Deconfined quantum criticality

T. Senthil (MIT)

P. Ghaemi, P. Nikolic, M. Levin (MIT)
M. Hermele (UCSB)

O. Motrunich (KITP), A. Vishwanath (MIT)
Competing orders and non-fermi liquids in correlated systems
``Classical” assumptions

1. NFL: Universal physics associated with quantum critical point between phases A and B.

3. Landau: Universal critical singularities ~ fluctuations of order parameter for transition between phases A and B.

Try to play Landau versus Landau.
Example 1: Cuprates

![Diagram showing a phase diagram with axes T and x, labeled regions include AF Mott insulator, Pseudo gap, NFL metal, dSc, and Fermi liquid.](image-url)
Example 2: Magnetic ordering in heavy electron systems

$\text{CePd}_2\text{Si}_2$, $\text{CeCu}_{6-x}\text{Au}_x$, $\text{YbRh}_2\text{Si}_2$, …

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(Radical) alternate to classical assumptions

- Universal singularity at some QCPs: Not due to fluctuations of natural order parameter but due to some other competing effects.

- Order parameters/broken symmetries of phases A and B mask this basic competition.

=>$ Physics beyond Landau-Ginzburg-Wilson paradigm of phase transitions.
Example 1: Cuprates

Mott insulator

NFL metal

Electrons delocalized
Example 1: Cuprates

- Competition between Fermi liquid and Mott insulator
- Low energy order parameters (AF, SC, ...) mask this competition.
Similar possibility in heavy electron systems

Critical NFL physics: fluctuations of loss of local moments from Fermi sea? Magnetic order – a distraction??

Local moments not involved in Fermi sea?

Metal: Local moments part of Fermi sea

AFM

NFL
This talk – more modest goal

- Are there any clearly demonstrable theoretical instances of such strong breakdown of Landau-Ginzburg-Wilson ideas at quantum phase transitions?
This talk – more modest goal

• Are there any clearly demonstrable theoretical instances of such strong breakdown of Landau-Ginzburg-Wilson ideas at quantum phase transitions?

Study phase transitions in insulating quantum magnets
- Good theoretical laboratory for physics of phase transitions/competing orders.
Highlights

- Failure of Landau paradigm at (certain) quantum transitions

- Emergence of `fractional’ charge and gauge fields near quantum critical points between two CONVENTIONAL phases.
  - ``Deconfined quantum criticality’’ (made more precise later).

- Many lessons for competing order physics in correlated electron systems.
Phase transitions in quantum magnetism

\[ H = J \sum_{<rr'>} S_r \cdot S_{r'} + \ldots \]

- Spin-1/2 quantum antiferromagnets on a square lattice.

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Possible quantum phases

• Neel ordered state
Possible quantum phases (contd)

QUANTUM PARAMAGNETS

• Simplest: Valence bond solids.

• Ordered pattern of valence bonds breaks lattice translation symmetry.

• Elementary spinful excitations have $S = 1$ above spin gap.
Possible phases (contd)

• Exotic quantum paramagnets – “resonating valence bond liquids”.

• Fractional spin excitations, interesting topological structure.
Neel-valence bond solid (VBS) transition

- Neel: Broken spin symmetry
- VBS: Broken lattice symmetry.

- Landau – Two independent order parameters.
  - no generic direct second order transition.
  - either first order or phase coexistence.

This talk: Direct second order transition but with description not in terms of natural order parameter fields.
Broken symmetry in the valence bond solid (VBS) phase

Valence bond solid with spin gap.

\[
= \left( \begin{array}{c}
\uparrow \\
\downarrow \\
\end{array} \right) / \sqrt{2}
\]
Discrete $\mathbb{Z}_4$ order parameter
Neel-Valence Bond Solid transition

- Naïve approaches fail
  Attack from Neel ≠ Usual O(3) transition in D = 3
  Attack from VBS ≠ Usual $Z_4$ transition in D = 3
  (= XY universality class).

Why do these fail?
Topological defects carry non-trivial quantum numbers!

This talk: attack from VBS (Levin, TS, cond-mat/0405702)
Topological defects in $\mathbb{Z}_4$ order parameter

- Domain walls – elementary wall has $\pi/2$ shift of clock angle
$Z_4$ domain walls and vortices

- Walls can be oriented; four such walls can end at a point.
- End-points are $Z_4$ vortices.
$Z_4$ vortices in VBS phase

Vortex core has an unpaired spin-1/2 moment!!

$Z_4$ vortices are ``spinons''.

Domain wall energy confines them in VBS phase.
Disordering VBS order

- If $Z_4$ vortices proliferate and condense, cannot sustain VBS order.

- Vortices carry spin $\Rightarrow$ develop Neel order
$Z_4$ disordering transition to Neel state

- As for usual (quantum) $Z_4$ transition, expect clock anisotropy is irrelevant.
  (confirm in various limits).

Critical theory: (Quantum) XY but with vortices that carry physical spin-1/2 (= spinons).
Alternate (dual) view

- Duality for usual XY model (Dasgupta-Halperin)
  Phase mode - ``photon”

Vortices – gauge charges coupled to photon.

Neel-VBS transition: Vortices are spinons
=> Critical spinons minimally coupled to fluctuating U(1) gauge field*.

*non-compact
Proposed critical theory
``Non-compact CP\textsubscript{1} model''

\[ S = \int d^2x d\tau \left| (\partial_\mu - i a_\mu) z \right|^2 + r \left| z \right|^2 + u \left| z \right|^4 \]
\[ + \left( \epsilon^{\mu \nu} \partial_\nu a_\lambda \right)^2 \]

\( z = \) two-component spin-1/2 spinon field

\( a_\mu = \) non-compact U(1) gauge field.

Distinct from usual O(3) or Z\textsubscript{4} critical theories.

Theory not in terms of usual order parameter fields but involve spinons and gauge fields.
Renormalization group flows

Clock anisotropy

Deconfined critical fixed point

Clock anisotropy is "dangerously irrelevant".
Precise meaning of deconfinement

- $\mathbb{Z}_4$ symmetry gets enlarged to $\mathbb{X}\mathbb{Y}$

$\Rightarrow$ Domain walls get very thick and very cheap near the transition.

$\Rightarrow$ Domain wall energy not effective in confining $\mathbb{Z}_4$ vortices (= spinons).

Formal: Extra global $U(1)$ symmetry not present in microscopic model.
Two diverging length scales in paramagnet

``Critical” \( \xi \) `` spin liquid” \( \xi_{\text{VBS}} \) VBS L

\( \xi \): spin correlation length
\( \xi_{\text{VBS}} \): Domain wall thickness.

\( \xi_{\text{VBS}} \sim \xi^\kappa \) diverges faster than \( \xi \)

Spinons confined in either phase but `confinement scale’ diverges at transition.
Extensions/generalizations

• Similar phenomena at other quantum transitions of spin-$1/2$ moments in $d = 2$
  (VBS- spin liquid, VBS-VBS, Neel – spin liquid, …)

Apparently fairly common

• Deconfined critical phases with gapless fermions coupled to gauge fields also exist in 2d quantum magnets (Hermele, Senthil, Fisher, Lee, Nagaosa, Wen, ‘04)
  - interesting applications to cuprate theory.
Summary and some lessons-I

- Direct 2nd order quantum transition between two phases with different broken symmetries possible.

Separation between the two competing orders not as a function of tuning parameter but as a function of (length or time) scale.

Loss of magnetic correlations

Onset of VBS order

``Critical” ξ “spin liquid” ξ_{VBS} VBS L
Summary and some lessons-II

- Striking ``non-fermi liquid” (morally) physics at critical point between two competing orders.

  Eg: At Neel-VBS, magnon spectral function is anomalously broad (roughly due to decay into spinons) as compared to usual critical points.

Most important lesson:

Failure of Landau paradigm – order parameter fluctuations do not capture true critical physics.

Strong impetus to radical approaches to NFL physics at heavy electron critical points (and to optimally doped cuprates).