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Traffic Flow Theory

Macroscopic perspective on traffic flow fundamentals

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1.041/1.200/11.544 Transportation: Foundations and Methods

References

- Many slides adapted from Carolina Osorio
- Chap 4. of Prof. Carlos Daganzo's book *Fundamentals of Transportation and Traffic Operations* (2007)
  Available online via MIT Librairies Catalog: Barton
- Prof. Nikolas Geroliminis' lecture *Fundamentals of Traffic Operations and Control*, Spring 2010 EPFL
- Lecture notes: *Principles of Highway Engineering and Traffic Analysis* (2009) by Fred Mannering, Scott Washburn and Walter Kilareski
- Lecture notes: 9^{th} *Dynamic Traffic Flow Modeling and Control* (2010) by Prof. Markos Papageorgiou
- Fred Hall (1997) *Traffic stream characteristics*. (available online)

Outline

1. Motivation
2. Main traffic stream variables, data collection
3. Basic relationship
4. Fundamental diagrams (FDs)
5. FDs versus time-space diagrams
6. Highway delay problem

Introduction

- **Traffic flow**: study of the movement of individual drivers and vehicles between two points and the interactions they make with one another.
- Also called traffic streams. Overlap in techniques used to study fluid mechanics (e.g. partial differential equations).
- **Traffic flow theory**: models and hypotheses for explaining traffic flow, i.e. what would happen to traffic streams if they were to flow on roads under different conditions, potentially not yet observed
- A better understanding of traffic flow will enable us to:
  - design roads with improved level of service
  - improve the performance of existing transportation systems (e.g. operations)
  - understand how the system might respond to potential engineering changes
- Introduction to main traffic stream variables used to describe such systems

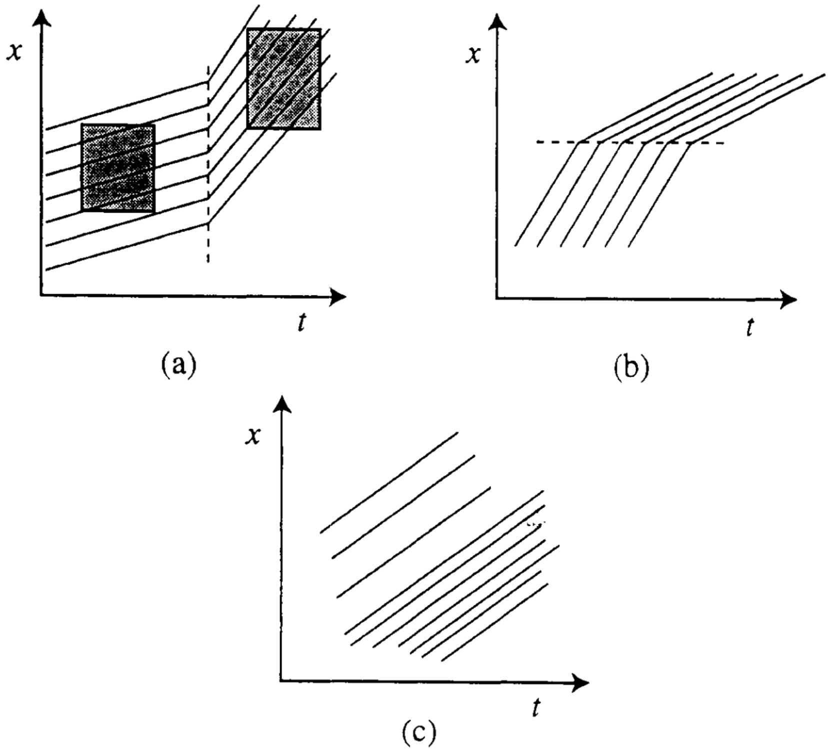
Context and assumptions

1. Study of a single traffic stream, flowing on a facility with a single entrance and a single exit
2. Uninterrupted traffic
   - Traffic regulated by interactions between vehicles, as opposed to being regulated by external means
   - E.g. on a highway or at unsignalized intersections, as opposed to traffic lights, stop signs.
3. Stationary traffic conditions (vs. time and space-varying dynamics)

Non-stationary traffic

Stationary traffic conditions (vs. time and space-varying dynamics):

- You cannot get any information as to what time it is or where you are by inspecting the time-space diagram through a small window in a template, regardless of where it is placed
- Traffic is stationary if it is a superposition of families of trajectories that are each parallel and equidistant.



Outline

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2. **Main traffic stream variables, data collection**
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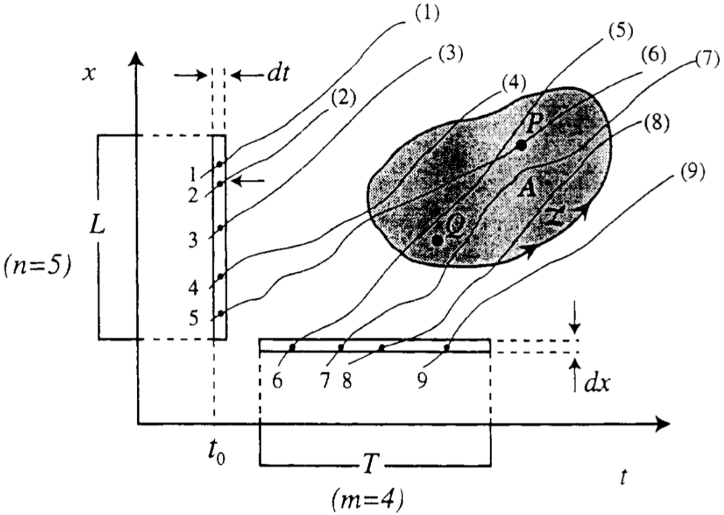
Traffic stream variables

- Main variables
  - Flow
  - Time headway
  - Density
  - Spacing
  - Speed (space-mean, time-mean)



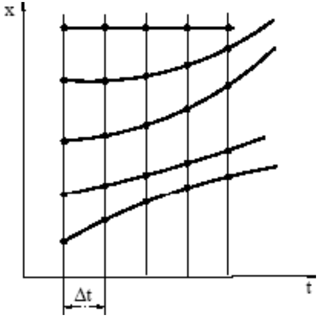
- Obtain relationships that hold "on average"; i.e. for large stationary time-space regions containing many vehicles

Time and space means

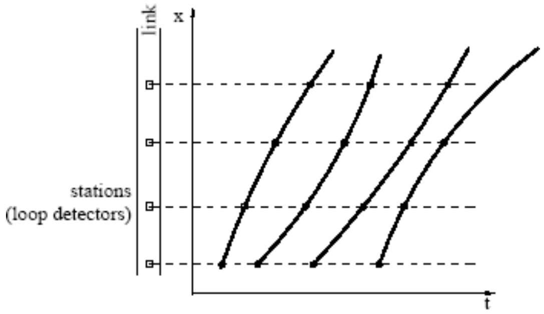
- Space-mean: averages taken at an instant over a space interval
- Time-mean: averages taken at a specific location (with time-varying over an interval)
- Speed:
  - $\bar{v}_s$: space-mean speed
  - $\bar{V}_t$: time-mean speed

Recall (time-space diagrams):

1) Aerial surveys



2) Traffic detectors



- Other *vehicle characteristics* can be averaged across space or time. E.g., occupancies (number of persons per vehicle), energy consumption, emissions, etc.
- There is no a priori reason to expect averages taken across space or time to be the same.
- You own two cars, they are both driven an equal distance of 100 miles. One gets 20 miles per gallon (mpg), the other 50 mpg. Is the average mpg 35 (i.e. $\frac{50+20}{2}$)?

Time and space averages

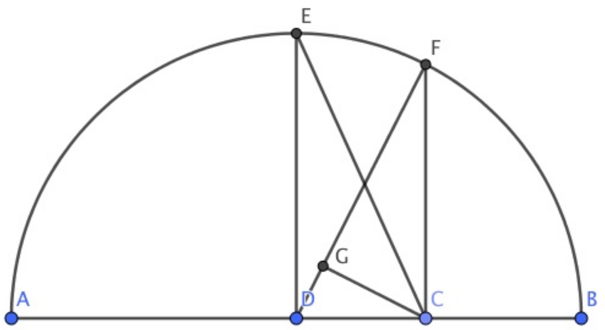
- Typically $\bar{v_t} \geq \bar{v_s}$
- Which is the average to use? It depends on the problem at hand.
- Space-mean speed is more useful in the context of traffic analysis and is determined on the basis of the time necessary for a vehicle to travel some known length of roadway.
- Time-mean speed good for?

The $n = 2$ case [edit]

When n=2, the inequalities become

$$\frac{2}{\frac{1}{x_1}+\frac{1}{x_2}} \leq \sqrt{x_1 x_2} \leq \frac{x_1+x_2}{2} \leq \sqrt{\frac{x_1^2+x_2^2}{2}}$$ for all $x_1, x_2 > 0$, which can be visualized in a semi-circle whose diameter is [*AB*] and center *D*.

Suppose $AC=x_1$ and $BC=x_2$. Construct perpendiculars to [*AB*] at *D* and *C* respectively. Join [*CE*] and [*DF*] and further construct a perpendicular [*CG*] to [*DF*] at *G*. Then the length of *GF* can be calculated to be the harmonic mean, *CF* to be the geometric mean, *DE* to be the arithmetic mean, and *CE* to be the quadratic mean. The inequalities then follow easily by the Pythagorean theorem.



Ref: HM-GM-AM-QM inequalities, Wikipedia

Time and space averages

Table 4.1.. Generalized formulas for various traffic characteristics using two observation methods. Boxed expressions correspond to the original definitions introduced in Chapter 1:

	Method of Observation	
	Instantaneous photograph (section length, L)	**Observation from a fixed location (duration, T)**
Density, k(**A**)	$\boxed{n/L}$	$\frac{1}{T}\sum_{j=1}^{m} p_j$
Flow, q(**A**)	$\frac{1}{L}\sum_{i=1}^{n} v_i$	$\boxed{m/T}$
Space-mean speed, v(**A**)	$\boxed{\frac{1}{n}\sum_{i=1}^{n} v_i}$	$[\frac{1}{m}\sum_{j=1}^{m} p_j]^{-1}$
Time-mean speed	$\sum_{i=1}^{n} u_i^2 \Big/ \sum_{i=1}^{n} u_i$	$\underline{\frac{1}{m}\sum_{i=1}^{m} v_i}$

<u>Notation:</u>

v_i = velocity (mi/hr)

p_i = pace (min/mi), $1/v_i$

$u_i == v_i$

- If traffic is stationary, then the definitions of the 1st column are equivalent to those of the 2nd column (i.e. same values).
- So, for instance, density (conventional definition) can be measured by counts at a given location, i.e. density can be obtained from easily observable data.

Source: C. Daganzo

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4. Fundamental diagrams (FDs)
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Basic relationship

$$q = v_s k$$

where:

- q: flow [veh/h]
- v_s: speed (space-mean speed) [mi/h] or [km/h]
- k: density [veh/mi] or [veh/km].

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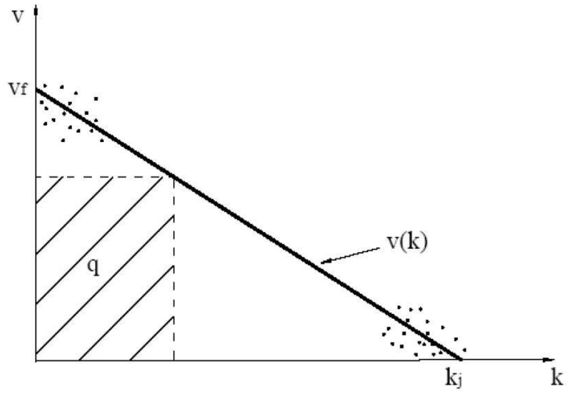
Traffic stream models

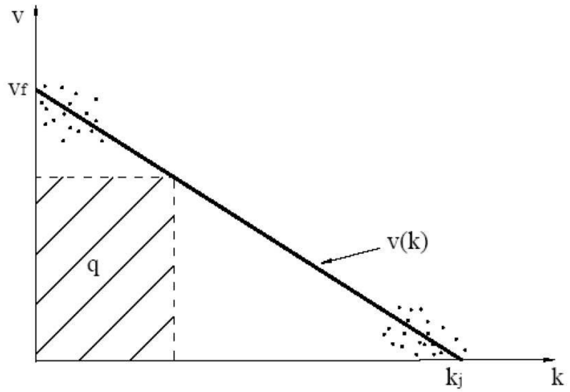
(1) Speed-density model

- Greenshields (1935), seminal work, assumed a linear relationship between speed and density
- From experimental data:
  - Light traffic → high speed
  - Heavy traffic → low speed (near zero).



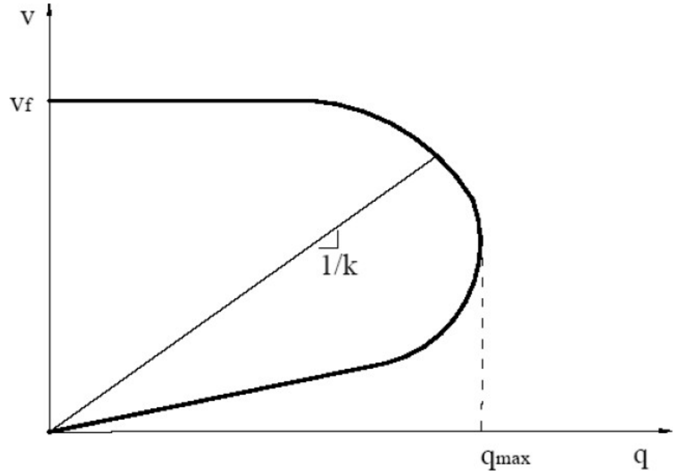
Traffic stream models

- The diagram is a property of the specific road
- Points on the diagram describe possible traffic conditions
- These relationships are postulated to be true “on average”



Traffic stream models

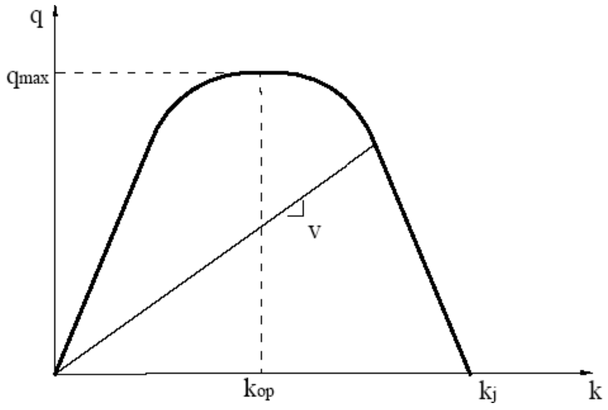
(2) Speed-flow model



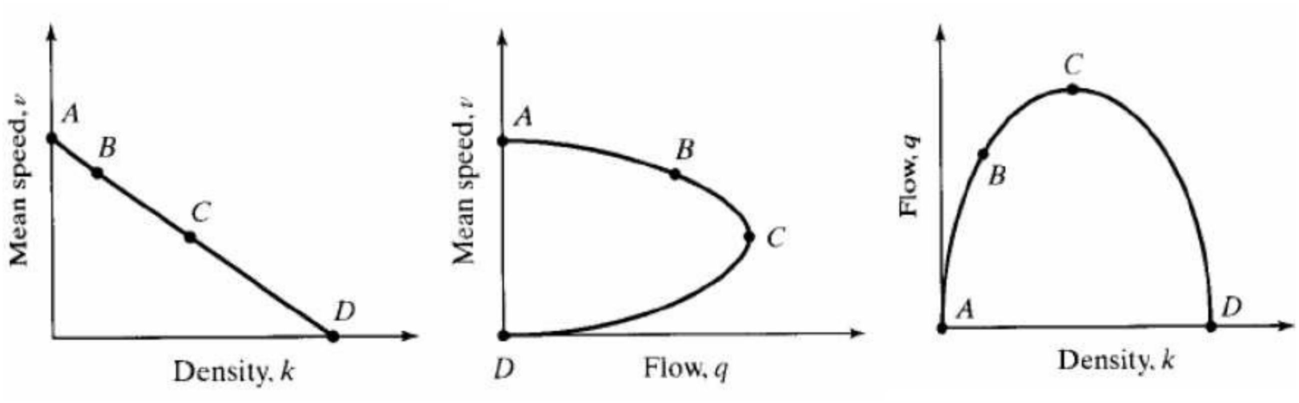
Traffic stream models

(3) Flow-density model

- v_f : free flow speed
- k_{cap} = k_j : jam density
- q_{cap} = q_{max}: capacity (maximum flow)

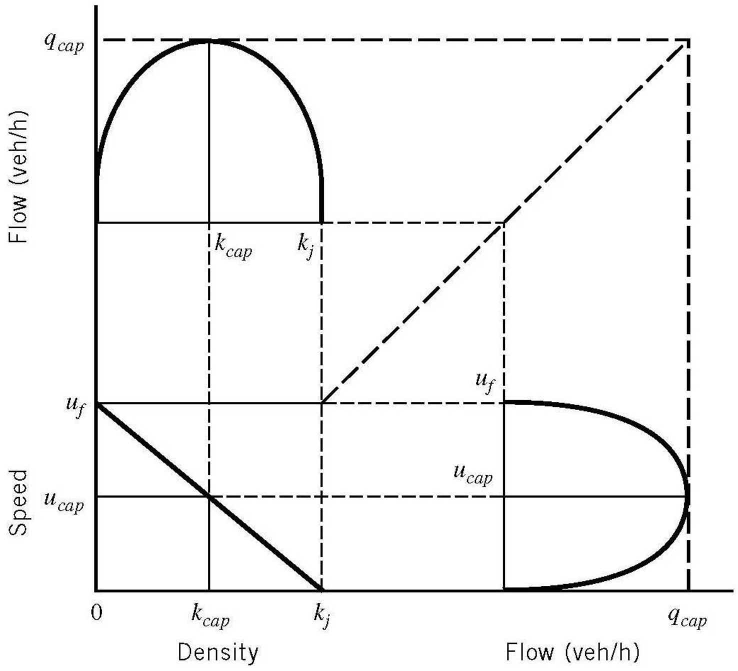


Traffic stream models



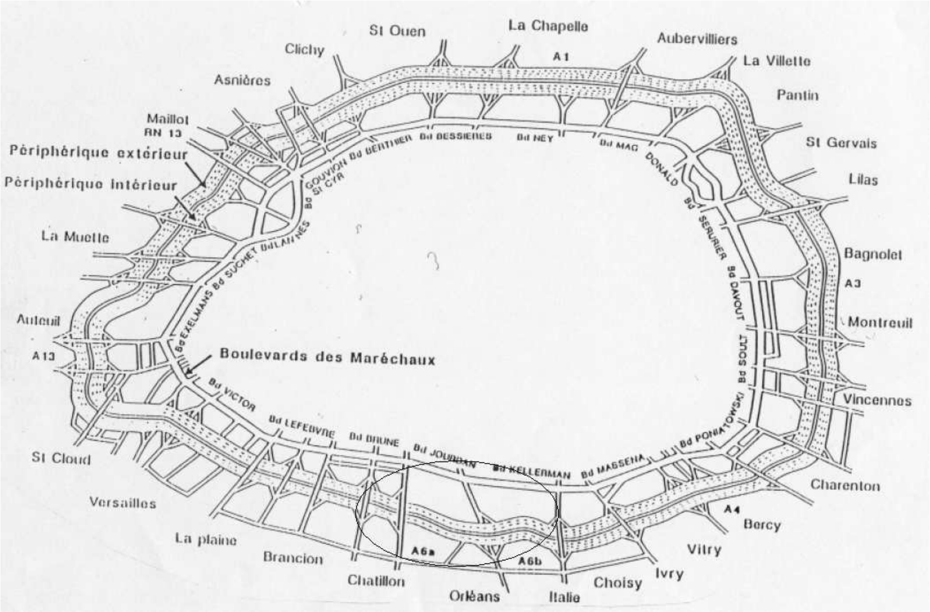
Traffic stream models

- Each diagram relates the three fundamental variables, and are therefore called fundamental diagrams.
- For a given road, a fundamental diagram is fitted based on measurements



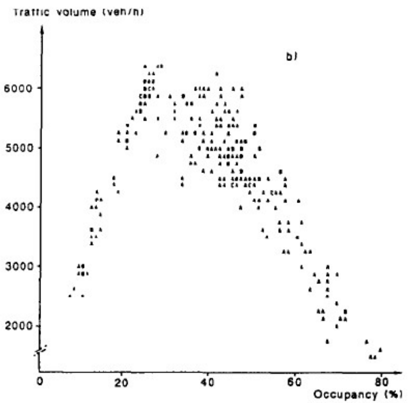
Fundamental Diagram in practice

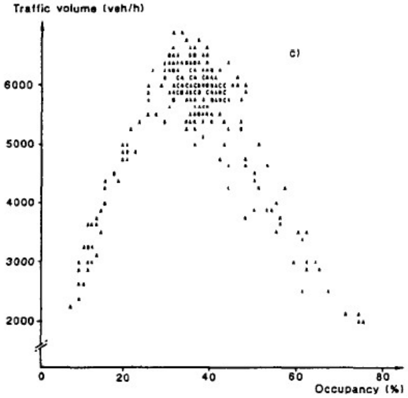
- Paris: boulevard periphérique
- Three locations (detectors)
- Measurements of car passages (traffic counts) and occupancies for each detector over a chosen time interval
- Source: Papageorgiou et al. (1990) Modelling and real-time control of traffic flow on the southern part of Boulevard Peripherique in Paris: Part I: Modelling, Trans. Res. Part A

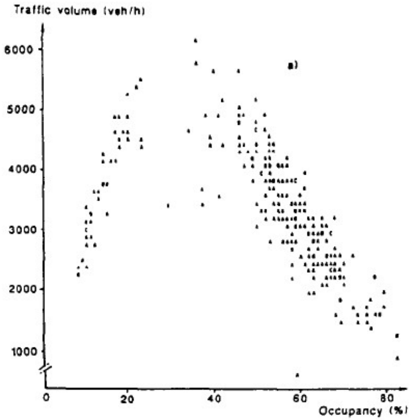


Fundamental Diagram in practice

- Flow-density model
- Measurements of car passages (traffic counts) and occupancies for each detector over time interval: 6am-10am of Nov. 27 1987
- Flow-density (volume-occupancy) diagrams taken over one-minute-intervals for a given location
- Traffic Occupancy (in %) (Def.: % of loop occupancy in a given time period)
- Conditions: heavily congested traffic after 7:40am, rainy

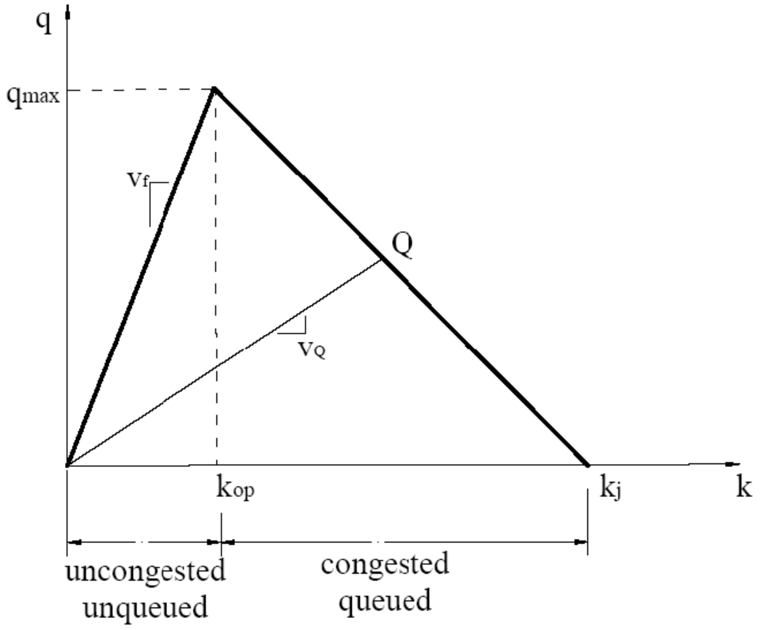






Traffic stream models

(3) Flow-density model



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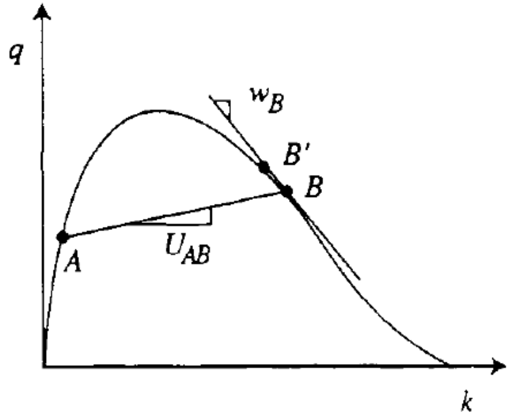
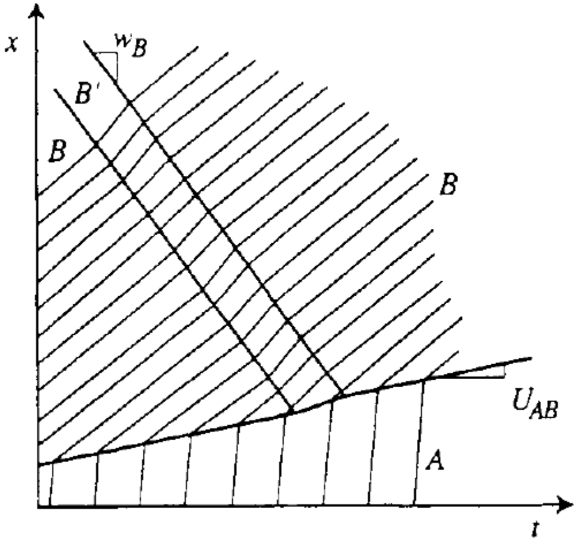
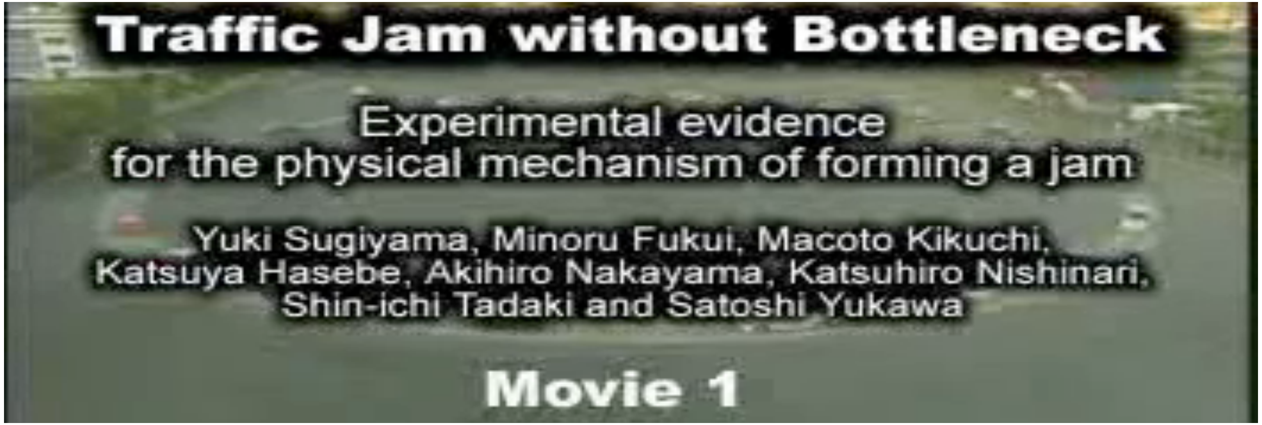
Fundamental diagram - (t,x) diagram



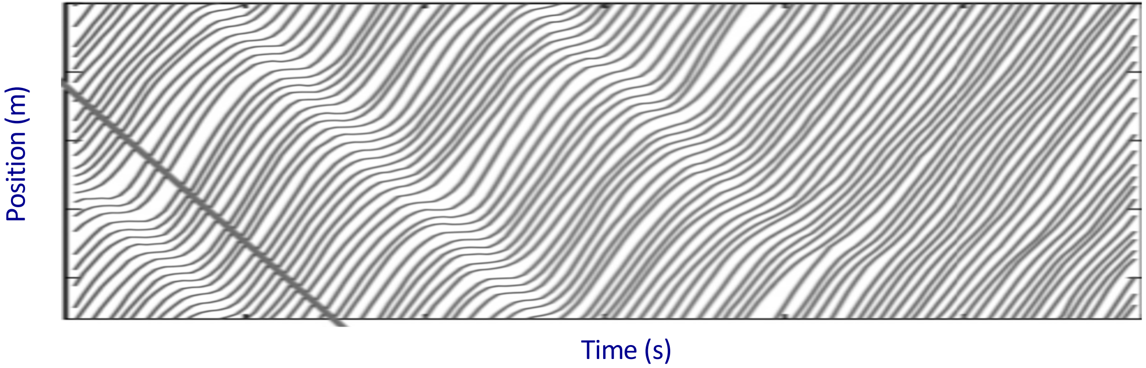
Source: C. Daganzo (1997)



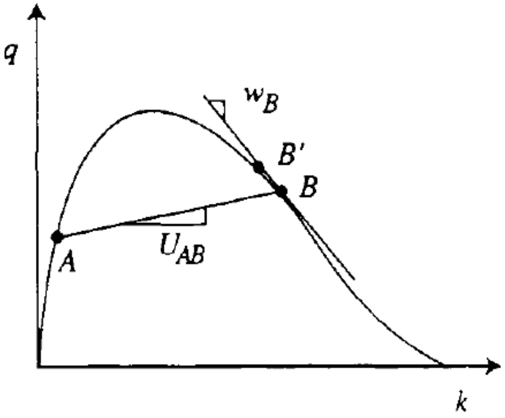
Recall: traffic waves



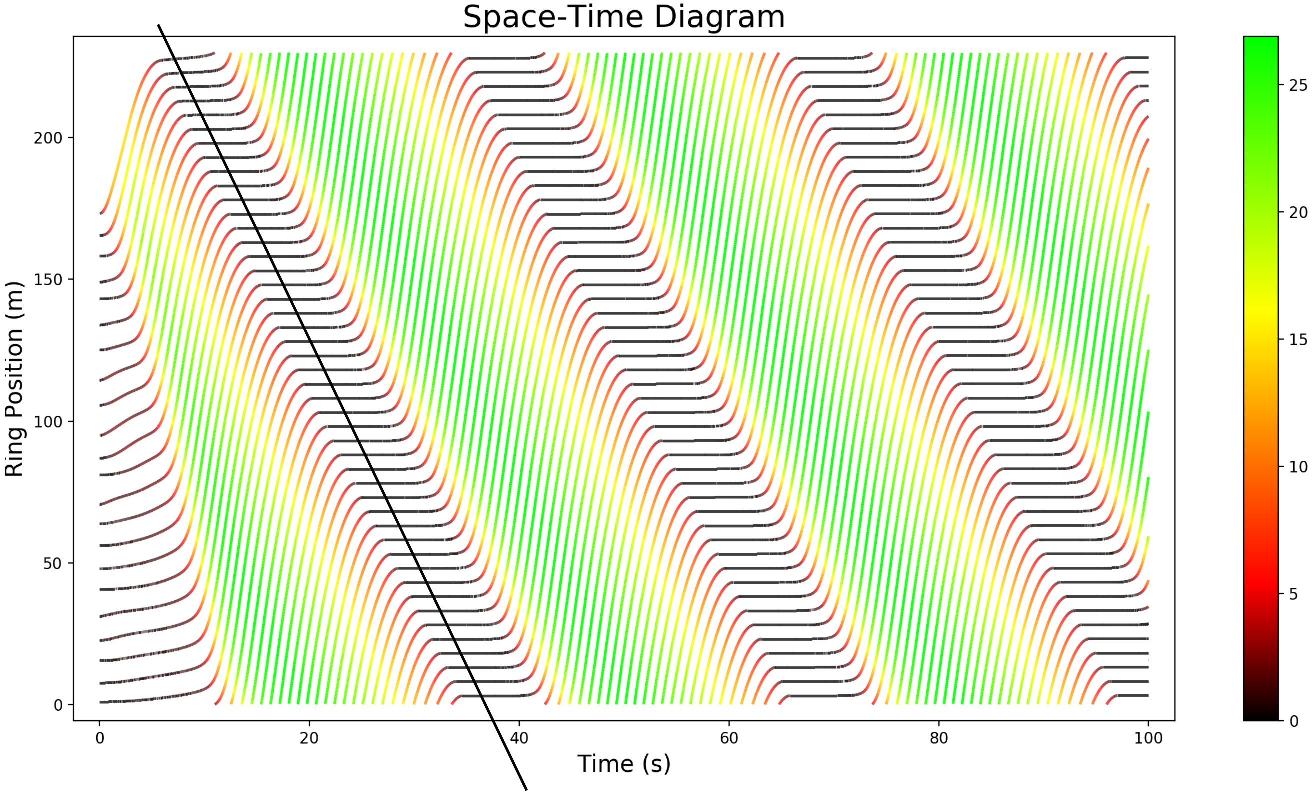
Vehicle trajectories (Sugiyama et al. 2008)



Traffic waves: fundamental diagram characterization



Source: C. Daganzo (1997)



Rate of backwards wave propagation

Wu, et al. IEEE T-RO, in review.

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Highway delay problem

- A freeway exhibits a triangular flow-density relation with parameters: v_f (free flow speed), q_{max} (capacity) and k_j (jam density)
  1. Plot the fundamental diagram and derive an expression for the function that gives the (space-mean) speed as a function of density inside a queue. Don't forget to specify the range of k for which the equation holds.
  2. If $k_j = 600$ veh/mile, $v_f = 1$ mile/min and $q_{max} = 100$ veh/min. Determine the delay experienced by a vehicle that joins a 2 mile queue caused by a bottleneck that flows at $q = 50$ veh/min.

Source: C. Daganzo