with a distributive reading, does not force one\textsuperscript{40}:

(363) "Napoleon and Squealer sold Boxer to the knacker" does not imply that Napoleon sold Boxer to the knacker, nor does it imply that Squealer did so. It entails that both of them sold Boxer to the knacker. (von Stechow, 1980:91)

Since both does not force a distributive reading, it cannot be analyzed like the Part operator with a universal quantifier, as in (360). As can be seen from (364) below, both shares this behavior as a sometime distributivity operator with all:

(364) Each of the boys made a cupcake, Ted made the chocolate pudding, Rita made the candied apples, and we all made the popcorn.

What exactly determines when a distributive reading is forced by all and when not is the subject of Dowty (1987).

7.4 Conclusion

As advertised in the introduction, this chapter has the character of a collection of notes related to quantifiers that float. We have conjectured that floated quantifiers are at least some of them not VP modifiers, hence they differ from the Part operator even though the inspiration for the Part

\textsuperscript{40}Does both\textsubscript{[\textit{Conj}]} also allow for a non-distributive reading? My intuition is that (\textit{i}) below makes no sense:

(i) "Napoleon and Squealer sold Boxer to the knacker" does not imply that Napoleon sold Boxer to the knacker, nor does it imply that Squealer did so. It entails that both Napoleon and Squealer sold Boxer to the knacker.

If both\textsubscript{[\textit{Conj}]} unlike both\textsubscript{det}, is always a distributive marker, then a) we have more support for the ambiguity hypothesis and b) as suggested in chapter 2, both...and would be a candidate expression to be translated as Hoeksema's (1983,7a) intersective conjunction.
operator was a VP modifier analysis of floated quantifiers. Concerning both, we argued following earlier work in generative grammar that this quantifier occurs in various categories. There is a pre-NP both which is a determiner that can occur alongside the just like all. There is another pre-NP both which is part of a complex cross-categorial conjunction. In addition, there is a floated quantifier both. What is common to all uses of both (including some we have not mentioned, cf. the both of them) is the notion of duality. In the case of determiner both, there is a duality presupposition and this was used here to argue against the sets theory. In the case of both_{Conj}, duality arises in the syntax. If the analysis here is correct, then the notion of duality is very general, including both syntactic and semantic instantiations. This raises interesting questions about the nature of lexical ambiguity and relatedness of words.

Chapter 7

Sorts:

8.1 The entity domain

to study the application of tests of cross-categorial duality in the verb domain and the different formalisms. The entity domain of entities as individuals might be an appropriate domain, focusing on the participation of entities in the compositional domain.

comes from "logic of" (what is what)?
Chapter 8
Sorting the Domain

8.1 Types of Pluralities

Our focus in this section will be on the difference in the variety of entities that the sets and the union approaches are committed to. In order to study this question, we need to have a way of characterizing variety and of testing for its activity in the grammar. We will start with the idea that the variety of a set of objects is established by dividing that set into different categories. If a semantic theory posits a domain of entities having a wide variety of entities in it, and if this variety is really relevant to the grammar, then it should make reference to the categories into which the domain is divisible. For example, a predicate may apply felicitously only to entities of a certain category.

There are many ways that a domain of individuals comprising singularities and pluralities can be organized. For example, individuals might be sorted into those that are animate, inanimate or mixed (e.g. the individual consisting of you and your coat). However, not all ways of dividing up the domain are relevant to the differences between our two approaches to plurals or to the semantics of plurals in general. We will focus on mathematical or logical ways of dividing the domain. In particular, the subjects of our interest will be the classification of entities in the domain by their cardinality and the classification of entities by position in a set-theoretical hierarchy, that is, by some logical typing of the domain.

To see why these classifications are relevant, I will explain how I come to call them logical. Following the logician's use of the term "logical" in describing quantifiers, a sorting or classification is considered logical if it is "permutation invariant," as follows:

Let perm be a one-one mapping of $D$ onto a set $D'$. A set $D'^\ast$ is constructed from $D'$ by the same method that $D^\ast$ is constructed from $D$ (what that method is depends on the particular theory of plurals). $\text{PERM}$ is a one-one function from $D^\ast$ to $D'^\ast$ defined as follows:
1. for $x \in D$, $\text{PERM}[x] = \text{perm}[x]$
2. for elements $X, Y$ in $D^*, D^*$ respectively,
   $$\text{PERM}[X] = Y \text{ iff } \forall x \in X \exists y \in Y \left( \text{PERM}[x] = y \right) \text{ and } \forall y \in Y \exists x \in X \left( \text{PERM}[x] = y \right)$$

A classification is permutation invariant if for any $X \in D^*$ and any permutation of $D$, perm, $X$ and $\text{PERM}[X]$ are in the same class. Given the way $\text{PERM}$ is defined, $X$ and $\text{PERM}[X]$ will always be of the same cardinality and of the same logical type, so these are logical classifications. An example of a non-logical sorting would be one that put all individuals having John as member in one category and all others in another category (John could turn out to be a member of $D$ but not of $D'$). Animacy provides another more natural example of a criterion for sorting that is non-logical ($\text{PERM}$ may map all animate elements of $D$ into inanimate elements of $D'$). The limitation to logical classifications expresses the requirement that the sorting criteria not distinguish between different elements of $D$ (cf. Mostowski 1957 for similar point regarding quantifiers). This guarantees that the categories involved are not simply inherited from characteristics of individuals that are unrelated to pluralization.

In this section then we will contrast the two most obvious logical classifications, cardinality and type-hierarchy, in terms of their activity in the grammar. Looking ahead a bit, notice that a type-hierarchy provides a rather impoverished method for sorting the individuals of the union theory, whereas that is certainly not the case for the sets theory. This should be clear from the relationship between $D$ and $D^*$ on the two approaches:

Union theory: $D^*$ is the set of all non-empty subsets of $D$.

Sets theory$^{41}$:

$$D_0 = D$$
$$D_{n+1} = D_n \cup \text{POW}_{\geq 2}(D_n)$$
$$D^* = \bigcup_{n \leq \omega} D_n$$

$^{41}$ $\text{POW}_{\geq 2}(X)$ is the set of all the non-empty non-singleton subsets of $X$.  

8.2 Classifications that are number-invariant

that are number-invariant. In the discussion that follows, we shall use expressions that are number-invariant in a more precise sense. For example, expressions like two more than, a dog, and a man with a beard will be more precisely classified as two more, a dog, and an individual, respectively.

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8.2 Cardinality

By sorting the domain in terms of cardinality we mean quite simply that individuals (pluralities and singularities) are distinguished by the number of members they have. Such a classification of the individuals in the domain does appear to be reflected in the grammar. This criterion can be used to describe the morphology of number. It is fairly accurate to say that morphologically plural predicates are true of entities having two or more members while the singular is true of entities having a single member. Some languages have dual forms. These are true of entities having exactly two members. And in these languages, the plural is sometimes defined only for entities having three or more members. For example, in American Sign Language, a sentence whose main predicate is an "agreement verb" inflected with the "exhaustive" inflection is ungrammatical if the agreeing noun phrase refers to an individual having only one or two members.\(^{43}\) In short, there is some evidence from morphology that languages presuppose a domain of individuals sorted in terms of cardinality.

In addition to number restrictions imposed by inflection, often lexical items themselves semantically select their arguments in terms of cardinality. A spectrum of verb types, though limited, is perceptible. Thus whereas the verb *stand* can felicitously be applied to a term denoting individuals with one or more members, the predicate *meet* requires two or more people:

(365)  #John met.

and the predicate *gather* requires somewhat more than that:

(366)  ? John and Mary gathered in the park.

and finally *swarm*, can be true or false only of large groups. Except in jest, we cannot say, referring to three insects, that:

(367)  #Pangur and Little One and Chubaka were swarming around the living room when I walked in.

\(^{42}\) This section owes much to Dougherty (1970).

\(^{43}\) This information was provided me by Karen Petronio. The verbal inflection "exhaustive" is discussed in Klima E. and U. Bellugi, 1979, *The Signs of Language*. Harvard University Press, Cambridge MA.
Another example of this type is the German verb *strömen* [to pour/stream in] which Bartsch (1973:77) notes “is not applicable to small groups but rather to masses of individuals."

While it seems that verbs in the *gather* and *swarm* classes do not always have two-place counterparts, the verbs in the *met* class all seem to, for example:

(368)  
\[\begin{array}{ll}
\text{a.} & \#\text{John met.} \\
\text{b.} & \text{John and Mary met.} \\
\text{c.} & \text{John met Mary.}
\end{array}\]

(369)  
\[\begin{array}{ll}
\text{a.} & \#\text{Line A intersects.} \\
\text{b.} & \text{Line A intersects line B.} \\
\text{c.} & \text{Lines A and B intersect.}
\end{array}\]

These verbs, in their intransitive incarnations, are sometimes described as covert reciprocals. I do not know why these should be the only members of the *met* class, if in fact that is the case.

This phenomenon is not limited to verbs. Adverbs and adjectives, for example, impose similar restrictions. Verb phrases formed by combining a verb with *together* are of the *met* type while those containing *en masse* are of the *swarm* type. The adjective *parallel* is of the *met* type while *unanimous* seems to require some larger group:

(370)  
\[\begin{array}{ll}
\# & \text{John and Bill were unanimous in their disapproval.} \\
\# & \text{My parents were unanimous in their disapproval.}
\end{array}\]

Although a description of this spectrum of predicate types in terms of cardinality seems right, it is not clear exactly how it should be characterized formally. Taking the semantic value of a 1-place predicate to be the characteristic function of a set, one might think to incorporate in the grammar statements such as:

(371)  
\[X \text{ is in the domain of the function } \parallel \text{swarm} \parallel \text{ iff the cardinality of } X \text{ is large.}\]

The problem with such a characterization is that many of these predicates combine nicely with singular collectives and bare-plurals, which on some accounts denote singularities. We will not attempt a formalization here. Another question which must remain open is the following. In the morphological domain, we had inflections, singular and dual, which place a maximum on the size of the individuals in the domain of predicates with that inflection, whereas the plural and exhaustive inflections enforce a
minimum but no maximum. On the other hand, in the case of selectional restrictions there were no examples of predicates that apply exclusively to groups below a certain size. What then is the status of this asymmetry between maxima and minima?

Our focus so far has been presuppositions introduced by certain predicates concerning the cardinality of their arguments. However, the actual meanings of predicates include reference to cardinality as well. Most obvious examples are the numerals themselves as well as verbs such as outnumber and other comparative constructions.

Finally, we note that there is a limited sorting of variables in terms of cardinality. In English this occurs in the difference between bound plural and singular pronouns. If both is functioning as a bound pronoun in the next example then we require a more fine grained sorting: one, two, two or more.

(372) Two people who truly like each other are happy only if both are successful.

Probably a better place to look for this type of thing would be in languages, like American Sign Language, which actually have dual (and trial) pronouns.

This completes our survey of cardinality related phenomena. The data here is meant mainly to be suggestive of how a logical sorting of the domain might be reflected in the grammar. Reference by the grammar to the cardinality of individuals is more or less equally expected on either of our two theories, and so we have not delved deeply into this question.

8.3 Set-theoretic Hierarchy

In contrast with cardinality, reference in the grammar to a hierarchical sorting of individuals is to be expected on the sets theory alone. Given the domain \( D^* \) as defined for the union theory, a type-theoretic sorting would at most distinguish singularities from pluralities. It could not be used to distinguish pluralities. Thus the one purpose a hierarchical classification of individuals could serve, to distinguish singularities and pluralities, is achieved with reference to cardinality, which is probably needed anyway. On the other hand, position in a set-theoretic hierarchy would provide a meaningful criterion for the sorting of \( D^* \) as defined in the sets theory. To clarify this claim further, we will settle on a specific hierarchy that would make sense on the sets approach. To do this we repeat the definition for \( D^* \):
(373) Sets theory:
\[ D_0 = D \]
\[ D_{n+1} = D_n \cup \text{POW}_{\geq 2}(D_n) \]
\[ D^* = \bigcup_{n \leq \omega} D_n \]

There are a number of different hierarchies that could be specified. One obvious possibility is a hierarchy in which level \( n \) corresponds to \( D_n \) in (373). One property of this hierarchy is that the same individuals can occur at various levels. Thus for example elements of \( D \) occur at every level. While this may be useful for some purposes, it seems like it would be difficult to find linguistic evidence for such a hierarchy.

Another possibility would be a hierarchy in which each level \( n+1 \) individual is composed entirely of individuals from level \( n \), for \( 0 \leq n \leq \omega \). This is a natural candidate for a hierarchy; however, it has a serious defect. "Mixed individuals" are left out. To see this, imagine that \( a \in D \) and \( \{c,b\} \in D^* \) and that they are from different levels. If this last requirement is not met then we do not even distinguish pluralities and singularities. The problem now is that we have no level at which \( \{a,\{c,b\}\} \) occurs since it is not composed solely of individuals from any one level.

If the grammar were indeed sensitive to some hierarchy then, it would have to be one that includes nonoverlapping levels, but levels that together included all elements of the domain. Viewing the construction in (373) dynamically, what we would like is a hierarchy in which at level \( n \) of the hierarchy, we find all and only the individuals that are newly created in the formation of \( D_n \). In (373), \( D_{n+1} \), defined as \( (D_n \cup \text{POW}_{\geq 2}(D_n)) \), includes all the new individuals created at \( D_{n+1} \) along with old ones. What about \( \text{POW}_{\geq 2}(D_n) \)? This also contains all the new individuals created at \( D_{n+1} \), but it still includes some individuals from \( D_n \). As the following shows this:

(374) \[
\begin{align*}
\text{a. } D_n &= D_{n+1} \cup \text{POW}_{\geq 2}(D_n) \\
\text{b. } \text{POW}_{\geq 2}(D_n) &= \text{POW}_{\geq 2}(D_{n+1} \cup \text{POW}_{\geq 2}(D_n)) \\
\text{c. For any } A, B, \text{POW}_{\geq 2}(A) &\subseteq \text{POW}_{\geq 2}(A \cup B) \\
\text{d. } \text{POW}_{\geq 2}(D_{n+1}) &\subseteq \text{POW}_{\geq 2}(D_n) \\
\text{e. } D_n \text{ and } \text{POW}_{\geq 2}(D_n) &\text{ overlap.}
\end{align*}
\]

What we need then is to remove the elements of \( D_n \), from \( \text{POW}_{\geq 2}(D_n) \).

This is the idea behind the following hierarchy (for any \( n \), \( D_n \) is given by

(375)
(375) Hierarchy of elements of $D^*$ for the sets-theory.

Level $0 = D$

Level $1 = \text{POW}_{\geq 2}(D)$

For any $n \geq 2$, Level $n = \text{POW}_{\geq 2}(D_{n-1}) - \text{POW}_{\geq 2}(D_{n-2})$.

The appeal of this hierarchy is its correspondence to the syntax and morphology. Noun phrases in which pluralization is nested to depth $n$ denote level $n$ entities, where by "pluralization" we mean an instance of plural marking or term conjunction. Thus, for example, John involves no pluralization and it denotes a level 0 individual. The boys contains one pluralization and it denotes level 1 entities. Finally, the boys and the girls and John and the girls both contain a pluralization within a pluralization and they both denote level 2 entities.

Assuming now that the sets theory is correct, and therefore that the hierarchical sorting of the domain given in (375) is available to the grammar, we set about to see if in fact the grammar makes any reference to it. The kind of thing we are looking for is a predicate that can be felicitously applied only to certain noun phrases depending on their level and therefore on the depth of embedding of pluralization in the noun phrase. In fact, as far as I could tell, there is no evidence either from morphology or from semantic selection of the essential use of a hierarchical sorting of the individuals in the domain.\footnote{Link(to appear:20) makes a similar point.}

J. Hoeksema suggested to me in public that the predicate be equally numerous may not fit this characterization, that is, it may apply only to (some) noun phrases denoting at level 2 and higher. However, I think the following piece of discourse is well-formed:

\begin{itemize}
  \item[i.] After the earthquake the community was divided into two independent groups, one in the west and the other in the east. Each developed its own culinary style and music tradition. A recent study of population statistics revealed a strange development. After about 100 years of separation, the women from the western community far outnumbered the women from the eastern community, whereas the men remained just about equally numerous.
\end{itemize}

In (i) the predicate remain equally numerous applies to the men which
English, there do not seem to be predicates whose meaning makes essential reference to our hierarchy. Nor is there any evidence that the language makes essential use of a range of variable types in quantifying over pluralities. Thus the variety characteristic of the sets-domain seems largely to be ignored by the grammar.

8.4 Conclusion

The motivation behind the discussion here is as follows. Our theories differ in the variety of entities found in $D$*. If entities are characterized hierarchically, then the union theory assumes there to be at most two kinds of entities, singularities and pluralities, whereas the sets theory assumes there to be an infinite range of entities. If there was a link between the variety manifested in the domain of individuals and grammatical phenomena, then this difference could be exploited to compare the theories. A look at reference to cardinality in the grammar suggested a plausible link. Our conclusion was that there doesn’t seem to be similar reference to a set-theoretic hierarchy in the grammar. This in turn is taken as a challenge to the sets approach. Why is it that the semantics requires distinctions among entities concerning which the grammar is silent? One might even try to phrase this question in terms of learnability.

Of course this is not a decisive argument against the sets approach. That approach is not committed to the specific sort of hierarchy that I have considered here. Furthermore, just because the individuals in the domain can be sorted in certain way, doesn’t mean that the grammar will necessarily make reference to that criterion of sorting.

denotes a level 1 type entity.