Vector Calculus Independent Study

Unit 7 Sample Test

1. [25 points] Find the surface area of the portion of the cylinder

   \[ y^2 + z^2 = 9 \]

   that is above the rectangle \( 0 \leq x \leq 2, -3 \leq y \leq 3 \).

2. [25 points] Evaluate the surface integral

   \[ \iint_S x^2 z + y^2 z \, dS, \]

   where \( S \) is the portion of the sphere \( x^2 + y^2 + z^2 = 9 \) above the plane \( z = 2 \).

3. [25 points] Find the flux of the vector field

   \[ \vec{F}(x, y, z) = (x, y, 2z) \]

   through the surface described by \( z = 1 - x^2 - y^2 \), \( z \geq 0 \).

4. [25 points] A certain surface is described parametrically by

   \[ \vec{S}(u, v) = (u \cos v, u \sin v, u^2). \]

   Convert this to an implicit description (one just mentioning \( x, y, \) and \( z \)), and also find a normal to the surface at \( u = \sqrt{2}, v = \pi/4 \).