External Financing and Customer Capital: A Financial Theory of Markups

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Abstract

We propose a dynamic structural corporate model in which firms face imperfect capital markets and frictional product markets. We highlight the importance of the endogeneity of firms’ short-term cash flows and the endogeneity of the marginal value of liquidity in determining the interactions between investment, financing and product price setting decisions. Our primary goal is to develop a financial theory of markups to advance the understanding of two related questions in Macro Finance. One is how financial frictions affect firms’ markups, and the other is how nominal frictions impact managers’ financial decisions and firms’ values. The model implies several testable predictions: (1) financially constrained firms are more inclined to increase their desired markups of products; (2) firms facing larger price stickiness tend to issue less external equity and conduct less big payouts; (3) a large part of the cost from price stickiness is induced by financial frictions; and (4) the impact of price stickiness on firms’ investment ratios is ambiguous, due to the countervailing forces of precautionary cash holdings and the change in the cost of capital.

Key words: markups, financial frictions, customer markets, price stickiness, cash holdings

JEL codes: E31, E32, G1, G3, L21.

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1 Introduction

We provide a unified theoretical framework based on financial frictions and imperfect product markets to rationalize the impact of financial slack on markups and offer a set of joint predictions on firms’ financing, investment and product price setting behaviors. Our financial theory of markups is mainly motivated by the following observation. During the 2007 – 2009 Great Recession, especially at the height of the financial crisis of 2008, there was a lack of deflationary pressure on product prices while investment and output suffered large declines in the United States. This phenomenon is believed to be deeply linked to the controversial debate on the cyclicality of markup dynamics. Despite extensive debates on various factors causing markups to vary either procyclically or countercyclically, there are few formal theoretical models that analyze the effect of firms’ financial slack on markup dynamics. We construct an analytically tractable dynamic investment model integrating customer markets with financial constraints to explicitly link markups to firms’ financial slack.

In our model, the manager is knowledgeable about choosing product prices endogenously in an imperfect product market to balance the tradeoff between current profits and future customer base. The imperfect product market is a major distinction from standard corporate structural models with financial frictions (e.g. Leland, 1994). In fact, it has been argued that the product price choice is indeed a relevant factor in tilting a firm’s cash flows in both the short term and the long term. To set up the frictional environment of product price setting for the manager, on top of the short-term demand effect of product prices (i.e. the intra-temporal demand effect) emphasized in most macroeconomic models, we incorporate the inter-temporal demand effect by introducing

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1 One recent relevant empirical study is Gilchrist et al. (2014) which exploits firm-level data, constructed by merging the U.S. Producer Price Index (PPI) and Compustat datasets, and finds that firms with weak balance sheets increased their product prices significantly relative to industry averages during the height of the crisis in late 2008.

2 In a recent review article on advances in macroeconomics, Blanchard (2009) emphasizes “How markups move, in response to what, and why, is almost terra incognita for macro. A number of theories exist. One of the most plausible may be that of consumer markets […], in which firms think of the stock of consumers as an asset and choose prices accordingly.” The most convincing evidence for heterogeneous desired markups comes from the pass-through effects of exchange rate movements. Gopinath, Itskhoki and Rigobon (2010) find that exchange rate movements have large and heterogeneous effects on the markups charged by importers.

3 A few exceptions include Chevalier and Scharfstein (1996) and Gilchrist et al. (2014). However, the existing theories are not rich enough, and the endogenous investment and endogenous cash hoarding are not allowed.

4 There are numerous examples of imperfect product markets which arise from customers consumption inertia or imperfect information. Chintagunta, Kyriazidou and Perktold (2001) reveal that several yogurt brands exist large habit effects. Bhattacharya and Vogt (2003) provide both theoretical justification and empirical evidence that the drug industry has consumption inertia. Prices of a new drug are kept low and advertising levels are high early in the life cycle in order to build public knowledge about the drug. As knowledge grows, prices rise and advertising falls. Collado and Browning (2007) present evidence that food outside home, alcohol and tobacco are associated with consumption inertia.

5 For example, Okun (1981) recognized the effect of a manager’s strategic choice of firms’ product prices on cash flows, by stating “Managers prefer to report to shareholders a record of profit that is less volatile over the cycle, so that in bad times a firm will disinvest in customer base and raise price, while in good times (when profits are high anyway) the firm cuts price to invest in customer base and so shift profits into the future.”
the frictional customer market that is originated and formulated in the seminal work of Phelps and Winter (1970). In order to build or keep its customer base, the firm sacrifices its average current profits by strategically reducing its product prices, because a lower product price is more likely to retain existing customers and attract new ones, which could increase its long-term average profits. By contrast, by increasing its product prices, the firm can increase its current profits at the expense of losing customer base, which puts future profits and growth at risk. As emphasized by Rotemberg and Woodford (1993), Gilchrist et al. (2014) and Gourio and Rudanko (2014), the long-term nature of customer base and the upfront-paid costs for customer acquisition render customer base a form of intangible assets held by the firm. Hence, the manager’s product price setting decisions are in part investment decisions.\(^6\) Based on this key intuition, we show that product market frictions have nontrivial implications for the firm’s dynamics, and in turn, the firm’s price setting behavior is also affected by its financial slack. More precisely, on the one hand, due to the complementarity and the imperfect substitutability between tangible and intangible assets, the firm’s investment decisions involving both types of assets are tightly coordinated. On the other hand, since the seminal work of Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999), there have been extensive studies on how liquidity constraints compromise corporate or intermediary tangible asset investment in a macroeconomic framework. At the same time, quantitative structural corporate models have also been developed to understand the relationship between liquidity constraints and corporate investment decisions. The basic intuition in this literature is that the marginal value of liquidity introduces a substantial wedge between the direct marginal cost of investment and the effective marginal cost of investment when the marginal financing cost is high. This intuition is shown by Bolton, Chen and Wang (2011) in a “unified q theory”. We emphasize that the price setting behavior, as a form of investment in intangible assets, is also determined by a q theory with marginal financing costs incorporated. In this sense, our work extends the “unified q theory” to intangible asset investment.

Our model provides a range of novel testable empirical predictions. The first important result concerns how a firm’s financial slack affects its product price setting decisions. When setting its product price, the firm in a customer market always balances the tradeoff between current operating revenue and future growth in customer base. A lower product price generates a lower operating revenue but enables the firm to attract customers from its competitors, which increases its profits in the future. By contrast, a higher product price raises the firm’s current cash flows at the expense of

\(^6\)In fact, the strategy of building up customer base through a lower price is prevalent in the automobile and airline industries. Any airline that recovers from a problem announces discounts to bring back fliers. We have seen Air India and Jet Airways doing it in the past post the pilots’ strike. More recently, in February 2015, an India airline company, SpiceJet, announced the launch of a new low fare offer as an attempt to win back the trust of the passengers after recovering from financial distress. In March 2010, Toyota fired the first volley when it announced a variety of discount financing and special lease deals to try to regain market share in the U.S. after a series of recalls. To penetrate in the electric car market, in 2014, Nissan boasts the lowest-price electric car in the U.S., after dropping the base price of a Nissan Leaf by $6,400 earlier this year.
of losing its customer base. When the firm’s financial condition is weak, the marginal value of cash is high, which motivates it to set a higher product price in order to mitigate liquidity problems. On the other hand, with abundant internal funds, the firm will stick to a relatively low price in order to build up its customer base. This mechanism is reminiscent of the empirical findings in Chevalier and Scharfstein (1996), which documents that during regional and macroeconomic recessions, more financially constrained supermarket chains raise their prices relative to less financially constrained chains. In addition, a recent paper by Gilchrist et al. (2014) uses confidential product price data and finds that during the “Great Recession” in the United States, firms with “weak” balance sheets increased their prices relative to industry averages, while firms with “strong” balance sheets chose lower product prices relative to industry prices. In sum, our model demonstrates that time-varying financial slack can generate a strong incentive for firms to manipulate their product prices.

A second key result concerns the impact of price stickiness\(^7\) on the firm’s cash holdings, investment, financial decisions, and the firm’s value. Like the price setting behavior in standard New Keynesian models (see, e.g. Galí, 2008), nominal stickiness prevents the firm from setting its product price to the desired markup immediately, and the firm resets to the desired markup once it receives a Calvo price resetting opportunity.\(^8\) Our model predicts that the firm facing larger price stickiness is more precautionary in its financial decisions. In particular, it tends to delay the payment of dividends or equity repurchases and issue less equity, resulting in more cash holdings on its balance sheet. Stickier prices increase the marginal value of cash, as the option of raising product prices to boost up current cash revenue becomes more costly. This implies that the firm will have the incentive to hold more cash, delaying the payment of dividends in order to cushion against negative demand shocks. However, although the marginal value of cash is high for the firm facing larger price stickiness, this does not imply that the firm will issue more equity when it is running out of cash. On the contrary, our model predicts that in most cases, the firm has already set a high product price by the time of pursuing external financing. When the product price is stickier, the firm anticipates that it is less likely/more costly to lower its price in the near future when it is out of liquidity problems. Therefore, it issues less equity since the demand for cash (mainly from investment) is low when the product price is high due to a decreasing customer base.

We provide empirical evidence that is consistent with these predictions by exploiting 18 industries within the manufacturing sector. We show that the industries that change prices less frequently\(^9\)

\(^7\)Price stickiness can arise endogenously from the “menu cost” in our model. In fact, we show that, in our model with external financing constraints and endogenous cash holdings, only a reasonably small menu cost is needed to produce the average product price change frequency and the average corporate investment rate observed in the data. This is due to the “amplification effect” of financial constraints on the impact of menu costs.

\(^8\)Under the menu cost framework, it is equivalent to the case that once the gain from resetting the product price to the desired markup is equal to the menu cost incurred as a result of price adjustment.

\(^9\)As shown in Nakamura and Steinsson (2008), product price stickiness is a persistent industry characteristic. If an industry has a high price stickiness index, it means that the firms within that industry, on average, face larger price stickiness.
issue less equity and conduct less repurchases.

Our model’s predictions about the impact of price stickiness on the firm’s value is, at first glance, counter-intuitive. We show that the firm facing a stickier price has a larger value in steady state because it endogenously chooses to hold more cash on its balance sheet. However, this does not imply that price stickiness is good in terms of boosting the firm’s value. In fact, holding too much cash is costly in our model. Particularly, our model shows that the firm’s enterprise value, which is more relevant for operating efficiency and growth (see, e.g. Wernerfelt and Montgomery, 1988; Lang and Litzenberger, 1989; Chen and Lee, 1995; Bharadwaj, Bharadwaj and Konsynski, 1999), is lower when its product price is stickier. This readily implies that the cost of price stickiness comes from three major channels: (1) compromised operating efficiency, as the firm may not be able to adjust its price to the optimal level in a timely way; (2) the cash holding cost, as the return on the firm’s cash holdings is lower than the return that outside investors can obtain; and (3) the direct cost of price adjustment, which exists in the models where price stickiness endogenously arises from menu costs (see Section 4).

Moreover, our theory advances the understanding of the impact of price stickiness on investment. The traditional view is that the firm with a stickier price invests less due to a higher cost of capital (see, e.g. Weber, 2014). Our model suggests that the endogeneity of cash holdings is missing in this argument. In our model, the firm facing larger price stickiness indeed faces a higher cost of capital, but because of this, it has a strong incentive to hold more cash on its balance sheet. This, on the one hand, reduces the cost of capital. On the other hand, it boosts the firm’s value and increases the return on investment, motivating the firm to invest more. The force of the endogenous cash holdings channel works oppositely to the cost of capital channel, generating an ambiguous impact of price stickiness on the investment rate.10

A third new result is that our model implies that the cost of price stickiness is higher when external financing costs are larger. We call it the “amplification effect” of financial frictions on the cost of price stickiness. In fact, Ball and Romer (1990) study the interaction between real price rigidity based on customer markets and nominal price stickiness. They show that significant nominal rigidities can be explained by a combination of real rigidities and small frictions in nominal adjustment, an amplification mechanism via the real rigidity channel. Our model stresses an additional financial channel. Intuitively, the firm loses more value when it is facing more frictions in external financing because a cash-constrained firm tends to rely more on raising the price to boost cash revenue when facing higher costs of external financing. Consider an extreme case when external financing costs are zero, the firm would always prefer to use external financing to replenish cash and the product price will never be distorted. In this case, the cost of price stickiness is zero.

10It should be noted that the impact of price stickiness on the firm’s enterprise value is exactly consistent with the interesting empirical findings of Weber (2014) because hoarding cash is costly for the shareholder in our model (i.e. the deadweight cost of holding cash).
in our model since the firm would never have the incentive to reset its price. If external financing
costs are infinite, the firm would have to raise its price when running out of cash. Since raising
prices is more costly for firms with stickier prices, the decrease in the enterprise value is larger.

The remainder of the paper is organized as follows. We review the connections of our results
with existing literatures and highlight our contributions in the next section. Section 3 sets out
the model and two irrelevance benchmarks to illustrate the importance of customer markets and
financial frictions. Section 3.3 discusses the calibration and quantitative results. Section 4 provides
an alternative model with menu costs to endogenize sticky prices and quantify the size of menu
costs in generating the observed price stickiness under financial frictions. In this section, we also
emphasize that the main results of the paper do not depend on the way we model nominal rigidity.
Finally, Section 5 concludes and provides a brief discussion on the robustness of our main results in
a general equilibrium model.

2 Related Literature

Our theory provides new insight into the long-lasting debate on the cyclicality of markups. In our
model, financial distress serves as a robust force which drives firms to increase their product prices.
Firms go into financial distress especially during recessions, when negative aggregate demand shocks
or higher external financing costs are prevalent across the whole economy. Therefore, our model
generates countercyclical markups through the channel of endogenous marginal value of liquidity.11
This contributes to the theoretical work that examines markup fluctuations over business cycles.
Green and Porter (1984) and Haltiwanger and Harrington (1991), for example, predict that markups
are procyclical with respect to demand shocks using game-theoretic models. But countercyclical
markups are predicted by many more papers, featuring either implicit collusion (see, e.g. Rotemberg
and Saloner, 1986; Rotemberg and Woodford, 1992; Athey, Bagwell and Sanchirico, 2004; Koszegi
and Heidhues, 2008) or customer markets (see, e.g. Phelps and Winter, 1970; Bils, 1989; Gottfries,
1991; Klemperer, 1995). Those models offer a lot of important insight, but they all presume that
firms are operating in a perfect capital market. A few exceptions are Chevalier and Scharfstein
(1996) and Gilchrist et al. (2014). However, they study stylized financial constraints which ignore
the endogeneity of firms’ retained earnings. By relaxing this assumption, the scope for testing the
theory of the cyclicality of markups is widened, as our model offers testable predictions concerning
how markups vary with firms’ financial slack.

Concerning the cyclicality of markups, empirical evidence so far are still mixed due to the lack
of good measures of price-cost margins. For example, Domowitz, Hubbard and Petersen (1986) find
that price-average variable cost markups are more procyclical in highly concentrated industries.

11In Appendix B.1, we present numerical experiments for countercyclical markups delivered by our model.
Machin and Van Reenen (1993) find margins to be strongly procyclical using British firm-level data. Chirinko and Fazzari (1994) and Ghosal (2000) also find markups to be procyclical. More recently, Hall (2012) finds that advertising is procyclical, which implies that profit margins should also be procyclical according to standard advertising theories. Braun and Raddatz (2012) find that markups are more procyclical in environments with tighter financial constraints using a sample of manufacturing industries in 59 countries. Nekarda and Ramey (2013) present evidence on the cyclicity of markups conditional on various types of shocks. They find that markups are procyclical conditional on technology shocks. However, they are either procyclical or acyclical conditional on demand shocks. In contrast to these studies, Bils (1987) finds markups to be countercyclical in most U.S. two-digit industries using the marginal wage cost as a proxy for marginal cost. Murphy, Shleifer and Vishny (1989) show that output prices move countercyclically relative to input prices in many industries. Rotemberg and Saloner (1986) and Rotemberg and Woodford (1991) provide evidence of countercyclical markups at the two-digit level. Chevalier and Scharfstein (1996) provide evidence from the supermarket industry that supports the countercyclical markups. More recently, Galí, Gertler and López-Salido (2007) show that the gap between the marginal product of labor and the household’s consumption leisure relates to the reciprocal of the markup of price over social marginal cost. The resulted markup measure is highly countercyclical and accounts for the efficiency costs of business fluctuations. Gilchrist et al. (2014) also find countercyclical markups using more detailed product-level price data underlying the PPI index. Mazumder (2014) derives a generalized markup index by relaxing the assumption that labor can be costlessly adjusted at a fixed wage rate. His estimation uses U.S. manufacturing data, and finds that markups are highly countercyclical and decreasing in trend since the 1960s. Our major contribution to this literature is that we link the cyclicity of markups to the marginal value of liquidity by introducing external financing frictions in a customer market model. This improves the existing work (e.g. Gottfries, 1991; Chevalier and Scharfstein, 1996; Gilchrist et al., 2014) as in our model cash holdings are endogenous and totally under the control of the firm’s manager.

A key feature of our model is that it incorporates customer markets into a structural corporate model with financial frictions. In contrast to the model presented here, standard New Keynesian models have constant desired markups, because there it is assumed that (1) the elasticity of short-term demand is constant as a result of no exit or entry; (2) customer flows are not allowed by abstracting out the customer market as firms’ intangible assets; and (3) the Modigliani-Miller theorem is valid. We would like to emphasize that both customer markets and financial frictions are important to obtain the results of this paper.\textsuperscript{12} If the firm is operating in a monopolistically

\textsuperscript{12}In other words, our model focuses on analyzing the implications of relaxing the assumption (2) on customer markets and the assumption (3) on financial frictions. In fact, in the macroeconomic literature, researchers have been extensively studying the markup dynamics based on relaxing assumption (1) on entry and exit (see, e.g. Jaimovich and Floetotto, 2008; Loecker and Warzynski, 2012; Bilbiie, Ghironi and Melitz, 2012). In particular, a recent work by Loualiche (2014) also uses time-varying entries and exits to generate time-varying elasticity of short-term demand.
competitive market, the product price is entirely determined by the cost of production, and there is no tradeoff between short-term revenue and long-term profits. Therefore, the firm would stick to some optimal markup (as dictated by any model with monopolistic competition) forever, irrespective of its financial slack. A more detailed discussion is in Section 3.2.1. On the other hand, if there is no financial friction, the firm would always rely on external financing to solve liquidity problems and price setting would never be influenced by its financial slack. In this case, the optimal markup is lower than the one implied by a monopolistic competition model without customer markets, but is still constant over time (a more detailed discussion is provided in Section 3.2.2). In sum, our model incorporates both and delivers new insight on the joint impact of product-market imperfections and financial frictions on firms’ price setting, investment, and financing decisions.

Moreover, in the standard corporate theory of investment and external financing/capital structure, the product market is typically assumed to generate exogenous stochastic cash flows, and firms’ financial decisions are usually independent of their decisions in the product market (see, e.g. Fischer, Heinkel and Zechner, 1989; Bolton and Scharfstein, 1990; Leland, 1994; DeMarzo et al., 2012; Bolton, Chen and Wang, 2011, 2013). A key contribution of our paper to the standard financial theory on corporate decisions is that our model features endogenous cash flows which are affected by the optimal choice of product prices. As a result, product market decisions are interlinked to investment and financial decisions, in an analytical and quantitative way.

At last, the customer base acts effectively as an intangible asset of firms. Thus, our paper is also related to the literature on the relevance of intangible assets, including Hall (2001) on the impact of intangible assets on the stock market, Atkeson and Kehoe (2005) on the rents from organizational assets over the firm’s life cycle, McGrattan and Prescott (2010a) on the large contribution of returns on intangible capital to U.S. foreign investment returns, McGrattan and Prescott (2010b) on the explanatory power of intangible assets in the 1990s boom, Eisfeldt and Papanikolaou (2013) on the impact of managers’ talent on firms’ riskiness, Ai, Croce and Li (2013) on the asset pricing implication of investment options, Belo, Lin and Vitorino (2014) on the relationship between brand capital and firms’ riskiness, and Gourio and Rudanko (2014) on the impact of product market frictions on firm dynamics. In the asset pricing literature, the growth option as a special form of intangible assets has been studied extensively to understand stock prices and return patterns (see, e.g. Hobijn and Jovanovic, 2001; Jovanovic, 2009; Gomes, Kogan and Zhang, 2003; Papanikolaou, 2011; Kogan and Papanikolaou, 2013, 2014; Ai and Kiku, 2013, among others.)

and hence time-varying markups. And, Loualiche (2014) analyzes the asset pricing implications of the time-varying markups.
3 Model

Our model combines the customer market in the literature of imperfectly competitive product markets (see, e.g. Phelps and Winter, 1970; Phelps, 1992), investment in neoclassical growth models (see, e.g. Hayashi, 1982; Gomes, Kogan and Zhang, 2003), and the imperfect capital market in structural corporate models (see, e.g. Bolton, Chen and Wang, 2011, 2013). In the following, we first describe the firm’s demand dynamics and investment behavior, and then we characterize the evolution of the firm’s customer base following traditional customer market models. Next, we introduce the firm’s external financing costs, cash holding costs and dynamics of cash holdings. Lastly, we formulate the firm’s optimization problem.

A. Demand and Investment

Consider a large and mature firm whose gross incremental operating revenue $dY_t$ during a small interval $[t, t + dt]$ is

$$dY_t = A_t dQ_t,$$  \hspace{1cm} (3.1)

where $A_t$ represents “effective firm size” and $dQ_t$ is the nominal incremental demand over the interval $[t, t + dt]$ per unit of effective size. Effective firm size $A_t$ can also be interpreted as the firm’s average sales. The nominal incremental demand $dQ_t$ is exogenous and following a diffusion process with drift term$^{13}$:

$$dQ_t = (p - c)\mu \left( \frac{p}{\bar{p}} \right) dt + \sigma dZ_t,$$  \hspace{1cm} (3.2)

where $Z_t$ is a standard Brownian motion under the risk neutral measure. We assume the shock $dZ_t$ to the incremental nominal demand $dQ_t$ to be exogenous. The variable $p$ is the product price charged by the firm, the term $c$ is the marginal cost of production, and $\bar{p}$ is the industry average price of the product. Note that neither the product price $p$ nor the marginal cost $c$ loads on the nominal shock. Without loss of generality, we normalize the industry average price to one, i.e. $\bar{p} \equiv 1$. The average intratemporal demand (i.e. the demand curve faced by the firm in the short run) is characterized by

$$\mu \left( \frac{p}{\bar{p}} \right) = \mu_A \left( \frac{p}{\bar{p}} \right)^{-\eta}, \text{ with } \eta > 1.$$  \hspace{1cm} (3.3)

This functional form has been widely adopted in the models with monopolistic pricing such as standard New Keynesian models (see, e.g. Phelps and Winter, 1970; Galí, 2008). Basically, it means that setting a higher product price $p$ relative to the industry average lowers the average demand

$^{13}$The modeling of demand shocks follows the long history in the literature, such as Caballero (1991). More precisely, the modeling of cash flows $dQ_t$ is very similar to the models of dynamic investment under dynamic optimal incentive contracting (see, e.g. DeMarzo et al., 2012). In those models, managers can control the drift of cash flows by choosing effort level, and here by choosing the product price level. Moreover, those models assume that managers cannot control the volatility of the cash flow process.
Moreover, in the spirit of Phelps and Winter (1970), we assume that the firm’s nominal profits in the short-run \((p - c)\mu(p)\) are increasing, on average, in the product price \(p\). That is, we require \(p \leq p^* \equiv \frac{\eta}{\eta - 1}\bar{c}\), where \(p^*\) is the optimal static monopolistic product price charged by the firm and \(\frac{\eta}{\eta - 1}\bar{c}\) is the static monopolistic markup. This assumption is innocuous, as argued by Phelps and Winter (1970), which states that firms often charge lower markups relative to the static monopolistic one.

We assume that the firm’s effective size depends on two major factors including customer base \(m_t\) and capital \(K_t\), i.e.,

\[
A_t = a(m_t, K_t).
\] (3.4)

In particular, we assume the functional form of the aggregator \(A(\cdot, \cdot)\) to be Cobb-Douglas:

\[
a(m, K) \equiv m^\alpha K^{1 - \alpha},
\] (3.5)

where \(\alpha\) is the share of customer base in determining the sales of the firm, conditional on the nominal incremental demand \(dQ_t\). Our model is an extension of the traditional customer market model (see, e.g. Phelps and Winter, 1970; Rotemberg and Woodford, 1993), since we incorporate the firm’s capital as a factor influencing sales in addition to the firm’s customer base. The Cobb-Douglas aggregator is adopted mainly for tractability.

Capital accumulation follows the standard investment model with quadratic adjustment costs. In particular, we assume

\[
dK_t = (I_t - \delta K_t)dt, \quad \text{for } t \geq 0,
\] (3.6)

where \(I_t\) is the gross investment rate on \([t, t + dt]\) and \(\delta\) is the rate of capital depreciation.

With the gross investment adjustment cost, the firm’s incremental net profits after paying the investment cost (denoted by \(dN_t\)) over the time incremental \(dt\) is given by

\[
dN_t = A_t dQ_t - \Gamma(I_t, K_t, A_t)dt, \quad \text{for } t \geq 0,
\] (3.7)

where \(\Gamma(I, K, A)\) is the total adjustment cost of investment. We assume that the adjustment cost is homogeneous of degree one in \(I\) and \(K\), in line with the neoclassical investment literature (see, e.g. Hayashi, 1982). That is, we assume \(\Gamma(I, K, A) = g(i)A\), where \(i \equiv I/K\) is the firm’s investment capital ratio and \(g(i) = \frac{1}{\theta} (1 + \theta i)^\varsigma - \frac{1}{\theta}\) is an increasing and convex function. We use the standard investment adjustment cost function in the neoclassical investment literature (see, e.g. Papanikolaou,

\footnote{This effect can be rationalized using a search model where customers conduct sequential search for the cheapest product and the distribution of search costs is uniform across buyers (see, e.g. Carlson and McAfee, 1983).}
Particularly, we take $\varsigma = 2$, and the functional form of $g(i)$ simply becomes

$$g(i) \equiv i + \frac{\theta i^2}{2}, \quad (3.8)$$

where $\theta$ captures the degree of the adjustment cost.

B. The Customer Market and Sticky Price Setting

In addition to the short-term demand effect of product prices (i.e. the intra-temporal demand), we incorporate the inter-temporal demand based on the customer market. The key idea is that reducing the product price not only reduces current profits, as $p\mu(p)$ is increasing in $p$. At the same time, the lower product price is more likely to retain existing customers and attract new ones, hence increasing the firm’s future profits. Conversely, by increasing the product price, the firm can raise current profits at the cost of losing customer base, hence jeopardizing its future profits and growth.

Following the literature on the customer market (Phelps and Winter, 1970; Rotemberg and Woodford, 1993), we assume that customers gradually learn about prices elsewhere overtime and probably need to overcome some brand switching costs. Therefore, customers drift toward the cheapest sellers slowly. In particular, we postulate that the evolution of the firm’s customer base follows the “customer flow equation”:

$$dm_t = h\left(\frac{p_t}{\bar{p}}\right) m_t dt, \quad \text{with } h'(\cdot) < 0 \text{ and } h(1) = 1, \quad (3.9)$$

where $p_t$ is the product price charged by the firm and $\bar{p}$ is the industry average price. First, note that the customer flow function $h$ in (3.9) captures the rate at which customers drift from one firm to others when the firm’s product price differs from the industry average. Second, the slow-moving assumption in (3.9) captures information frictions faced by customers when searching for the cheapest price\textsuperscript{15} or brand switching costs, in line with the search models of product markets, including Gottfries (1986), Klemperer (1987), Farrell and Shapiro (1988), Beggs and Klemperer (1992) and Farrell and Klemperer (2007), among many others. Third, as in Phelps and Winter (1970), the change in customer base is proportional to existing customer base, which implies that a temporal change in relative product prices can bring a permanent effect on the firm’s customer base. Fourth, combining (3.2), (3.3) and (3.9), we see that the long-run elasticity of demand, which measures the percentage response of the eventual demand to a permanent increase in the product price, is larger than the short-run elasticity of demand $\eta$.

For simplicity, we adopt the following functional form to model the customer flow function $h$, which is also widely used by other customer market models (see, e.g. Rotemberg and Woodford,\textsuperscript{15}Phelps and Winter (1970) suggest that customers exchange information about the prices charged by different firms through random encounters.)
1993; Choudhary and Orszag, 2007):

\[
h \left( \frac{p}{\bar{p}} \right) \equiv \kappa - \kappa \left( \frac{p}{\bar{p}} \right)^\nu \quad \text{with } \kappa > 0, \nu > 0. \tag{3.10}
\]

The relative price \( p/\bar{p} \) determines the growth rate of customer base. Given the innocuous normalization \( \bar{p} \equiv 1 \), the marginal change in customer base is \( -\nu \kappa p^{\nu-1} \) when the product price varies. Thus, the quantity \( \nu \kappa \) measures how sensitive customers are to changes in the relative price, which can be interpreted as an inverse measure of information frictions or brand switching costs faced by customers.

Price setting follows the continuous-time version of the staggered price-setting model originally developed by Calvo (1983). We assume that the firm’s price resetting opportunities arrive randomly following a Poisson process with intensity \( \xi \).\(^{16}\) Intuitively, within any given period \([t, t + \Delta t]\), the firm can reset its product price with probability \( 1 - e^{-\xi \Delta t} \), independent of the time elapsed since the last adjustment. Thus, the average duration between two consecutive price resetting opportunities is \( \xi^{-1} \). Therefore, the intensity parameter \( \xi \) captures the price change frequency which constitutes a natural index of price stickiness. When price resetting opportunities arrive, the firm is free to reset its product price to either \( p_L \) or \( p_H \).\(^{17}\)

C. Cash Holdings, External Financing and Liquidation

The firm has access to an imperfect capital market. For simplicity, we assume that the firm uses outside equity as the only source of external funds for investment (see, e.g. Bolton, Chen and Wang, 2011, 2013). The cost of external financing is captured by a fixed cost and a variable cost which is proportional to the amount of issued equity. We assume that the fixed cost is given by \( \phi A \), where \( \phi \) is the fixed cost parameter. The fixed financing cost plays a crucial role in generating an option-exercising type of external financing decisions. This not only produces severe nonlinearity in investment and the marginal value of cash, but also strongly incentivises the firm to increase its product price when the firm is financially constrained. The fixed cost is proportional to effective firm

\(^{16}\)Ball and Romer (1990) jointly model the real price rigidity based on customer markets and nominal price stickiness based on small adjustment costs. They emphasize that substantial nominal rigidity can arise from a combination of real rigidities and small nominal frictions. We would like to point out that the main mechanism of the model that the firm increases its price when being financially constrained does not depend on nominal rigidity. Introducing nominal rigidity enables us to analyze its impact on the firm’s value, financing, and investment decisions, etc. We adopt the Calvo rule for technical convenience and to follow the convention in the New Keynesian literature. In Section 4, we propose an alternative model based on menu costs, which shows that our main results remain valid even if nominal rigidity is modeled in a different way.

\(^{17}\)We assume that the firm can only choose two different prices for simplicity and clarity. We are able to solve the model with continuous product prices. However, the problem becomes more complicated, as a PDE with free boundary conditions has to be solved. We find that the qualitative results are unchanged. Therefore, enabling the firm to choose among two prices is sufficient to convey the main idea of our theory and also helps clarify the model’s mechanism in a coherent way.
size since this ensures that the firm does not grow out of its fixed cost of issuing equity. Technically, the proportional fixed cost also helps to keep the model homogeneous. In addition to the fixed cost, the firm needs to pay a variable financing cost $\gamma A$, for each incremental dollar raised from the capital market.

The firm can also file for bankruptcy, resulting in a liquidation value $L = \ell A$, which is proportional to effective firm size. In the event of bankruptcy, shareholders obtain $L + W$, where $W$ is the amount of cash holdings when the firm files for bankruptcy.

The firm optimally chooses the timing and the amount of external equity financing. When the gain from external financing is smaller than the value of liquidation, the firm will file for bankruptcy when running out of cash. Otherwise, the firm will pursue external financing.

Combining the firm’s cash inflows from the incremental operating profits net the investment expenditures ($dN_t$ in (3.7)) with cash inflows from financing policies (given by the cumulative payout $U_t$ and the cumulative external financing $H_t$), the firm’s cash inventory $W_t$ evolves according to the following equation:

$$dW_t = dN_t + (r - \lambda)W_t dt + dH_t - dU_t,$$

(3.11)

where the term $(r - \lambda)W_t dt$ represents the interest income net of the cash carrying cost; the term $dH_t$ refers to the cash inflows from external financing; and the term $dU_t$ refers to the cash outflows to investors.

We define the cash ratio as $w_t \equiv W_t/A_t$. We show below that the cash ratio $w_t$ plays an important role as an endogenous state variable in characterizing the equilibrium. Using Ito’s lemma, the law of motion for $w_t$ within the internal financing region is

$$dw = -w [\alpha h(p) + (1 - \alpha) (i(w,p) - \delta)] dt$$

$$+ \left[ (p - \bar{c}) \mu(p) - i(w,p) - \frac{\theta}{2} i(w,p)^2 + (r - \lambda) w \right] dt + \sigma dZ_t.$$

(3.12)

It shows that the product price $p$ affects the cash ratio dynamics through three channels. First, a higher $p$ has a positive effect on the cash ratio through the “current profit channel”, because the term $(p - \bar{c}) \mu(p)$ is increasing in $p$ on the support $(0, p^*)$. Second, a higher $p$ has a positive effect on the cash ratio through the “long-run growth channel” by changing customer base, because the term $-w \alpha h(p)$ is increasing in $p$. More intuitively, a higher $p$ leads to a smaller growth rate in the firm’s effective size, hence making the cash ratio easier to catch up. Third, the product price $p$ affects the cash ratio through the “investment channel” as reflected by the term $-w [w(1 - \alpha) + 1] i(w,p) - \frac{\theta}{2} i(w,p)^2$. In fact, the significance of this channel depends on the impact of the product price on investment $i(w,p)$, which is determined both by the current profit channel and the long-run growth channel. As we show in Section 3.2, without external financing costs, the firm will always focus on the long-run
growth channel and investment will not be affected by the current profit channel. However, when there are external financing costs, the firm concerns more about the current profit channel if its financial slack is not sound.

The firm maximizes shareholders’ value, as below, by optimally choosing its investment \( I \), its product price \( p \), payout policy \( U \), external equity financing policy \( H \), and liquidation time \( \tau \):

\[
\mathbb{E}_0 \left[ \int_0^\tau e^{-rt} (dU_t - dH_t - dX_t) + e^{-r\tau} (\ell K_\tau + W_\tau) \right],
\]

where the expectation is taken under the risk-neutral measure. The term \( dU_t - dH_t - dX_t \) is the discounted value of net payouts to shareholders. The quantity \( X_t \) is the cumulative costs of external financing up to time \( t \), and \( dX_t \) is the incremental costs of raising incremental external equity \( dH_t \). The term \( \ell K_\tau + W_\tau \) is the liquidation value paid to shareholders at the time of bankruptcy \( \tau \).

### 3.1 Model Solution

Let \( U(A, W, p) \) be the value function of the firm. The firm needs to endogenously and simultaneously make three kinds of decisions, namely, investment decisions, financing/liquidation decisions, and price setting decisions. Since both financing/liquidation decisions and price setting decisions are discrete in our model, they can be sufficiently characterized by “decision boundaries”. Figure 1 elaborates this idea: Basically, the firm’s decision-making depends on which of the following four regions the firm finds itself lying in: (1) an external financing/liquidation region within which the firm pursues external financing (\( dH > 0 \)) or liquidation; (2) an internal financing region within which the firm chooses the high product price (\( p_H \)) once price resetting opportunities arrive; (3)
an internal financing region within which the firm chooses the low product price \( p_L \) once price resetting opportunities arrive; and (4) a payout region within which the firm chooses to payout dividends \( (dU > 0) \). More precisely, it is optimal for the firm to hoard up cash to finance future investment as a result of precautionary motives. When exogenous nominal demand shocks drive cash holdings \( W \) gradually to some low level \( W \) (i.e. the “external finance boundary”) such that the current financing costs and the discounted future financing costs are equal, the firm will decide to raise outside equity. The product price setting decision essentially depends on the tradeoff between long-run customer base buildup and short-run profits. When cash holdings \( W \) are lower than \( W_0^p \) (i.e. the “price setting boundary”), the marginal value of cash is large enough so that the marginal value of short-run profits dominates the marginal value of developing future customer base. Thus, the firm desires to raise the product price to increase current profits. Lastly, because holding cash is costly (captured by \( \lambda > 0 \)), the firm chooses to pay out cash when exogenous shocks drive cash holdings \( W \) beyond some high level \( W \) (i.e. the “payout boundary”).

### 3.1.1 Internal Financing Region

The equilibrium dynamics within the internal financing region can be described by the following Hamilton-Jacobi-Bellman (HJB) equation:

\[
\begin{align*}
    rU(A, W, p) &= \max_{I, p^* \in \{p_L, p_H\}} \left[ \alpha h(p) + (1 - \alpha)(I/K - \delta) \right] AU_A \\
    &+ \left[ (r - \lambda)W + A(p - \tau)\mu(p) - \Gamma(I, K, A) \right] U_W + \frac{1}{2}\sigma^2 A^2 U_{WW} \\
    &+ \xi \left[ U(A, W, p^*) - U(A, W, p) \right],
\end{align*}
\]

(3.14)

where \( A \) is the effective firm size defined in (3.4). The term \( U_A \) represents the marginal effect of increasing effective firm size on the firm’s value, while effective firm size is changing due to net investment \( (I/K - \delta) \) and the change in customer base \( (h(p)) \). The term \( U_W \) represents the effect of the firm’s expected savings and profits on the firm’s value, the term \( U_{WW} \) represents the effect of the volatility of cash holdings on the firm’s value, and the jump term represents the jump in the firm’s value caused by the change in the product price.

Price resetting decisions amount to comparing the value functions \( U(A, W, p_L) \) and \( U(A, W, p_H) \). The optimal price is chosen to be \( p_L \) if and only if \( U(A, W, p_L) \geq U(A, W, p_H) \). The optimal investment rate \( i = I/K \) is pinned down by the following first order condition:

\[
1 + \theta i = (1 - \alpha) \frac{U_A(A, W, p)}{U_W(A, W, p)}. 
\]

(3.15)

A key simplification in our setup is that the firm’s three-state optimization problem can be reduced to a two-state problem by exploiting homogeneity. We define the function \( u(w, p) \) on
\( \mathcal{D} = [0, +\infty) \times \{p_L, p_H\} \) such that

\[
U(A, W, p) \equiv Au(w, p), \quad \text{with } w = W/A.
\]

Therefore, by taking out the scaling factor \( A \), the HJB equation in (3.14) can be rewritten into a system of coupled ordinary differential equations:

\[
ru(w, p_L) = (u(w, p_L) - wu_w(w, p_L)) [\alpha h(p_L) + (1 - \alpha)(i(w, p_L) - \delta)] + \xi \max\{u(w, p_H) - u(w, p_L), 0\}
+ u_w(w, p_L) \left[ (p_L - \bar{c})\mu(p_L) - i(w, p_L) - \frac{\theta}{2} i(w, p_L)^2 + (r - \lambda)w \right] + u_{ww}(w, p_L) \frac{\sigma^2}{2},
\]

and

\[
rw(w, p_H) = (u(w, p_H) - wu_w(w, p_H)) [\alpha h(p_H) + (1 - \alpha)(i(w, p_H) - \delta)] + \xi \max\{u(w, p_L) - u(w, p_H), 0\}
+ u_w(w, p_H) \left[ (p_H - \bar{c})\mu(p_H) - i(w, p_H) - \frac{\theta}{2} i(w, p_H)^2 + (r - \lambda)w \right] + u_{ww}(w, p_H) \frac{\sigma^2}{2},
\]

where

\[
i(w, p) \equiv \left[ \frac{u(w, p)}{u_w(w, p)} - w \right] \frac{1 - \alpha}{\theta} - \frac{1}{\theta'}, \quad \text{for } p \in \{p_L, p_H\}.
\]

In fact, the coupled ordinary differential equations can be separated into two groups of coupled ordinary differential equations on two sub-regions. Denote the price resetting boundary as \( w_0^P \) such that

\[
u(w, p_L) < u(w, p_H) \quad \text{whenever } w > w_0^P, \quad \text{and } u(w, p_L) > u(w, p_H) \quad \text{whenever } w < w_0^P,
\]

and \( u(w, p_L) = u(w, p_H) \) if and only if \( w = w_0^P \).

When \( w \in [w_0^P, \bar{w}(p_L)] \), the ODE for \( u(w, p_L) \) is

\[
rw(w, p_L) = (u(w, p_L) - wu_w(w, p_L)) [\alpha h(p_L) + (1 - \alpha)(i(w, p_L) - \delta)]
+ u_w(w, p_L) \left[ (p_L - \bar{c})\mu(p_L) - i(w, p_L) - \frac{\theta}{2} i(w, p_L)^2 + (r - \lambda)w \right] + u_{ww}(w, p_L) \frac{\sigma^2}{2},
\]

and when \( w \in [w_0^P, \bar{w}(p_H)] \), the ODE for \( u(w, p_H) \) is

\[
rw(w, p_H) = (u(w, p_H) - wu_w(w, p_H)) [\alpha h(p_H) + (1 - \alpha)(i(w, p_H) - \delta)] + \xi (u(w, p_L) - u(w, p_H))
+ u_w(w, p_H) \left[ (p_H - \bar{c})\mu(p_H) - i(w, p_H) - \frac{\theta}{2} i(w, p_H)^2 + (r - \lambda)w \right] + u_{ww}(w, p_H) \frac{\sigma^2}{2},
\]
where
\[ i(w, p) = \left[ \frac{u(w, p)}{u_w(w, p)} - w \right] \frac{1 - \alpha}{\theta} - \frac{1}{\theta} \text{ for } p \in \{p_L, p_H\}. \tag{3.17} \]

When \( w \in [0, w_0^p] \), the ODEs are
\[
ru(w, p_L) = (u(w, p_L) - wu_w(w, p_L)) [\alpha h(p_L) + (1 - \alpha)(i(w, p_L) - \delta)] + \xi(u(w, p_H) - u(w, p_L))
+ u_w(w, p_L) \left[ (p_L - \bar{c})\mu(p_L) - i(w, p_L) - \frac{\theta}{2}i(w, p_L)^2 + (r - \lambda)w \right] + u_{ww}(w, p_L) \frac{\sigma^2}{2},
\]
and
\[
ru(w, p_H) = (u(w, p_H) - wu_w(w, p_H)) [\alpha h(p_H) + (1 - \alpha)(i(w, p_H) - \delta)]
+ u_w(w, p_H) \left[ (p_H - \bar{c})\mu(p_H) - i(w, p_H) - \frac{\theta}{2}i(w, p_H)^2 + (r - \lambda)w \right] + u_{ww}(w, p_H) \frac{\sigma^2}{2},
\]
where
\[ i(w, p) = \left[ \frac{u(w, p)}{u_w(w, p)} - w \right] \frac{1 - \alpha}{\theta} - \frac{1}{\theta} \text{ for } p \in \{p_L, p_H\}. \tag{3.18} \]

### 3.1.2 Payout Region

The characterization of the payout boundary is mainly based on the work of Dumas (1991). The firm starts to pay out cash when the marginal value of cash held by the firm is less than the marginal value of cash held by shareholders which is one. Thus, the value matching condition gives the following boundary condition:
\[ u_w(\overline{w}(p), p) = 1. \tag{3.19} \]

The payout region is characterized by \( w \geq \overline{w}(p) \) for each \( p \). Whenever the cash ratio is beyond the boundary, it is optimal for the firm to payout all the extra cash \( w - \overline{w}(p) \) in a lump-sum manner and return its cash holdings back to \( \overline{w}(p) \). Thus, the firm’s value in the payout region has the following form:
\[ u(w, p) = u(\overline{w}(p), p) + (w - \overline{w}(p)), \text{ when } w \geq \overline{w}(p). \tag{3.20} \]

Lump-sum payouts can occur mainly because payout boundaries are different for different product prices. In our quantitative analysis, we show that \( \overline{w}(p_L) > \overline{w}(p_H) \) under the parameter calibration of interest (see Section 3.3.1). Moreover, the first-order condition for maximizing the firm’s value over constant payout boundaries leads to the smooth pasting or the super contact condition
\[ u_{ww}(\overline{w}(p), p) = 0, \tag{3.21} \]
where optimization is achieved at \( \overline{w}(p) \).
3.1.3 External Financing/Liquidation Region

Although the firm can raise outside equity any time, it is optimal for the firm to raise equity only when it runs out of cash, which means the external financing boundary \( w(p) = 0 \). This is due to the following reasons. First, cash within the firm earns a lower interest rate \( r - \lambda \) due to the cash holding cost. Second, the firm’s investment is continuous. Third, financing costs have smaller present value when they are paid further in the future.

Once the firm hits the financing boundary and decides to raise external equity, the optimal financing amount is also endogenously determined. The value matching condition for the issuance amount \( w^*(p) \) is

\[
\begin{align*}
    u(0, p) &= u(w^*(p), p) - \phi - (1 + \gamma)w^*(p). \\
    \text{Eq. (3.22)}
\end{align*}
\]

The left-hand side of equation (3.22) is the firm’s value per unit of effective size right before the issuance. The right-hand side of equation (3.22) is the firm’s value per unit of effective size minus both the fixed and variable costs of equity issuance per unit of effective size. The first-order optimality condition for the issue amount leads to the smooth pasting condition

\[
    u_w(w^*(p), p) = 1 + \gamma. \\
    \text{Eq. (3.23)}
\]

Since \( w^*(p) \) is the optimal equity issuance, the marginal value of the last dollar raised by the firm must equal to one plus the marginal cost of external financing \( \gamma \).

Now, we characterize the liquidation boundary \( w^L \) and the decision of liquidation. It is easy to see that \( w^L = 0 \), since as long as \( w > 0 \), the firm can still invest. In this case, both the marginal value of effective size and the marginal value of cash within the firm are larger than one, thus the firm’s value is strictly larger than the value of liquidation. In the model, when the firm uses up all its cash, it needs to decide whether to issue equity or to file for bankruptcy. If it is optimal for the firm to choose filing bankruptcy instead of raising external equity at the boundary \( w^L = w(p) = 0 \), the liquidation value per unit of effective size gives

\[
    u(0, p) = \ell. \\\n    \text{Eq. (3.24)}
\]

3.2 Benchmark Cases with Constant Markups

We conduct theoretical experiments by setting two benchmark cases in order to illustrate the role of financial frictions in shaping the dynamics of markups desired by the firm.
3.2.1 The Optimal Static Monopolistic Price

Under the common setup adopted by the New Keynesian literature (see, e.g. Galí, 2008; Dou et al., 2014, for a review), the firm has no external financing costs and there are no customer flows among sellers. By mapping these assumptions onto our model, we have $\phi = \gamma = 0$ (i.e. no external financing costs), $\lambda = 0$ (i.e. no cash holding costs), and $\alpha = 0$ (i.e. no customer flows among sellers). Thus, on the financial side, the Modigliani and Miller (1958) Theorem holds, and on the product side, the price elasticity of demand only shows up in the short run. The intra-temporal profit optimization, as in traditional New Keynesian (DSGE) models, leads to the equilibrium where the firm chooses $p = p^* \equiv \frac{\eta}{\eta - 1}c$ once it gets the chance to reset its price and keeps this price forever. The desired markup is constant over time and purely determined by the intra-temporal elasticity of demand.\(^{18}\)

In other words, we have $w_0^r = \infty$. Since there are no external financing costs, the marginal value of cash held by the firm is one. Thus, it is reasonable to guess that the value function of the firm has the following form

$$u(w, p) \equiv u(p) + w.$$ \hspace{1cm} (3.25)

By plugging (3.25) into the coupled ODEs, we can get

$$u(p_H) = \theta(\delta + r) - \theta \sqrt{(\delta + r)^2 - 2[\mu_H - (r + \delta)]/\theta} + 1,$$ \hspace{1cm} (3.26)

$$u(p_L) = \theta(\delta + r + \xi) - \theta \sqrt{(\delta + r + \xi)^2 - 2[\mu_L + \xi u(p_H) - (r + \delta + \xi)]/\theta} + 1,$$ \hspace{1cm} (3.27)

where $\mu_H \equiv (p_H - c)\mu(p_H)$ and $\mu_L \equiv (p_L - c)\mu(p_L)$ are expected current profits for the firm with $p_H$ and $p_L$, respectively. For illustrative purposes, we assume that $p_H$ is the optimal static monopolistic product price, $p_H = p^*$. It implies that $\mu_H > \mu_L$. Therefore, the optimal investment is

$$i(w, p_H) = (r + \delta) - \sqrt{(r + \delta)^2 - 2[\mu_H - (r + \delta)]/\theta},$$ \hspace{1cm} and

$$i(w, p_L) = (r + \delta + \xi) - \sqrt{(r + \delta + \xi)^2 - 2[\mu_L + \xi u(p_H) - (r + \delta + \xi)]/\theta}.$$ 

In this case, the steady-state price is deterministic with $p \equiv p_H$. We highlight the following implications arising from this simple benchmark case. First, it is apparent that the firm’s investment decisions only focus on keeping investment on the optimal growth path, while its price setting decisions focus on maximizing expected current profits. This is called the “growth-profit separation effect”. Second, the desired markup is constant at $\eta / (1 - \eta)$. Third, price stickiness has no impact on

\(^{18}\)Note that in traditional New Keynesian DSGE models (see, e.g. Galí, 2008), although the desired markup is constant over time, the realized product price can fluctuate over time which is purely driven by the time-variation in marginal costs. In order to highlight the forces driving time-varying desired markups, we postulate a fixed marginal cost. This helps us to clearly illustrate the key mechanisms of the model.
the firm’s value or its decisions since the optimal price $p_H$ is constant overtime. Fourth, the efficiency cost of price stickiness is just a pass-through from whatever cost (e.g. menu costs) resulting a sticky product price, if the initial price is not $p_H$. That is, the value function is deteriorated exactly by the same amount of menu costs.

At last, in order to make the above equilibrium solution rigorous, we show, in Proposition 1, that it is indeed the case that $u(p_H) > u(p_L)$. The proof can be found in Appendix A.1.

**Proposition 1.** Suppose the parameters satisfy $(\delta + r)^2 - 2 [\mu_H - (r + \delta)] / \theta > 0$, then $u(p_L)$ in (3.27) is well defined and $u(p_H) > u(p_L)$.

### 3.2.2 The Optimal Inter-temporal Monopolistic Price in Customer Market Models

We incorporate the customer market (see, e.g. Phelps and Winter, 1970; Rotemberg and Woodford, 1993) into traditional New Keynesian models. The only difference from the previous irrelevance benchmark is that now the firm’s customer base affects its profits and customer flows are allowed among sellers, i.e. $\alpha > 0$ and $h'(p) < 0$. The Modigliani and Miller (1958) Theorem still holds, but in this case the price elasticity of demand shows up both in the short run and in the long run. As a result, price setting decisions not only affect the firm’s current profits but also influence the long-run growth rate in customer base, thus leading to an equilibrium price lower than the optimal static monopolistic price $p^* = \frac{\eta}{\eta - 1}e$. In equilibrium, the firm sets $p_L$ once it gets the chance to reset its price, and the firm keeps the product price at $p_L$ forever.\(^{19}\) In other words, we have $W_0^P = 0$. As in traditional New Keynesian models, the desired markup is constant over time, however, it is lower than $p^*$ as emphasized by the traditional customer market model (see, e.g. Phelps and Winter, 1970; Rotemberg and Woodford, 1993). Since there are no external financing costs, the marginal value of cash is one. Thus, it is reasonable to guess that the value function of the firm has the following form:

$$u(w, p) \equiv u(p) + w. \quad (3.28)$$

By plugging (3.25) into the coupled ODEs, we can get

$$u(p_L) = \frac{\theta(\delta(1 - \alpha) + r)}{(1 - \alpha)^2} + \frac{(1 - \alpha)}{(1 - \alpha)^2} - \theta \sqrt{\frac{[\delta(1 - \alpha) + r] - 2(1 - \alpha)[\mu_L - r - \delta(1 - \alpha)]}{(1 - \alpha)^2}},$$

\(^{19}\)We require $p_L < p^*$ but $p_L$ should not be very small so that, for example, the condition (3.31) holds.
and
\[
  u(p_H) = \frac{\theta [\delta(1 - \alpha) + r + \xi - \alpha h(p_H)] + (1 - \alpha)}{(1 - \alpha)^2} \\
  - \frac{\theta \sqrt{[\delta(1 - \alpha) + r + \xi - \alpha h(p_H) + (1 - \alpha)/\theta]^2 - (1 + 2\theta \mu_H + 2\theta \xi u(p_L))(1 - \alpha)^2/\theta^2}}{1 - \alpha}.
\]

Therefore, the optimal investment is
\[
i(w, p_L) = \frac{\delta(1 - \alpha) + r}{1 - \alpha} - \sqrt{[\delta(1 - \alpha) + r]^2 - 2(1 - \alpha)[\mu_L - r - \delta(1 - \alpha)]/\theta}, \quad \text{and}
\]
\[
i(w, p_H) = \frac{\delta(1 - \alpha) + r + \xi - \alpha h(p_H)}{(1 - \alpha)^2} \\
  - \sqrt{[\delta(1 - \alpha) + r + \xi - \alpha h(p_H) + (1 - \alpha)/\theta]^2 - (1 + 2\theta \mu_H + 2\theta \xi u(p_L))(1 - \alpha)^2/\theta^2}}{1 - \alpha}.
\]

If customer flows are fast enough so that the deterioration of customer base due to a high product price is relatively more significant than the gain from current profits, i.e.
\[
(1 - \alpha)^2(\mu_H - \mu_L) + \alpha h(p_H) [\theta \delta(1 - \alpha) + \theta r + (1 - \alpha)] \leq 0.
\]

In this case, the steady-state price is deterministic with \( p \equiv p_L \). We highlight four important aspects of this simple benchmark case with the customer market. First, the firm’s investment decisions only focus on the optimal growth path, but price setting decisions are affected by both short-run profits and long-run growth rates. When the firm weights more on long-run growth, it is optimal to choose the low price \( p_L \) in order to build up customer base at the cost of reducing current profits. Second, the desired markup is constant at \( p_L < p_H \equiv \eta(\frac{\mu_H}{\mu_L} \equiv \frac{\eta}{1 - \eta}) \), which is lower than the optimal static monopolistic markup \( \frac{\eta}{1 - \eta} \) due to the existence of customer flows. Third, price stickiness has no impact on the firm’s value or its decisions. Lastly, the efficiency cost of price stickiness is just a pass-through to the firm’s value from whatever cost (e.g. menu costs) resulting a sticky product price, if the initial price is not \( p_L \).

Similarly, in order to make the above equilibrium solution rigorous, we show, in Proposition 2, that it is indeed the case that \( u(p_H) > u(p_L) \). The proof can be found in Appendix A.2.

**Proposition 2.** Suppose the parameters satisfy the restriction (3.31) and \( [\delta(1 - \alpha) + r]^2 - 2(1 - \alpha)[\mu_L - r - \delta(1 - \alpha)]/\theta > 0 \), then \( u(p_H) \) in (3.29) is well defined and \( u(p_L) > u(p_H) \).
3.3 Quantitative Results

3.3.1 Parameter Choices and Calibration

In parameter choices and calibration, we discipline ourselves by choosing parameter values based on existing calibration and empirical evidence.

The liquidation parameter is set to be $l = 0.9$ following the estimates provided by Hennessy and Whited (2007). We choose the variable cost of financing to be $\gamma = 6\%$ based on the estimates reported by Altinkilic and Hansen (2000) and the fixed cost of financing is $\phi = 2\%$. The interest rate is taken to be $r_f = 3\%$, which is within the range of broad empirical evidence in the United States. The volatility of demand shocks is set to be $\sigma = 12\%$, which is consistent with the parameters estimated by Eberly, Rebelo and Vincent (2009). The rate of depreciation is set to be $\delta = 2\%$. The cash holding cost is assumed to be $\lambda = 0.9\%$. The adjustment cost parameter is $\theta = 1.5$ (Whited, 1992). We set the Calvo price intensity to $\xi = 2.8$ to generate a median price duration of 4.3 months as reported in Bils and Klenow (2004). We set $\eta = 1.5$ as used in Backus, Kehoe and Kydland (1994) and Zimmermann (1997). We set $c = 0.78$, $p_L = 0.95$, and $p_H = 2.34$, implying that $p_H$ is the price that maximizes cash revenue, namely, $p_H = \frac{\eta}{\eta - 1} \bar{c}$, and the industrial average price is about 1.21 There is no direct empirical evidence on the growth rate of customer base for different prices, we choose $\kappa = 0.73$ and $\nu = 1.3$ to reflect that the firm sets its price to $p_L$ when cash is abundant, in line with the main implication of the customer market literature (Phelps and Winter, 1970; Phelps, 1992).

In the end, we are left with two parameters, the expected nominal demand, $\mu_A$, and the capital share in effective firm size, $1 - \alpha$. We calibrate them to match the relevant moments for U.S. public mature large firms during the period of 1998-2012. We interpret effective firm size $A$ as total sales and capital $K$ as total assets. We set $\mu = 1.02$ and $\alpha = 0.145$ to match the mean cash-sales ratio (18.45%), and the mean investment-asset ratio (3.01%). Table 1 summarizes the symbols for the key variables of the model and the parameter values in the benchmark case.

3.3.2 Basic Mechanism: Financial Drivers of Markups

In this section, we elaborate on the rich interactions among cash holdings, product prices, investment, and financing decisions. Note that in our model, there are three channels that the firm can raise short-term cash inflows: increasing product prices, disinvesting, or external financing. However, there are costs associated with each channel either directly incurred or indirectly reflected as a loss

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20 It is the same as Bolton, Chen and Wang (2013) which interprets the cash holding cost as a result of tax disadvantage or agency frictions. Under the simple tax disadvantage interpretation, compared to borrowing the fund, holding cash as retained earnings bears an additional cost 0.9% because the marginal tax rate is about 30% and the interest rate is 3%.

21 The simulation results indicate that about 3.6% of the time the firm is setting its price to $p_H$, implying that the industry average price is equal to 1.
Table 1: Summary of key variables and parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital stock</td>
<td>$K$</td>
<td>Risk-free rate</td>
<td>$r$</td>
<td>3%</td>
</tr>
<tr>
<td>Cash holding</td>
<td>$W$</td>
<td>Rate of depreciation</td>
<td>$\delta$</td>
<td>2%</td>
</tr>
<tr>
<td>Effective firm size</td>
<td>$A$</td>
<td>Mean nominal demand</td>
<td>$\mu_A$</td>
<td>1.02</td>
</tr>
<tr>
<td>Customer base</td>
<td>$m$</td>
<td>Volatility of demand shocks</td>
<td>$\sigma$</td>
<td>12%</td>
</tr>
<tr>
<td>Investment</td>
<td>$I$</td>
<td>Adjustment cost parameter</td>
<td>$\theta$</td>
<td>1.5</td>
</tr>
<tr>
<td>Cumulative nominal demand</td>
<td>$Q$</td>
<td>Share of customer base</td>
<td>$\alpha$</td>
<td>0.145</td>
</tr>
<tr>
<td>Cumulative gross operating revenue</td>
<td>$Y$</td>
<td>Marginal cost of production</td>
<td>$\zeta$</td>
<td>0.78</td>
</tr>
<tr>
<td>Cumulative external financing</td>
<td>$H$</td>
<td>Fixed financing cost</td>
<td>$\phi$</td>
<td>2%</td>
</tr>
<tr>
<td>Cumulative payout</td>
<td>$U$</td>
<td>Variable financing cost</td>
<td>$\gamma$</td>
<td>6%</td>
</tr>
<tr>
<td>Price resetting boundary</td>
<td>$W_0^p$</td>
<td>Demand elasticity</td>
<td>$\eta$</td>
<td>1.5</td>
</tr>
<tr>
<td>External financing boundary ($p_H$)</td>
<td>$W(p_H)$</td>
<td>Customer base growth parameter</td>
<td>$\kappa$</td>
<td>0.73</td>
</tr>
<tr>
<td>External financing boundary ($p_L$)</td>
<td>$W(p_L)$</td>
<td>Customer base growth parameter</td>
<td>$\nu$</td>
<td>1.3</td>
</tr>
<tr>
<td>Payout boundary ($p_H$)</td>
<td>$W^*(p_H)$</td>
<td>Proportional cash-carrying cost</td>
<td>$\lambda$</td>
<td>0.9%</td>
</tr>
<tr>
<td>Payout boundary ($p_L$)</td>
<td>$W^*(p_L)$</td>
<td>Liquidation parameter</td>
<td>$l$</td>
<td>0.9</td>
</tr>
<tr>
<td>Optimal financing amount ($p_H$)</td>
<td>$W^*(p_H)$</td>
<td>Calvo parameter</td>
<td>$\xi$</td>
<td>2.8</td>
</tr>
<tr>
<td>Optimal financing amount ($p_L$)</td>
<td>$W^*(p_L)$</td>
<td>High price</td>
<td>$p_H$</td>
<td>2.34</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Low price</td>
<td>$p_L$</td>
<td>0.95</td>
</tr>
</tbody>
</table>

in the firm’s future revenue. The strategy for a liquidity constrained firm is to choose an optimal combination of the three choices to avoid the possibility of liquidation in the short run, while at the same time taking into account long-run growth opportunities.

**Enterprise Value** The firm’s enterprise value is defined as the value of all the firm’s marketable claims minus cash, $U(A,W,P) - W$, which can be considered as the value of the firm’s total tangible and intangible capital stock. We normalize the enterprise value by effective firm size, and obtain $u(w,p) - w = \frac{U(A,W,P) - W}{A}$, where $w = W/A$ denotes the cash-size ratio. This normalized enterprise value $u(w,p) - w$ can be considered as a measure of the firm’s average $q$, thus reflecting operating efficiency and growth (see, e.g. Wernerfelt and Montgomery, 1988; Lang and Litzenberger, 1989; Chen and Lee, 1995; Bharadwaj, Bharadwaj and Konsynski, 1999).

Panel A of Figure 2 plots the normalized enterprise value as a function of the cash-size ratio for the two product prices, $p_L$ and $p_H$, respectively. The solid line represents the normalized enterprise value when the product price is set at $p_L$. It is concave and increasing in the region between zero and the payout boundary $\overline{w}_{p_L} = 0.26$ (the vertical dotted line), and becomes flat (with slope zero) beyond the payout boundary ($w \geq \overline{w}_{p_L}$). The dashed line represents the normalized enterprise value when the product price is set at $p_H$, which has a similar shape as the solid line but is associated with a lower payout boundary $\overline{w}_{p_H} = 0.22$ (the vertical dotted line).

The two curves capturing the normalized enterprise value intersect with each other at the price resetting boundary, $w_0^P = 0.095$ (the vertical solid line). For $w > w_0^P$, the normalized enterprise
value is higher if the product price is set at \( p_L \); while for \( w < w_0^P \), the normalized enterprise value is higher for \( p_H \). This implies that when price resetting opportunities arrive (with Poisson intensity \( \xi \)), the firm will set its price to \( p_H \) if the cash-size ratio is less than \( w_0^P \), and \( p_L \) if the cash-size ratio is larger than \( w_0^P \). The fact that the optimal product price varies with the cash-size ratio is generated by two forces underlying our model. The product price not only affects short-term operating revenue (the “current profit channel”, captured by \( dQ_t = (p_t - c)\mu(P_t)dt \)), but also determines the growth rate in customer base (the “growth channel”, captured by \( dm_t = h(P_t)m_tdt \)). Therefore, there exists a trade-off between \( p_L \) and \( p_H \). Setting the price to \( p_H \) enables the firm to increase its short-term operating revenue, but customer base will be gradually diminishing. By contrast, the firm builds up its customer base over time by setting the price to \( p_L \), at the cost of lowering short-term operating revenue. The current profit channel is more crucial when the firm is liquidity constrained (i.e. with a low cash-size ratio), as in this case the marginal value of cash is high. Thus the firm is inclined to set its price to \( p_H \) for \( w < w_0^P \), relying on the current profit channel to accumulate cash. When cash is abundant, the “growth channel” plays a dominating role in determining the firm’s product price setting, and \( p_L \) would be chosen to build up customer base. This mechanism is reminiscent of the empirical findings in Chevalier and Scharfstein (1996) and Gilchrist et al. (2014) that firms under weak/strong financial conditions tend to increase/decrease product prices relative to industry average prices during a recession.

As we have elaborated before, the firm issues equity only when its cash holdings hit the zero lower bound because of the proportional cash carrying cost and continuous investment flows (see Bolton, Chen and Wang, 2011, for more detailed explanations). At the financing boundaries (i.e. \( w_{p_L} = w_{p_H} = 0 \)), the firm’s normalized enterprise value is strictly higher than its liquidation value. Therefore, external financing is always preferred to liquidation under our model parameterization.

**Marginal Value of Cash** Panel B plots the marginal value of cash \( u_w(w, p) \) for the two product prices \( p_L \) and \( p_H \). When the cash-size ratio is beyond the payout boundary, the marginal value of cash is equal to one. The marginal value of cash is higher when the firm becomes more liquidity constrained due to the frictions in external financing. This induces the firm to hoard cash in order to reduce the likelihood of external financing, although holding cash itself is costly, as captured by \( \lambda > 0 \). The frictions in external financing effectively generate “risk aversion” for the firm, a point emphasized by Bolton, Chen and Wang (2011).

To economize on the fixed external financing cost (\( \phi = 2\% \)), the firm issues equity in lumps. Conditional on issuing equity and having paid the fixed cost, the amount of equity issued returns the cash-size ratio to the point where the marginal value of cash \( u_w(w, p) \) is equal to the marginal cost \( 1 + \gamma \). The firm’s optimal issuance amount is \( w_{p_L}^* = 0.103 \) (the vertical dashed line) for \( p_L \), and \( w_{p_H}^* = 0.066 \) (the vertical dashed line) for \( p_H \). To the left of the optimal issuance amount, the marginal value of cash is higher than \( 1 + \gamma \), reflecting the fact that the fixed external financing cost
in raising equity increases the marginal value of cash. If there is no fixed external financing cost \((\phi = 0)\), the firm’s optimal issuance amount is zero, as the firm raises just sufficient funds to keep positive \(w\) and to avoid incurring the cash carrying cost.

Note that within the internal financing region \((w < w < \overline{w})\), the marginal value of cash is always higher when the product price is \(p_L\). As a result, the firm with \(p_L\) delays the payout \((\overline{w}_{p_L} > \overline{w}_{p_H})\) and replenishes more cash when issuing equity \((w^*_{p_L} > w^*_{p_H})\) compared to the firm with \(p_H\). The firm with \(p_L\) faces a higher marginal value of cash because in general it receives less cash inflows. As mentioned above, conditional on the same demand shock, setting a lower product price generates less short-term operating revenue. Since price resetting opportunities arrive in a Poisson fashion, the firm may find itself unable to adjust its product price to \(p_H\) immediately after the cash-size ratio \(w\) drops below the price resetting boundary \(w_{0P}^p\). This implies that the firm with \(p_L\) is more likely to hit the financing boundary after experiencing a sequence of negative demand shocks. The higher likelihood of executing costly external financing drives up the marginal value of cash. As a result, the firm with \(p_L\) is motivated to delay the payout of equity and endogenously chooses to hold more cash.

Figure 2: The firm’s value, marginal value of cash, and optimal investment rates.
Panel C of Figure 2 plots the normalized firm value, which is equal to the enterprise value plus the value of cash.

**Investment**  Panel D of Figure 2 plots the firm’s optimal investment-capital ratio. The investment-capital ratio is increasing in cash between zero and the payout boundary, and becomes flat beyond the payout boundary. Notably, the firm disinvests when the cash-size ratio is low. This is to move away from the financing boundary to avoid costly external financing. However, disinvesting is costly not only because of its effect on lowering the growth rate of effective firm size, but also due to the convex capital adjustment cost. Since external financing bears a fixed cost, the firm only issues equity when the cash-size ratio hits the zero lower bound. To avoid paying the fixed cost, the firm starts to raise cash through disinvesting and price adjustment before the cash-size ratio hits zero. Once the firm is completely running out of cash, it raises sufficient cash in lump-sum through the external financing channel. This mechanism delivers a pecking-order solution to liquidity problems: the firm holds cash on its balance sheet to cushion against negative demand shocks. When shocks are small or only last for a few periods, the firm can run down cash and partially rely on raising its price or disinvesting to refill its cash reserves. However, if shocks are large or long lasting, the firm will eventually run out of cash, in which case it would fill up the cash reserve through the external financing channel, which is most costly due to the fixed cost.

Moreover, compared to the firm with $p_H$, the firm with $p_L$ invests more when cash is sufficient; however, it invests less (disinvests more) when cash is constrained. This can be explained by the interaction between the current profit channel and the growth channel. When cash is abundant, the marginal value of cash is one, and the firm purely focuses on the growth channel. The firm with $p_L$ enjoys a higher growth rate in customer base. Since investment has a complementary effect in boosting the growth rate of effective firm size, the firm with $p_L$ optimally chooses to invest more. However, if the firm is cash constrained, the current profit channel kicks in and starts to play a more important role in determining the firm’s investment-capital ratio. With price stickiness, the firm with $p_L$ cannot adjust its price to $p_H$ immediately after its cash-size ratio drops below the price resetting boundary $w_0^P$. Therefore, it anticipates less incremental operating revenue in the short term, and an increased likelihood of pursuing costly external financing. Being aware of this anticipation, the firm’s investment decision is also more precautionary. In Section 3.3.3, we analyze the impact of price stickiness and show that a higher degree of price stickiness intensifies the precautionary investment motive, leading to less investment.

\[ A = m^\alpha K^{1-\alpha} \] indicates that the marginal return of increasing capital stock $K$ increases with the value of customer base $m$.  

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\(^{22}\) To see the complementarity between investment and customer base in boosting effective firm size, note that the growth rate of effective firm size, $g_A$, is determined by both investment, $i$, and the growth rate of customer base, $g_m$. The Cobb-Douglas form, $A = m^\alpha K^{1-\alpha}$ indicates that the marginal return of increasing capital stock $K$ increases with the value of customer base $m$. 

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Stationary Distribution  Using the optimal policy rules solved from the model, we simulate the evolution of prices and cash holdings of a single firm for 100 years and plot the stationary distribution of normalized cash-size ratios and investment ratios in Figure 3. Not surprisingly, as shown in panel A, cash holdings are relatively high during most of the time because using the above mentioned three channels (i.e. raising price, disinvesting, and external financing) to raise cash is costly. As a result, the probability mass of the investment ratio, $i(w)$, is concentrated around the highest value in the relevant support of $w$ (see panel B). However, there are periods where the firm is liquidity constrained, and hence making negative investment. The stationary distribution of prices also reveals (not reported here) that the firm sets $p_L$ during most of the time, to attract customers and build up customer base; while $p_H$ is set for about 3.6% of the entire simulation period to overcome liquidity problems.

3.3.3 The Impact of Price Stickiness

In this section, we investigate the impact of price stickiness. Specifically, we analyze the impact of the Calvo parameter $\xi$ on the firm’s enterprise value, financing, payout, and investment decisions.

Price Stickiness on the Enterprise Value  Figure 4 compares the firm’s enterprise value when the Calvo parameter $\xi$ varies. Panel A of Figure 4 plots the benchmark case, with $\xi = 2.8$. The case with a more flexible price ($\xi = 40$) is shown in Panel B.

The enterprise value for the firm with either $p_L$ or $p_H$ is higher in panel B, indicating that price stickiness reduces the enterprise value for any cash-size ratio. This is intuitive since being able to adjust the price can be considered as an option for the firm. The firm always prefers to set its price to $p_H$ when the cash-size ratio is low ($w < w_0^P$) to take advantage of the current profit channel, and $p_L$ when the cash-size ratio is high ($w > w_0^G$) to benefit from the growth channel. The likelihood of exercising this option is dependent on the degree of price stickiness, which is captured by the Calvo
Figure 4: The firm’s enterprise values for different value of the Calvo parameter. Panel A is plotted for $\xi = 2.8$, and Panel B is plotted for $\xi = 40$.

parameter, $\xi$. The larger $\xi$ is, the lower the cost of price adjustment, and the higher the option value and the firm’s enterprise value.

Moreover, notice that the enterprise value for the firm with $p_L$ and the firm with $p_H$ converges to each other when the price becomes less sticky. This is because when the firm obtains more opportunities to adjust its price, the enterprise value is affected less by the inherited price from the previous instant. On the extreme, when the price is perfectly flexible (with $\xi = \infty$), the two curves coincide with each other, and the price set in the previous instant no longer matters for the enterprise value (i.e., the product price is no longer a state variable).

Price Stickiness on Financing and Payout As shown in Figure 4, for the firm with $p_L$, both the payout boundary (the vertical dotted line) and the issuance amount (the vertical dashed line) shift to the left when the price becomes less sticky, because the firm has a larger chance to adjust its product price to $p_H$ when it is running out of cash. This, as a result, would increase the incremental operating revenue through the current profit channel, and thereby mitigate liquidity problems and decrease the marginal value of cash. Hence, the firm is willing to hold less cash on its balance sheet when facing smaller price stickiness.

On the contrary, for the firm with $p_H$, both the payout boundary (the vertical dotted line) and the issuance amount (the vertical dashed line) shift to the right when the price becomes less sticky,
indicating that the firm is willing to hold more cash. This is because when the price is less sticky, the firm with \( p_H \) anticipates that, in the future, it is more likely to get the chance to adjust its price to \( p_L \) when its cash-size ratio exceeds the price resetting boundary \( (w_0^p) \). As mentioned above, setting the low price increases the investment demand for cash due to the complementarity between capital stock and customer base in determining effective firm size. This motivates the firm with \( p_H \) to accumulate more cash, in order to benefit from future investment opportunities through the complementarity channel, when price resetting opportunities arrive.

Price stickiness has diametrically different implications for the firm with \( p_L \) and the firm with \( p_H \) in their financing and payout decisions. This is essentially because the marginal value of cash for the firm with \( p_L \) and the firm with \( p_H \) converges to each other when the price becomes less sticky. As shown in panel B of Figure 2, for any cash-size ratio, the marginal value of cash for the firm with \( p_L \) is higher than that for the firm with \( p_H \). As a result, the convergence in the marginal value of cash triggered by a smaller price stickiness leads to a decrease in the marginal value of cash for the firm with \( p_L \) and an increase for the firm with \( p_H \). Since financing and payout decisions are intimately linked to the marginal value of cash, it is not surprising that the firm with \( p_L \) tends to payout more and finance less while the firm with \( p_H \) tends to do the opposite when the price becomes less sticky.

Our simulation results show that for the benchmark calibration, the firm is setting its price to \( p_L \) when repurchasing equity (or issuing dividends) with a probability of 99.5%, thus the model predicts that firms facing larger price stickiness tend to repurchase equity less frequently, as the payout boundary associated with \( p_L \) in Figure 7 shifts to the right when \( \xi \) decreases from 40 to 2.8. On the other hand, during 98.5% of time the firm is setting its price to \( p_H \) when pursuing external financing, thus the model predicts that firms facing larger price stickiness issue less equity, as the optimal issuance amount associated with \( p_H \) in Figure 7 shifts to the left when \( \xi \) decreases from 40 to 2.8.

**Price Stickiness on Investment** Figure 5 presents the firm’s optimal investment decisions when the Calvo parameter varies.

In panel A, the optimal investment ratio is plotted for the firm with \( p_L \). It is shown that the firm facing smaller price stickiness invests more especially when the cash-size ratio is low. As we noted above, when the cash-size ratio is low, the “current profit channel” constrains the firm with \( p_L \) from investing more. Smaller price stickiness dampens the impact of this channel, as it becomes more likely for the firm to adjust its product price to \( p_H \), hence boosting short-term operating revenue. Therefore, the liquidity-constrained firm with \( p_L \) increases its investment (or disinvests less) when the price becomes less sticky. For a cash-abundant firm, investment is determined mostly by the “growth channel”, which is not affected much by the degree of price stickiness. Therefore, there is no significant difference in investment among the cash-abundant firms with \( p_L \) when price
In panel B, we plot the optimal investment ratio for the firm with $p_H$. Again, it is shown that the firm facing smaller price stickiness invests more for any cash-size ratio. This is mainly due to the increase in the normalized enterprise value for the firm with $p_H$ when the price becomes less sticky (see Figure 4). To see this, note that the firm’s value is equal to the sum of the enterprise value and cash, which is equal to the sum of the normalized enterprise value and the cash-size ratio multiplied by the firm’s effective size. Since investment increases effective firm size, its return is higher when either the normalized enterprise value or the cash-size ratio is larger. Therefore, a higher normalized enterprise value due to smaller price stickiness motivates the firm to make more investments.

To illustrate the impact of price stickiness on investment more clearly, in Figure 6, we plot both the optimal investment ratio and the steady-state distribution of the cash-size ratio. For expositional purposes, we also mark the simulated average investment ratio on the figure, and place the up-arrow at the position representing the average cash-size ratio. It shows that the average investment ratio for the baseline calibration ($\xi = 2.8$) is 0.030, whereas it increases to 0.034 if the firm faces a more flexible price ($\xi = 40$). These calculations are subject to numerical errors, but the observed small difference indicates that the impact of price stickiness on investment is not large.

There are three forces underlying our model which affect investment in different directions, and as a result, investment is on average not affected significantly by the degree of price stickiness faced by the firm. As we have said above, investment boosts effective firm size, and thus its marginal return is linked to the firm’s value per unit of effective size, which is equal to the sum of the normalized enterprise value and the cash-size ratio. The first force reduces investment as}

\footnote{Note that for the firm with $p_L$, when cash is abundant, investment ratios are marginally higher when the price is less sticky. This is due to the increase in the enterprise value generated by smaller price stickiness (Figure 4).}
the enterprise value is lower when the price is stickier, which decreases the marginal return on investment (see Figure 5). However, there is a countervailing force transmitted through endogenous cash holdings which pushes a stickier firm to make more investments. In fact, a stickier firm is more precautionary and holds more cash during most of the time, which increases its average cash-size ratio. Since the marginal return on investment also increases with the cash-size ratio, the firm with a stickier price tends to invest more due to more cash holdings. This can be directly seen from Figure 6, although investment for the firm with a stickier price ($\xi = 2.8$) is lower for any cash-size ratio, the distribution of cash holdings is more right skewed. Third, as shown in Figure 5, the difference in investment ratios when price stickiness varies is quantitatively large only for the firm with $p_L$ when the cash-size ratio is low and for the firm with $p_H$ when the cash-size ratio is high. However, during most of the time the firm with a low cash-size ratio is setting its price to $p_H$ and the firm with a high cash-size ratio is setting $p_L$. Therefore, the difference in the average investment ratio may not be significant. The latter two results are related to the steady-state distribution of the firm’s cash holdings, which has been largely ignored in the existing literature. Our result complements the traditional view that firms facing larger price stickiness invest less due to the higher cost of capital (see, e.g. Weber, 2014). In fact, the impact of price stickiness on investment could be ambiguous due to the channel of endogenous cash holdings.

**Quantifying the Cost of Price Stickiness** To shed light on the impact of price stickiness on the firm’s value, we simulate the model for 100 years and compute the average normalized firm value, cash-size ratio, and normalized enterprise value, respectively, over the whole simulation period. We focus on the steady-state outcomes and discard the simulated path of the first 10 years as burn in. Panel A of Figure 7 shows that the firm’s value increases with the degree of price stickiness (decreases with the Calvo parameter). The firm facing larger price stickiness bears a higher marginal value of cash, and is more precautionary in its payout decisions. As a result, it endogenously chooses to hold more cash on its balance sheet, boosting up the firm’s value. This is confirmed in panel B of Figure 7, which shows that cash holdings increase when the price becomes stickier. However, note that the firm facing larger price stickiness has a lower enterprise value (panel C of Figure 7). Since the enterprise value is equivalent to the average Tobin’s q in our model, this implies that the firm with a stickier price is less efficiently operated (see, e.g. Wernerfelt and Montgomery, 1988; Lang and Litzenberger, 1989; Chen and Lee, 1995; Bharadwaj, Bharadwaj and Konsynski, 1999) or riskier (see, e.g. Liew and Vassalou, 2000; Griffin and Lemmon, 2002). This result is consistent with Weber (2014), which finds that firms facing larger price stickiness are riskier and demand a higher risk premium.

Quantitatively, our simulation results shown in Figure 7 indicate that the firm facing a completely

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Note that the results presented in Panel A of Figure 7 are about the average firm’s value when the firm is in steady state. A stickier price indeed reduces the firm’s value at the impact of the change in price stickiness.
A. Investment and cash distribution, $\xi = 2.8$

B. Investment and cash distribution, $\xi = 40$

Figure 6: The firm’s investment and cash-size ratio distribution for different values of the Calvo parameter. Panel A is plotted for $\xi = 2.8$, and Panel B is plotted for $\xi = 40$. The left y-axis is for the firm’s investment and the right y-axis is for the steady-state distribution of the cash-size ratio. The up-arrow in each panel is positioned at the average cash-size ratio.

sticky price ($\xi = 0$) on average holds 14% more cash than the firm facing a perfectly flexible price ($\xi = \infty$), while the enterprise value is lowered by 0.5%. A sticky price is costly not only through its direct effect on reducing the enterprise value, but also by indirectly inducing the firm to carry more cash, which is costly as captured by $\lambda > 0$. Moreover, note that although the reduction in the enterprise value due to price stickiness has a smaller magnitude on average, it is magnified especially when the firm is liquidity constrained. As shown in Figure 4, when the cash-size ratio is near zero, the enterprise value of the firm with $p_L$ is decreased by about 1.5% when the value of the Calvo parameter decreases from $\xi = 40$ to $\xi = 2.8$.

3.3.4 The Impact of Financing Costs

In this section, we analyze the impact of the fixed and variable financing costs on the firm’s enterprise value, financing, payout, investment, and price setting decisions.

The Fixed Financing Cost  Figure 8 presents the firm’s normalized enterprise value when the fixed financing cost $\phi$ varies. When $\phi$ is increased from 1% to 2%, the enterprise value slightly decreases. Moreover, the payout boundaries and the issuance amounts shift to the right for both
Figure 7: The average steady-state normalized firm value, cash holdings and normalized enterprise value for different values of the Calvo parameter.

$p_L$ and $p_H$, indicating that the firm will hold more cash on its balance sheet. This is because a higher fixed financing cost increases the marginal value of cash by dampening the external financing channel. The price resetting boundary shifts to the right when the fixed financing cost increases. This implies that the firm is more likely to set its price to $p_H$ when external financing costs are high. The response of investment is similar to Bolton, Chen and Wang (2011), i.e. investment is lower when the fixed financing cost increases (see Figure 9).

Panel A of Figure 10 shows that the normalized firm value increases with the fixed financing cost. The underlying force is similar to the one driving the impact of price stickiness. That is, the firm increases its cash holdings (as shown in panel B) due to a higher marginal value of cash when the fixed financing cost increases.\(^{25}\) Panel C shows that the normalized enterprise value (or the

\(^{25}\)Note that the results presented in Panel A of Figure 10 are about the average firm’s value when the firm is in
A. Enterprise value: $u(w) - w, \phi = 2\%$

B. Enterprise value: $u(w) - w, \phi = 1\%$

Figure 8: The firm’s enterprise value for different values of the fixed financing cost. Panel A is plotted for $\phi = 2\%$, and Panel B is plotted for $\phi = 1\%$.

The average $q$ decreases as external financing becomes more costly.

**The Variable Financing Cost**

Figure 11 shows the response of the normalized enterprise value when the variable financing cost varies. Similar to the effect of the fixed financing cost, the normalized enterprise value is lower when the variable financing cost increases.

The firm is less likely to payout when $\gamma$ increases as the marginal value of cash is higher. This can be seen in Figure 11, which shows that the payout boundaries (the vertical dotted lines) shift to the right from panel A ($\gamma = 1\%$) to panel C ($\gamma = 16\%$). The optimal issuance amounts are located at the points where the marginal value of cash is equal to one plus the variable financing cost $\gamma$. Therefore, when $\gamma$ increases, the firm issues less equity when hitting the financing boundary (the vertical dashed lines shift to the left).

Notice that the relative locations of the issuance amounts and the price resetting boundary (the vertical solid line) is indeterminate. Panel A plots the case for $\gamma = 1\%$. The issuance amount for both the firm with $p_L$ and the firm with $p_H$ are to the right of the price resetting boundary ($w_{p_L}^* > w_{p_H}^* > w_P^0$). This suggests that when the firm runs out of cash and issues equity, it takes advantage of the low variable financing cost and raises sufficient external funds in order to not steady state. A larger fixed financing cost indeed reduces the firm’s value at the impact of the change in financing costs.
distort its pricing behavior (i.e. set the price to \( p_L \) after external financing). In panel B, the variable financing cost is set at \( \gamma = 6\% \). In this case, only the firm with \( p_L \) raises sufficient cash through external financing to go beyond the price resetting boundary \((w^*_P < w^*_0 < w^*_P)\). In panel C, the variable financing cost is high, \( \gamma = 16\% \). Both the firm with \( p_L \) and \( p_H \) remain in the high price region after external financing \((w^*_P > w^*_P > w^*_P)\).

3.3.5 The Interaction Between Price Stickiness and Financing Costs

Both price stickiness and external financing costs reduce the enterprise value and affect the firm’s marginal value of cash. In this section, we seek to understand the interaction between the two frictions and their joint impact on the firm’s value.

Panel A of Figure 12 presents the firm’s normalized enterprise value when the fixed financing cost varies from \( \phi = 0 \) to \( \phi = 3\% \) for the firm with \( \xi = 2.8 \) and \( \xi = 40 \), respectively. Confirming our previous results, the enterprise value is lower as the price becomes stickier. This is true regardless of the value of the fixed financing cost. As shown in panel A, the firm with \( \xi = 2.8 \) has uniformly a lower enterprise value (solid line) than the firm with \( \xi = 40 \) (dashed line).

Notably, the decrease in the enterprise value due to larger price stickiness increases with the value of the fixed financing cost. This implies that the firm loses more value when it is facing more frictions in external financing. This is because cash-constrained firms tend to rely more on raising prices to boost cash revenue when facing higher costs of external financing. Consider an extreme case when external financing costs are zero, the firm will always prefer to use external financing to replenish cash and stick to the optimal price forever. As shown in panel A, the two curves converge to each other as the fixed financing cost approaches zero, implying that the degree of price stickiness has no impact on the enterprise value. If external financing costs are infinite, the firm will have to raise its price when running out of cash since external financing is not feasible. The firm facing a
Figure 10: The average steady-state normalized firm value, cash holdings and normalized enterprise value for different values of the fixed financing cost.

A stickier price is more likely to hit the liquidation boundary, because it is less likely to find a chance to reset its price when the cash-size ratio is low. While the firm facing a more flexible price can timely increase its price and boost cash revenue to avoid liquidation. Hence, the difference in the enterprise value between the two firms, or the cost of price stickiness, is particularly large when external financing is not allowed.

In our model, the cost of holding cash is captured by parameter $\lambda > 0$. Panel B of Figure 12 presents the increase in the cash carrying cost when the Calvo parameter decreases from $\xi = 40$ to $\xi = 2.8$. When external financing costs are zero, the firm’s cash management policy is no longer affected by price stickiness, thus there is no change in the cash carrying cost when the price becomes stickier. However, for a relatively large fixed external financing cost, $\phi = 3\%$, the cost of holding cash increases by about 10% when $\xi$ is reduced from 40 to 2.8.
Figure 11: The firm’s enterprise value for different values of the variable financing cost. Panel A is plotted for $\gamma = 1\%$, panel B is plotted for $\gamma = 6\%$, and panel C is plotted for $\gamma = 16\%$.

In sum, the impact of price stickiness varies significantly with financing costs. Price stickiness becomes more costly for the firm when the frictions in external financing are more severe.

4 A Model with Menu Costs

In our baseline model, price stickiness is modeled as in Calvo (1983). In this section, we modify this aspect by assuming that price adjustment is totally under the control of the firm’s manager. However, whenever the price is adjusted, a fixed “menu cost”, $\zeta$ is incurred. This modification enables us to quantitatively measure the direct cost of price stickiness, and to address the concern that the qualitative predictions of the baseline model on the firm’s pricing strategy is driven by the mechanical Calvo pricing rule.
Introducing a fixed cost of price adjustment changes the HJB equation (3.14) to the following

\[
\begin{align*}
    rU(A, W, p) = & \max_{I, p^+ \in \{p_L, p_H\}} \left[ \alpha h(p) + (1 - \alpha)(I/K - \delta) \right] AU_A \\
    & + [(r - \lambda)W + A(p - \tau)\mu(p) - \Gamma(I, K, A)] U_W \\
    & + \left[ U(A, W, p^+) - U(A, W, p) \right] - \zeta 1_{p^+ \neq p},
\end{align*}
\]

where \(1_{p^+ \neq p}\) is an indicator function, which equals one if \(p^+ \neq p\) and zero otherwise. The financing and payout boundary conditions are the same as the baseline model.

### 4.1 Quantitative Results

We set the menu cost parameter \(\zeta = 0.002\) to match the firm’s average normalized enterprise value in the menu cost model with the one in the baseline model. The other parameters are set to be the same as the baseline model. Quantitatively, this implies that the menu cost amounts to about 0.15% of the average normalized enterprise value, or 1% of the firm’s average cash holdings.

Figure 13 plots the normalized enterprise value for the menu cost model. Similar to the baseline model (Figure 2), the normalized enterprise value of the firm with \(p_L\) is higher than the firm with \(p_H\) when the cash-size ratio is high, and the reverse is true when the cash-size ratio is low. However, the difference in the normalized enterprise value between the firm with \(p_L\) and the firm with \(p_H\) is bounded by the amount of the menu cost. This is intuitive since whenever the difference is larger than the menu cost, the firm will choose to reset its price immediately, which ensures that the resulting difference in the enterprise value cannot exceed the menu cost.

There are two price resetting boundaries in the menu cost model, as marked by the vertical solid lines in the figure. The right boundary \(u_0^{p_H \rightarrow p_L} = 0.13\) captures the threshold of the cash-size
Figure 13: The enterprise value in the model with menu costs.

The cash-size ratio at which the firm switches from $p_H$ to $p_L$, while the left boundary ($w_{0}^{PL\rightarrow PH} = 0.05$) captures the threshold where the firm switches from $p_L$ to $p_H$. When the cash-size ratio is between the two boundaries ($w_{0}^{PL\rightarrow PH} \leq w \leq w_{0}^{PH\rightarrow PL}$), the firm is in the “inaction” region and does not change its price at all because the benefit obtained from resetting the price is smaller than the menu cost. When the cash-size ratio is below the left price resetting boundary ($w < w_{0}^{PL\rightarrow PH}$), the firm always sets $p_H$, while when the cash-size ratio is above the right price resetting boundary ($w > w_{0}^{PH\rightarrow PL}$), the firm always sets $p_L$.

Compared to the baseline model, the menu cost model implies a one-to-one mapping from the cash-size ratio to the product price out of the inaction region. While in the baseline model, the mapping only exists between the cash-size ratio and the “desired” product price. Whether the firm can achieve the desired price or not depends on the arrival of price resetting opportunities, which is out of the manager’s control. Within the inaction region, the firm’s price is indeterminate, depending on the inherited price when the firm first enters the region. The firm facing a larger menu cost (or a stickier price) is associated with a wider inaction region and the shift in issuance amounts and payout boundaries are exactly consistent with the implications of the benchmark model (see Section 3.3.3) following the same intuitions.

Figure 14 plots the marginal value of cash (panel A) and the optimal investment-capital ratio (panel B). Consistent with the baseline model, the marginal value of cash is high for the firm with $p_L$. However, the difference only shows up within the inaction region. Outside this region, the difference in the enterprise value is locked by the menu cost, resulting in the same marginal value of cash irrespective of the firm’s price. Similarly, the firm with $p_H$ has a higher investment ratio (or a smaller disinvestment ratio) compared to the firm with $p_L$ only within the inaction region, reflecting the impact of the current profit channel.
5 Conclusion and General Equilibrium Discussion

We propose a tractable model which demonstrates price setting decisions when a firm is operated in an environment featuring both customer markets and financial frictions, and the impact of price stickiness on a firm’s investment and financing.

Our model offers several testable predictions on the interaction between price stickiness and financial frictions, and how they jointly affect a firm’s financial and investment decisions. In particular, our model predicts that financially constrained firms have a tendency to set a relatively higher markup, and such tendency is larger when prices are less sticky or external financing costs are high. Moreover, our model predicts that firms facing larger price stickiness are more precautionary in their financial decisions—they tend to delay the payment of dividends and issue less equity. Existing literature (see, e.g. Chevalier and Scharfstein, 1996; Gilchrist et al., 2014) provide empirical evidence in line with these predictions. Empirical tests based on more detailed firm-level price data set a research agenda of markup dynamics and the impact of price stickiness.

The interaction between price stickiness and financial frictions highlighted in our model can strengthen countercyclical markups. We analyze markup dynamics in Appendix B.1, which is only intended to be illustrative, since our model is a partial equilibrium model with both the industry average price and the interest rate being exogenously given. Yet, we believe its implications on the cyclicity of markups are robust even after taking into account the feedback effect from firms’ optimal decisions on the industry average price. During recessions, financially weak firms have a tendency to increase their product prices to boost revenue. In a symmetric Nash equilibrium, the financial distress can reinforce the motivation for “implicit collusion” during recessions (see Rotemberg and Saloner, 1986), since there is little loss in each firm’s customer base when all firms keep high prices or even raise prices all together at the same time. Even without “implicit collusion”, under a pure Walrasian equilibrium framework, the higher product prices charged by financially
weak firms increase the industry average price. This may further induce financially strong firms to increase their product prices since now they are facing a less elastic demand curve due to a higher industry average price.
Appendix

A Proofs of Propositions

A.1 Proof of Proposition 1

First, we show that

\[(r + \delta + \xi)^2 - 2[\mu_L + \xi u(p_H) - (r + \delta + \xi)]/\theta > \left\{\sqrt{(r + \delta)^2 - 2[\mu_H - (r + \delta)]/\theta + \xi}\right\}^2 > 0.\]

Rearranging the terms, we get

\[(r + \delta + \xi)^2 - 2[\mu_L + \xi u(p_H) - (r + \delta + \xi)]/\theta - \left\{\sqrt{(r + \delta)^2 - 2[\mu_H - (r + \delta)]/\theta + \xi}\right\}^2 = 2(\mu_H - \mu_L) > 0.\]

It is straightforward to see that \(u(p_L)\) converges to \(u(p_H)\) as \(\xi\) goes to infinity. The partial derivative of \(u(p_L)\) with respect to \(\xi\) is

\[\frac{\partial u(p_L)}{\partial \xi} = 1 - \frac{\sqrt{(r + \delta)^2 - 2[\mu_H - (r + \delta)]/\theta + \xi}}{\sqrt{(r + \delta + \xi)^2 - 2[\mu_L + \xi u(p_H) - (r + \delta + \xi)]/\theta}} > 0.\]

Therefore, the value function \(u(p_L)\) is monotonically increasing in \(\xi\), and \(u(p_H) > u(p_L)\).

A.2 Proof of Proposition 2

First, it is straightforward to see that

\[
[\delta(1 - \alpha) + r + \xi - ah(p_H) + (1 - \alpha)/\theta]^2 - [1 + 2\theta \mu_H + 2\theta \xi u(p_L)](1 - \alpha)^2/\theta^2 - \left\{\xi - ah(p_H) - \sqrt{[\delta(1 - \alpha) + r]^2 - (1 - \alpha)[\mu_L(1 - \alpha) - \delta(1 - \alpha) - r]/\theta}\right\}^2 \\
\geq -2\alpha h(p_H) [\delta(1 - \alpha) + r + (1 - \alpha)/\theta] + 2\frac{(1 - \alpha)^2}{\theta}(\mu_L - \mu_H) > 0. \tag{A.1}
\]

Obviously, \(u(p_H)\) converges to \(u(p_L)\) as \(\xi\) goes to infinity. The partial derivative of \(u(p_H)\) with respect to \(\xi\) is \(\frac{\partial u(p_H)}{\partial \xi}\) which has the following expression

\[\frac{\theta}{(1 - \alpha)^2} \left\{1 - \frac{\xi - ah(p_H) - \sqrt{[\delta(1 - \alpha) + r]^2 - (1 - \alpha)[\mu_L(1 - \alpha) - \delta(1 - \alpha) - r]/\theta}}{\sqrt{[\delta(1 - \alpha) + r + \xi - ah(p_H) + (1 - \alpha)/\theta]^2 - [1 + 2\theta \mu_H + 2\theta \xi u(p_L)](1 - \alpha)^2/\theta^2}}\right\}.\]
Based on the inequality (A.1) above, we know that $\frac{\partial u(p_H)}{\partial \xi} > 0$. Therefore, the value function $u(p_H)$ is monotonically increasing in $\xi$, and $u(p_L) > u(p_H)$.

B Additional Numerical Results

B.1 Countercyclical Markups

In this section, we present the economy’s transitional dynamics to illustrate that our model provides new forces which generate countercyclical markups. Since our model is a partial equilibrium model, without taking into account the feedback effect from firms’ optimal decisions on industry average prices or on the equilibrium interest rate, our analysis about countercyclical markups is only intended to be illustrative. A more systematic analysis of the cyclicality of markups based on a general equilibrium model with heterogeneous firms is left for future research.

Demand Shocks  As elaborated in Section 3.3, the firm’s desired product price is $p_H$ when it is financially constrained, since setting a higher price increases the incremental operating revenue. Consider an economy-wide negative demand shock which reduces all firms’ incremental operating revenue. Consequently, firms’ have to run down their cash holdings, and some of them become financially constrained. To prevent costly external financing, these financially constrained firms raise their product prices, resulting in a higher aggregate price level. This generates countercyclical markups/desired markups.

To illustrate this idea, we consider a simple scenario where the steady-state economy, under the benchmark calibration, is hit by an unexpected aggregate negative demand shock which lasts for one quarter.\(^26\) We assume that for firm $i$, the nominal shock $\sigma dZ^i_t$ in equation (3.2) consists of an aggregate component $\sigma_A dZ^A_t$ and an idiosyncratic component $\sigma_I dZ^i_t$, i.e., $\sigma dZ^i_t = \sigma_A dZ^A_t + \sigma_I dZ^i_t$ and $\sigma^2 = \sigma_A^2 + \sigma_I^2$. $dZ^A_t$ and $dZ^i_t$ are standard Brownian motion and are independent from each other. We generate the aggregate demand shock by assuming $dZ^A_t = -4$ in the first quarter, and $dZ^A_t = 0$ thereafter. Hence, the first quarter of the economy is hit by a negative aggregate demand shock. Figure B.1 shows the transitional dynamics for the average price and cash holdings of a large number of firms when $\sigma_I = \sigma_A = \sigma/\sqrt{2}$. During the first quarter, the negative aggregate demand shock reduces operating revenue and most of the firms have to run down cash to maintain investment. Moreover, firms who find themselves financially constrained increase their prices (i.e. set their prices to $p_H$), which drives up the economy’s price level and generates a higher average markup since the marginal cost of production is unchanged. When the demand shock subsides at the end of the first quarter, financially constrained firms start to accumulate cash and reset

\(^{26}\)The steady state of the economy is defined as the state where the joint distribution of firms’ cash holdings and product prices is invariant over time. To obtain this, we simulate 500,000 firms for 10 years.
Figure B.1: The dynamics of the average cash ratio and the price level after a demand shock.

their prices to $p_L$, allowing the economy’s price level to decrease and gradually converge to its steady-state value.

Notice that the underlying mechanism for the countercyclical markups is different from standard New Keynesian models. In New Keynesian models, a negative demand shock reduces the marginal cost, but as firms cannot adjust prices immediately, the markups would be temporarily high. This relies on the assumptions that the marginal cost of production is increasing in demand and that firms face nominal rigidity. In our model, these two assumptions are not necessary, and we emphasize the roles played by the customer market and financial frictions in generating countercyclical markups. Moreover, customer base can be considered as a force that generates real rigidity. Therefore, the desired markup is also countercyclical in our model, while it is constant in standard New Keynesian models. This addresses the concern raised by Blanchard (2009). Moreover, in contrast to the classical customer market models without financial frictions (see, e.g. Phelps and Winter, 1970; Phelps, 1992), the countercyclical markups are more robust in our model. As pointed out by Klemperer (1995), the customer market model can generate both countercyclical and procyclical markups depending on the nature of demand shocks. If positive demand shocks come from existing customers increasing their demand, then firms would be motivated to increase their prices to profit from locked-in customers. But, if demand shocks come from the arrival of new customers, then firms would decrease their prices to invest in customer base. In a related paper, Chevalier and Scharfstein (1996) show that the customer market model without financial frictions can only
generate procyclical markups.\footnote{This is considered as the benchmark model which motivates them to introduce financial frictions in order to generate countercyclical markups. However, in fact, enriching their benchmark model with multiple periods enables countercyclical markups when the increase in demand is from new customers or when demand shocks are persistent.} In our model, by introducing financial frictions, the endogenous marginal value of liquidity can interact with the customer market, which generates an additional force pushing towards countercyclical markups.

**Financial Shocks** As shown in Figure 8, the price resetting boundary shifts to the right when the fixed financing cost increases. This implies that the firm is more likely to set its price to $p_H$ when external financing costs are high. Since most recessions are associated with a credit crunch in the financial market (Claessens, Kose and Terrones, 2009), the increase in external financing costs provide an additional force pushing towards countercyclical markups.

To show this, we start from the steady-state economy with the benchmark calibration and consider an unexpected one-quarter financial shock which increases the fixed financing cost from 2\% to 3\%. The results are shown in Figure B.2.

The left three panels of Figure B.2 are based on the benchmark value of parameter $\xi = 2.8$. It shows that firms tend to raise their prices to accumulate more cash when the financial shock arrives in order to avoid costly external financing, resulting in an increase in the average price level. When the financial shock vanishes, both the price level and the average cash ratio converge to their steady-state values. Note that during the first quarter, the price level increases only gradually because firms are facing nominal rigidity. By contrast, when the price is more flexible, the increase in the price level is the highest at the impact of the financial shock, and the price level gradually decreases as firms accumulate more cash and become less financially constrained. This is shown in the right three panels of Figure B.2, where the Calvo parameter is set to $\xi = 40$. Therefore, in our model, nominal rigidity essentially dampens the response of the price level to a financial shock, and the immediate increase in the price level following a negative financial shock is larger when the price is more flexible. Hence, the mechanism delivered by our model has the potential to explain the lack of deflationary pressure during the “Great Recession” in the United States not by appealing to nominal rigidity or large unobservable shocks to the markup.

**B.2 Price Setting and Product Market Characteristics**

Following the discussion of equation (3.10), when $\kappa$ is fixed, parameter $\nu$ captures how sensitive customer flows are to the change in product prices. By varying the value of $\nu$, our model is able to capture markets with different degrees of competition, and further shed light on the firm’s behavior when the characteristics of the underlying product market change. Intuitively, a larger $\nu$ implies that customer base is more sensitive to the price, reflecting a competitive product market. On the contrary, a smaller $\nu$ captures the feature of a customer-based product market, which is associated
Figure B.2: The dynamics of the average cash ratio and the price level after a financial shock. The left three panels are for the case with $\xi = 2.8$, the right three panels are for the case with $\xi = 40$.

with a higher degree of consumption inertia (or higher switching/information costs). In a market with a relatively small $\nu$, the firm loses customer base slowly even if a high price is temporarily charged.

Our model implies that firms in a customer-based product market tend to set their prices to $p_H$ during a liquidity-constrained period. This is because setting a high price imposes less cost on the firm if the underlying product market is more customer based—the demand is less elastic in the short run and customer base only decreases slowly as consumers are reluctant to change their consumption habits or switch to other brands. However, setting a high price benefits the firm through the current profit channel by increasing short-term operating revenue. Thus, the firm is more inclined to set its price to $p_H$ when the cash-size ratio is low.

Figure B.3 illustrates this result. As parameter $\nu$ decreases from 1.32 to 1.3, the price resetting boundary (vertical solid line) shifts to the right, indicating that the firm is more likely to set its price to $p_H$ when experiencing a liquidity problem. This is consistent with the empirical findings in Gilchrist et al. (2014), which show that firms operating in a more customer-based market (as captured by high SG&A expenses or advertising expenses$^{28}$) raise their product prices relative to the industrial average prices during the recent U.S. financial crisis.

$^{28}$Gourio and Rudanko (2014) use SG&A ratios as an indicator for a customer market environment.
A. Enterprise value: $u(w) - w$, $\nu = 1.3$

B. Enterprise value: $u(w) - w$, $\nu = 1.32$

Figure B.3: The firm’s enterprise value in different product markets. Panel A is plotted for $\nu = 1.3$, and Panel B is plotted for $\nu = 1.32$.

### B.3 The Volatility of Productivity Shocks

In this section, we discuss the impact of the volatility of productivity shocks on the firm’s enterprise value, investment, payout boundaries, optimal issuance amounts, and the price resetting boundary. As shown in Figure B.4 and B.5, when productivity shocks become more volatile (parameter $\sigma$ increases from 12% to 14%), the firm has a lower enterprise value and lower investment. Moreover, the payout boundaries $\bar{w}_{pL}$ and $\bar{w}_{pH}$, and the optimal equity issuance amounts $w^*_p$ and $w^*_p$ shift to the right, reflecting a higher marginal value of cash. The price resetting boundary ($w^*_P$) shifts to the right, indicating that the firm is more likely to raise its price because a higher volatility of productivity shocks increases the chance of hitting the financing boundary.

### C Numerical Methods

Due to the large non-linearity introduced by price stickiness and financing costs, the solution of the coupled ODE problem is not robust and subject to large numerical errors. To mitigate this problem, we reformulate the continuous time coupled ODEs into a discrete recursive problem, which is solved using a standard dynamic programming method.

Time is discrete with interval $\Delta$. The aggregate demand shock $Q_t$ is i.i.d. and follows a normal
Figure B.4: The firm’s enterprise value for different volatilities of productivity shocks. Panel A is plotted for $\sigma = 12\%$, and Panel B is plotted for $\sigma = 14\%$.

distribution, $N(\mu \Delta, \sigma^2 \Delta)$. In period $t$, the state variables for the recursive problem are cash $W_t$ and effective size $A_t$.

Let $U^L(W, A)$ and $U^H(W, A)$ be the value functions for the low price and high price, respectively. Price resetting opportunities arrive with probability $\xi \Delta$. We assume that price resetting opportunities arrive before the realization of demand shocks, $Q$. The firm makes investment and financing decisions before they know whether they can adjust the price in the current period. After the realization of price resetting opportunities, if the firm obtains the chance to reset its price, either $p_L$ or $p_H$ will be chosen to maximize the objective function. Otherwise, the firm has to stick to the price inherited from the previous period. Demand shocks are realized after price setting decisions are made. Figure C.6 illustrates the timing.

Note that the timing assumption used here is to simplify calculations. Alternatively, we could assume that the firm makes optimal decisions after the realization of demand shocks. However, this would complicate computations because now the firm’s optimal decisions also depend on the realized value of demand shocks (see Dumas and Lyasoff, 2012). When $\Delta$ approaches zero, the timing assumption does not matter, and the solution will converge to the solution of the coupled ODEs. In our calculation, we set $\Delta = 0.02$. The resulted numerical errors are negligible.
Figure B.5: The firm’s investment for different volatilities of productivity shocks. Panel A/B plots investment for the $p_L/p_H$ case. Solid/Dashed lines refer to the $\sigma = 12\% / \sigma = 14\%$ case.

Figure C.6: Timing assumption for the recursive problem.

The recursive formulation for $U^L(W, A)$ is

$$U^L(W, A) = \max_{i, U} U + \frac{\xi \Delta}{1 + r \Delta} \max \{ E[U^L(W', A')|Q], E[U^H(W'', A'')|Q]\} + \frac{1 - \xi \Delta}{1 + r \Delta} E[U^L(W', A')|Q],$$

subject to

$$W' = W + p_L A Q - g(i) \Delta A + (r - \lambda) \Delta W - U - ((\phi A - \gamma U)1_{U<0}),$$

$$A' = (1 + h(p_L)\Delta)^\alpha [(1 - \delta \Delta) + i \Delta]^{1-\alpha} A,$$

$$W'' = W + p_H A Q - g(i) \Delta A + (r - \lambda) \Delta W - U - ((\phi A - \gamma U)1_{U<0}),$$

$$A'' = (1 + h(p_H)\Delta)^\alpha [(1 - \delta \Delta) + i \Delta]^{1-\alpha} A.$$

Similarly, for $V^h(W, A)$, the recursive formulation is
\[ U^H(W, A) = \max_{i, u} \left[ U + \frac{\xi \Delta}{1+r \Delta} \max \{ E[U^L(W', A')|Q], E[U^H(W'', A'')|Q] \} + \frac{1-\xi \Delta}{1+r \Delta} E[U^H(W'', A'')|Q] \right], \]

subject to

\[ W' = W + p_L AQ - g(i) \Delta A + (r - \lambda) \Delta W - U - ((\phi A - \gamma U)1_{U < 0}), \]
\[ A' = (1 + h(p_L) \Delta)^{\alpha}(1 - \delta \Delta) + i \Delta]^{1-\alpha} A, \]
\[ W'' = W + p_H AQ - g(i) \Delta A + (r - \lambda) \Delta W - U - ((\phi A - \gamma U)1_{U < 0}), \]
\[ A'' = (1 + h(p_H) \Delta)^{\alpha}(1 - \delta \Delta) + i \Delta]^{1-\alpha} A. \]

Since \( U(W, A) \) is homogeneous in \( A \), we write \( U^L(W, A) = u^L(w)A, U^H(W, A) = u^H(w)A \), and define \( w = \frac{W}{A}, \) and \( u = \frac{U}{A} \).

The recursive problems for the normalized value functions, \( u^L \) and \( u^H \) are

\[ u^L(w) = \max_{i, u} \left[ u + \frac{[1-\delta \Delta] + i \Delta]^{1-\alpha}}{1+r \Delta} \left\{ (1 - \xi \Delta)(1 + h(p_L) \Delta)^{\alpha} E[u^{L}(w')|Q] + \xi \Delta \max \{ (1 + h(p_L) \Delta)^{\alpha} E[u^{L}(w')|Q], (1 + h(p_H) \Delta)^{\alpha} E[u^{H}(w'')|Q] \} \right\}, \]

subject to

\[ w' = \frac{w + p_L Q - g(i) \Delta + (r - \lambda) \Delta w - u - ((\phi - \gamma u)1_{u < 0})}{[(1 - \delta \Delta) + i \Delta]^{1-\alpha}(1 + h(p_L) \Delta)^{\alpha}}, \]
\[ w'' = \frac{w + p_H Q - g(i) \Delta + (r - \lambda) \Delta w - u - ((\phi - \gamma u)1_{u < 0})}{[(1 - \delta \Delta) + i \Delta]^{1-\alpha}(1 + h(p_H) \Delta)^{\alpha}}, \]

and

\[ u^H(w) = \max_{i, u} \left[ u + \frac{[1-\delta \Delta] + i \Delta]^{1-\alpha}}{1+r \Delta} \left\{ (1 - \xi \Delta)(1 + h(p_H) \Delta)^{\alpha} E[u^{H}(w'')|Q] + \xi \Delta \max \{ (1 + h(p_L) \Delta)^{\alpha} E[u^{L}(w')|Q], (1 + h(p_H) \Delta)^{\alpha} E[u^{H}(w'')|Q] \} \right\}, \]

subject to

\[ w' = \frac{w + p_L Q - g(i) \Delta + (r - \lambda) \Delta w - u - ((\phi - \gamma u)1_{u < 0})}{[(1 - \delta \Delta) + i \Delta]^{1-\alpha}(1 + h(p_L) \Delta)^{\alpha}}, \]
\[ w'' = \frac{w + p_H Q - g(i) \Delta + (r - \lambda) \Delta w - u - ((\phi - \gamma u)1_{u < 0})}{[(1 - \delta \Delta) + i \Delta]^{1-\alpha}(1 + h(p_H) \Delta)^{\alpha}}. \]

Table C.1 lists the parameters for the discretization of state space.
Table C.1: Discretization of state space

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta$</td>
<td>0.017</td>
<td>Time interval</td>
</tr>
<tr>
<td>$n_w$</td>
<td>2000</td>
<td>Number of cash grid points (equally spaced)</td>
</tr>
<tr>
<td>$n_i$</td>
<td>1000</td>
<td>Number of investment grid points (equally spaced)</td>
</tr>
<tr>
<td>$n_u$</td>
<td>2000</td>
<td>Number of financing grid points (equally spaced)</td>
</tr>
<tr>
<td>$[w \bar{w}]$</td>
<td>$[0 \ 0.50]$</td>
<td>Cash range</td>
</tr>
<tr>
<td>$[\bar{i} \bar{i}]$</td>
<td>$[-1.00 \ 0.50]$</td>
<td>Investment range</td>
</tr>
<tr>
<td>$[u \bar{u}]$</td>
<td>$[-0.20 \ 0.20]$</td>
<td>Financing range</td>
</tr>
</tbody>
</table>
References


Gilchrist, Simon, Jae Sim, Raphael Schoenle, and Egon Zakrajsek. 2014. “Inflation Dynamics During the Financial Crisis.”


Loualiche, Erik. 2014. “Asset Pricing with Entry and Imperfect Competition.”


