Vector Calculus Independent Study

Unit 2 Sample Test

1. [20 points] Show that the arclength of the line segment connecting the points $P$ and $Q$ is the same as the distance between $P$ and $Q$.

2. [20 points] Say a particle starts at time 0 at the point $(1, 2, 3)$ and then moves such that its velocity vector at time $t$ is $(\sin t, t^2, e^t)$. Give the position function of the particle.

3. [20 points] Find the tangent line to the curve $\vec{\sigma}(t) = (1 - 5t, te^t, \log(t + 10))$ at $t = 1$.

4. [20 points] Find the arc length of the spiral

$$\vec{\sigma}(t) = (e^t \cos 2\pi t, e^t \sin 2\pi t)$$

for $t = 0 \ldots 2\pi$.

5. [20 points] Find the work done by a particle moving on the path $\vec{\sigma}(t) = (\cos t, \sin t, t)$, $0 \leq t \leq 2\pi$ through the force field $\vec{F}(x, y, z) = (-y, x, z)$.