Are the cuprates doped spin liquid Mott insulators?

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Meaning of the question?
How might we find answers?
Don’t we already know?

T.Senthil, Patrick Lee, cond-mat/0406066
Are the cuprates doped spin liquid Mott insulators?

- "Obvious" answer: No!

Undoped material has antiferromagnetic order – not a spin liquid.

However "obvious" answer may be too quick........
Aspects of underdoped phenomenology (at not too low doping or temperature)

- Charge transport is by holes
- No magnetic long range order (AF LRO quickly destroyed by hole motion)
- Existence of spin gap
Perhaps useful to view as doped paramagnetic Mott insulator.

Further theoretical bonus: Superconductivity a natural outcome of doping paramagnetic Mott states

(old RVB notion – Anderson, Kivelson et al, Kotliar-Liu,………..)
View as doped paramagnetic Mott insulator (a very old idea actually)

Theoretical path

Path of real material

Para-magnet

AF

g

g = frustration/ring exchange, ....
Questions

1. How to sharpen?

3. What paramagnet to dope?

5. How to test?
How to sharpen?

- Useful to consider phase diagram as a function of chemical potential rather than doping

More generally a paramagnet
Theoretical suggestion
(implicit in much previous work)

• Physics at moderately low doping:

Influenced by proximity to chemical potential tuned Mott transition between spin liquid Mott insulator and dSc.
Doping induced Mott criticality from a spin liquid

Path of system at fixed doping

QC: "quantum critical" region of Mott transition.
FS: Fluctuating superconductor associated with $T > 0$ superconducting transition
Comparison to cuprate phase diagram

AF Mott insulator

Nernst region

FS: Nernst region
QC: ``High-T'' pseudo-gap region
What paramagnet to dope?

Theoretical candidates

3. Valence bond solid (spin Peierls) states

7. Various kinds of RVB spin liquids
What paramagnet? Some hints from experiments

• Softening of neutron resonance mode with decreasing $x$
  - consider paramagnets proximate to Neel state
  i.e potentially separated by 2\textsuperscript{nd} order transition.

• Gapless nodal quasiparticles in dSC
  - consider paramagnets with gapless spin excitations.

Tight constraints

=> Only few candidates: ``gapless spin liquids''
Theory of spin liquids
(enormous progress in last few years due to several people)

Spin liquid = translation invariant paramagnetic Mott state with one electron per unit cell.

Excitation spectrum of all known examples – describe in terms of spin-1/2 neutral spinons.

Specific examples of interest – spinons are gapless at 4 nodal points with linear dispersion

=> Very appealing starting points to dope to get dSc with nodal quasiparticles.
Theoretical characterization of spin liquids*

- Topological structure:
  Extra ‘topological’ conservation law not present in microscopic spin model.

Conveniently viewed as a conserved gauge flux.

Different classes of spin liquids distinguished by nature of gauge flux.

Spinons couple minimally to corresponding gauge field.

*abelian
Gauge theories and spin liquids

- Conserved gauge flux – important PHYSICAL property of excitation structure of spin liquid phase

Effective theory – `deconfined’ gauge theory

=> Gauge description not just a calculational device

Conserved gauge flux in spin liquid can in principle be detected by experiments.
Example of spin liquid with nodal spinons

Gapless $\mathbb{Z}_2$ spin liquid:

Conserved $\mathbb{Z}_2$ gauge flux (= `vison`).

Doping a $\mathbb{Z}_2$ spin liquid – attractive theory of cuprates but apparently not supported by experiments

(eg: no evidence for visons or their consequences – Bonn-Moler flux-trapping and other experiments).

Are there any other alternatives??
Alternate possibility: gapless U(1) spin liquids

- Affleck-Marston ’88: Flux phase of large-N spin models.

Mean field: Half-filled tight binding band of fermionic spinons (f) with a staggered flux through each plaquette (no real breaking of lattice symmetry)

Band structure: four gapless Fermi points

Low energies: massless Dirac theory in \( D = 2+1 \).
Beyond mean field

Describe by fermionic massless Dirac spinons coupled to compact U(1) gauge field.

Compactness: Allow for monopole events in space-time where the gauge flux changes by $2\pi$.

Ultimate fate?? Confinement??

- Doped versions: Lee, Nagaosa, Wen, .......
  (1996 - ....)

Mostly ignore possibility of confinement.
Monopole events irrelevant for low energy physics – at least within a systematic $1/N$ expansion ($N$: number of Dirac species)

Low energy theory is critical with no relevant perturbations (non-compact QED$_3$):

conformally invariant with power law spin correlations.

\[
\mathcal{L} = \bar{\psi}_j \gamma^\mu (\partial_\mu + ia_\mu) \psi_j + \frac{1}{8\pi e^2} (\epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda)^2.
\]
Physics of the gapless U(1) spin liquid

Monopoles irrelevant ~ "deconfined" spinons.

Precise meaning of "deconfinement": extra global topological U(1) symmetry associated with gauge flux conservation.

Power-law correlations in various physical quantities (staggered magnetization, VBS order, etc…).

Gauge flux conservation is physical – can in principle be measured!
Doping the U(1) spin liquid

A natural possibility [realized in slave boson mean field calculations (Lee, Nagaosa, Wen, ...)]

Doped holes spin-charge separate
⇒ Spin of holes carried away by spinons leaving behind spinless-charged bosons which condense to give dSc.

[Alternate:
Doped holes retain spin and charge
⇒ At low doping get exotic small Fermi surface metal violating conventional Luttinger theorem]
Decay routes of hole

- Both $f_{\uparrow}$ and $f_{\downarrow}$ operators create an up spin

=> Expect

\[
c_{\uparrow}^+ = b_1 f_{\uparrow}^+ + b_2 f_{\downarrow}
\]

$b_{1,2} =$ spin $-0$ bosons with physical charge $e$ but with opposite gauge charges $\pm 1$.

$b_1 b_2 =$ gauge neutral spin $-0$ boson with physical charge $2e$

= identify with Cooper pair.
Physics at finite doping

- Superconductivity: Both bosons condense (=> physical Cooper pair is condensed).

- Nernst region: Bosons have local amplitude but phase fluctuates.

- High-T pseudogap: Bosons above their `degeneracy’ temperature (=> `incoherent’)

Is all this really correct?

Experimental tests

• Crucial ingredient

Conservation of gauge flux of undoped spin liquid approximately true at finite-T in doped normal state; justifies use of slave particle degrees of freedom.

=> Crucial experiment: directly detect the gauge flux.
An idea for a gauge flux detector

TS, Lee, cond-mat/0406066

Cuprate sample with spatially modulated doping as below
Gauge flux detection

• Start with outer ring superconducting and trap an odd number of $hc/2e$ vortices (choose thin enough so that there is no physical flux).

• Cool further till inner annulus goes superconducting.

• For carefully constructed device will spontaneously trap $hc/2e$ vortex of either sign in inner annulus.
How does it work?

• Odd $hc/2e$ vortex inside outer ring $\Rightarrow \pi$ flux of internal gauge field spread over the inner radius.  
  
  (Lee, Wen, 2001)

• If inner annulus sees major part of this internal flux, when it cools into SC, it prefers to form a physical vortex.

• For best chance, make both SC rings thinner than penetration depth and device smaller than roughly a micron.
Other possible tests

- Spin physics in high-T pseudogap region expected to be only weakly modified from undoped spin liquid

=> Approximate characteristic finite-T scaling in number of measurable correlators (eg: in $(\pi,\pi)$ spin response)
Summary-I

• View of underdoped cuprates as doped Mott paramagnets very appealing starting point.

• Experiments tightly constrain nature of paramagnet to dope.

Current understanding of Mott paramagnet allows for only one surviving candidate – the gapless U (1) spin liquid
Summary-II

• Gapless U(1) spin liquids potentially stable in two dimensions, have low energy “gauge fluctuations” characterized by extra conserved quantity (gauge flux) not present in microscopic model.

⇒ Gauge flux is physical.

Doped version: Proposal for experiment to detect gauge flux.

Large number of open questions
Some open questions-I
Phenomenology

• **Description of location of nominal “Fermi surface”?**
  (Why does leading edge more or less match band theory? How does node of dSc move away from spinon node at \((\pi/2, \pi/2)\)?)

• **Description of Fermi arcs?**
  (Early crude attempt (Wen, Lee) manages to get Fermi pocket with \(Z\) small in back portion)

• **Velocity anisotropy of nodal quasiparticles?**
  (At spin liquid fixed point, expect no velocity anisotropy for spinons – can this evolve into large anisotropy seen in dSc?)

• **Slope of penetration depth versus \(T\) inside dSc?**
  (Simplest calculations: slope \(~ x^2\) in disagreement with expt on moderately underdoped samples).
Some open questions-II
Basic theory

2. Stability of U(1) spin liquid at half-filling established for large enough $N > N_c$. Is $N_c < 2$ so that SU(2) spin models have such phases?
(Current numerical evidence: $N_c$ at least $< 4$)

Needed: numerics to determine $N_c$.

What if $N_c > 2$ – can theory be salvaged? (see TS, Lee, cond-mat/0406066 for a suggestion)

2. Better theoretical control on charge physics in doped spin liquids.

3. Better microscopic understanding for why doping might push spins into spin liquid state.
General lesson I

• Stable gapless U(1) spin liquids exist in $D = 2+1$ (at least for SU($N$) models and $N >$ some $N_{c1}$).

$N_{c1}$ possibly smaller than 2, not known at present*.

$N_{c1} < 2 \Rightarrow$ appealing description of cuprates as doped U(1) spin liquids.

*Indications from numerics: $N_{c1} < 4$ (Assaad, cond-mat/0406...)
Second order transition to Neel
(induced by increasing strength of quartic spinon interaction)

- Spin density wave of spinons
- Monopoles continue to be irrelevant at critical point to Neel.
- Spinons gapped in Neel phase => monopoles no longer irrelevant, cause confinement to yield conventional Neel state.
- Deconfined critical point with dangerous irrelevant monopoles, 2 diverging length scales, etc.
Summary, conclusions, etc - I

• Gapless spin liquids exist as stable phases in $D = 2+1$.

They may be accessed from conventional Neel by second order transitions.

Needed: Numerics to determine $N_{c1}, N_{c2}$
Summary, conclusions -II

- U(1) SL with "gapless Dirac spinons" apparently plays an important role whether it is stable or not.

Are the cuprates doped U(1) spin liquids?

How to tell?

Detect conserved U(1) gauge flux!