Graphical analysis II

Cumulative diagrams

Cathy Wu

1.041/1.200 Transportation: Foundations and Methods

References

1. Readings: Chap 2. of Prof. Carlos Daganzo's book Fundamentals of Transportation and Traffic Operations (2007)

2. Prof. Nikolas Geroliminis' lecture Fundamentals of Traffic Operations and Control, Spring 2010 EPFL

3. Some slides adapted from Profs. Carolina Osorio and Dan Work.

Outline

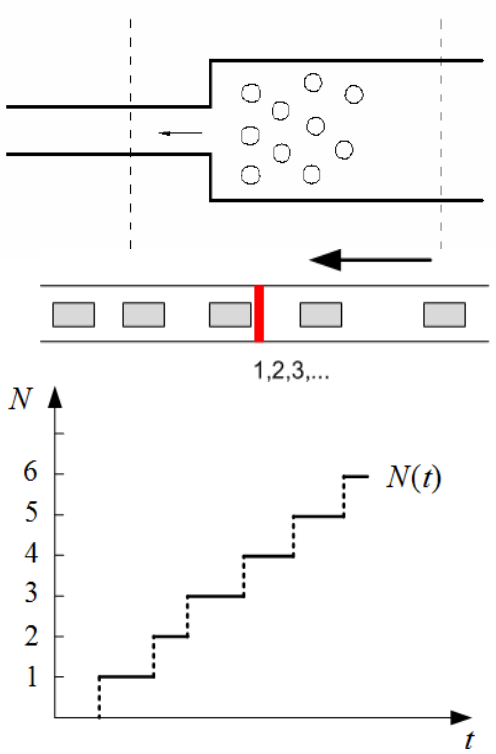
1. Cumulative diagrams
2. Reconstructing cumulative diagrams
3. Ramp metering problem

Outline

1. **Cumulative diagrams**
    a. **Visualizing queues and delays**
    b. Cumulative diagrams vs. time-space diagrams
    c. Working with cumulative diagrams
    d. Analysis & interpretation
    e. Little's Law
    f. Application: Signalized intersections
2. Reconstructing cumulative diagrams
3. Ramp metering problem

Cumulative diagram

- Cumulative diagrams help us analyze delay in transportation systems.
- The “cumulative count curve” started from a basic technique known as ‘mass curve analysis’ in hydrologic synthesis
- Introduced to transportation by Moskowitz (1954)[2] and Gazis and Potts (1965)[3]; Gordon Newell (1971,1982,1993)[1,4,5] demonstrated their full potential.
- The following diagram represents the cumulative number (count) of arrivals at a fixed location
- Cumulative diagram also called **cumulative count curve, N-curve, queueing diagram**

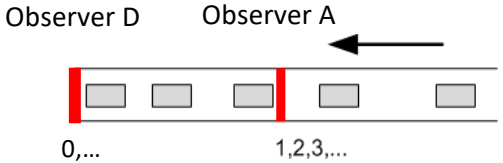


1. Newell GF. Applications of queueing theory (2nd edition). Chapman and Hall London, 1982.
2. Moskowitz, K. Waiting for a gap in a traffic stream. Proc. Highway Res. Board, 33, 385-395, 1954.
3. Gazis, D.C. and R.B. Potts (1965). The over-saturated intersection. Proc. 2nd Int. Symp. on the Theory of Road Traffic Flow.
4. Newell, G.F. Applications of queueing theory, Chapman Hall, London, 1971.
5. Newell, G.F. A simplified theory of kinematic waves in highway traffic, I general theory, II queuing at freeway bottlenecks, III multi-destination flows. Transportation Research Part B, 1993.

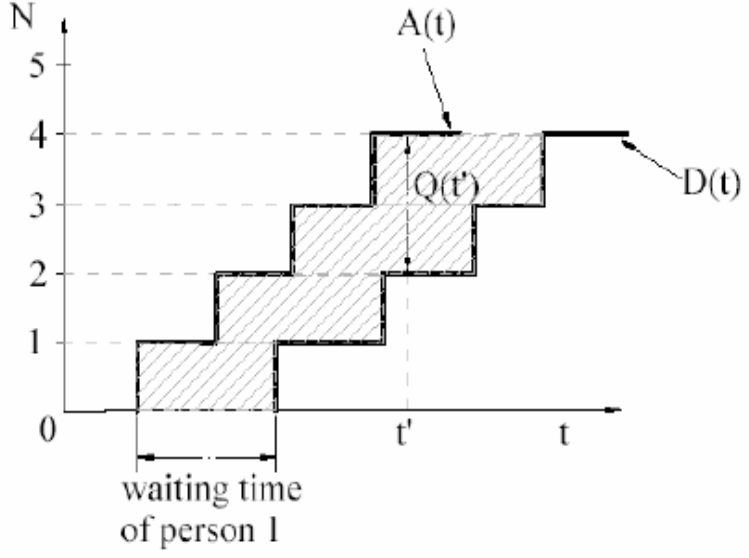
Slide adapted from Prof. Zhengbing He

Cumulative diagram: illustrating system delays

- We can use two observers:
  - Observer A looks at the arrivals to the system
  - Observer D looks at the departures from the system



- The following diagram represents the cumulative number of **arrivals and departures**
- Assumptions: FIFO, no passing / reordering



- $Q(t) =$ number of customers in queue at time t
- $w(n) =$ waiting time of person n
- $A_n =$ time of arrival of the nth customer
- $D_n =$ time of departure of the nth customer
- Area = total wait time in the system

Cumulative diagrams - Applications

- Mainly used to evaluate waiting time and queue lengths.
- Applications of this technique appear in all transportation modes.
- Examples:
  - Predict queues caused by scheduled maintenance or accidents
  - Design a suitable “waiting area”: e.g. length of an on-ramp
  - Control purposes: e.g. determine suitable traffic signal plans

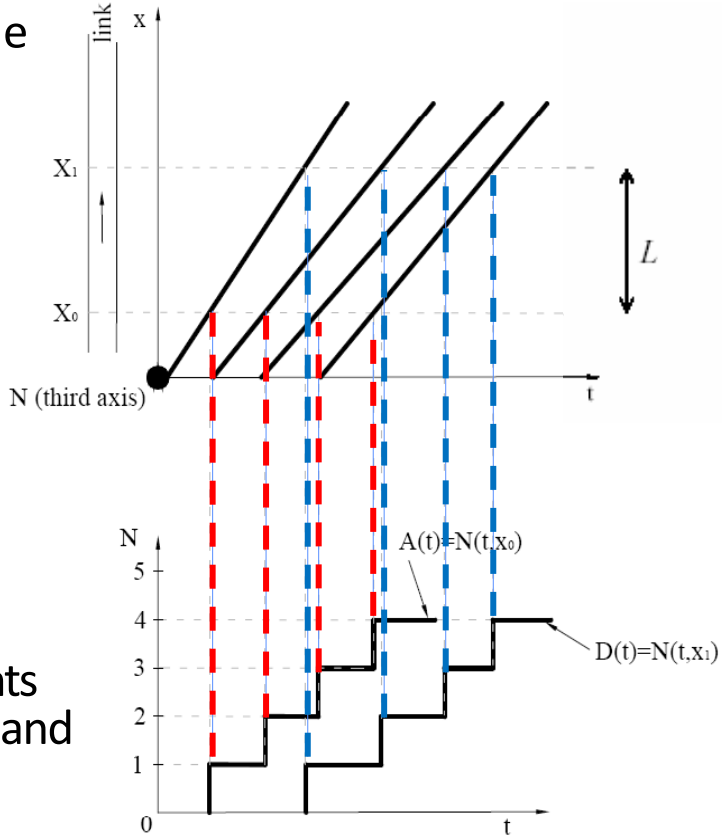
Outline

1. **Cumulative diagrams**
   a. Visualizing queues and delays
   b. Cumulative diagrams vs. time-space diagrams
   c. Working with cumulative diagrams
   d. Analysis & interpretation
   e. Little's Law
   f. Application: Signalized intersections
2. Reconstructing cumulative diagrams
3. Ramp metering problem

Cumulative diagram vs. time space diagram

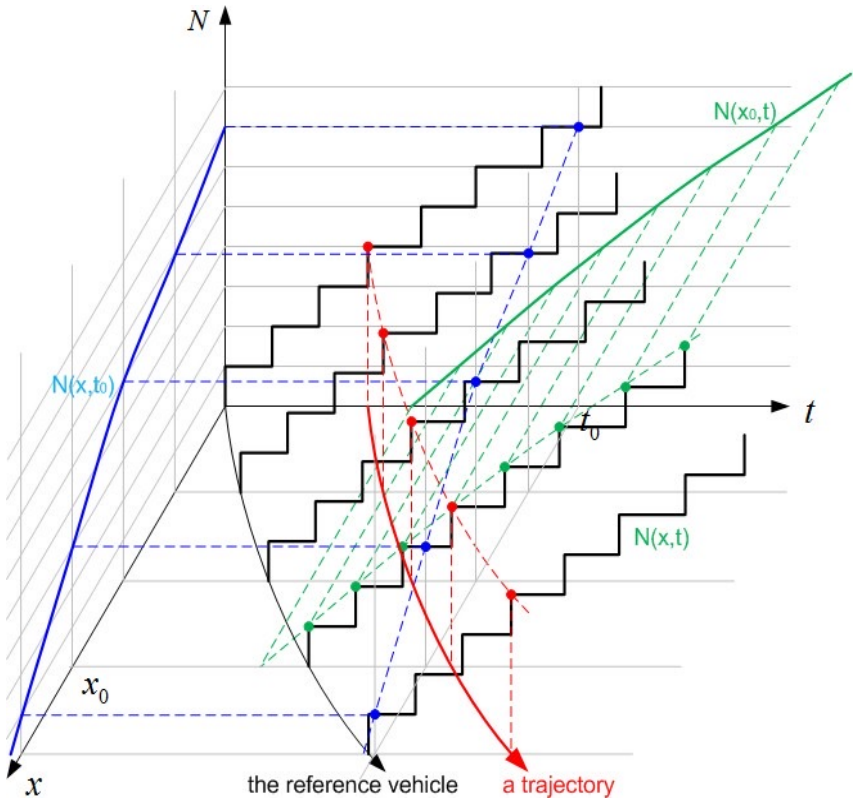
- Cumulative diagram can be obtained from the time-space diagram
- Time space diagram has complete info
- Cumulative diagram has a subset of the info
- Note:
  Slope of cumulative diagram = flow

- Use cases:
  - **Time-space diagram**: when depicting more than one vehicle trajectory;
  - **Cumulative diagram**: when depicting aggregations of multiple vehicles at fixed points (bottlenecks), and in particular the 'arrival' and 'departure' curves at a single bottleneck.



Three-dimensional N-curve: N(x, t)

- **Flow** at x: $q(x) = \frac{\partial N(x,t)}{\partial t}$
- **Density** at t: $k(t) = \frac{\partial N(x,t)}{\partial x}$



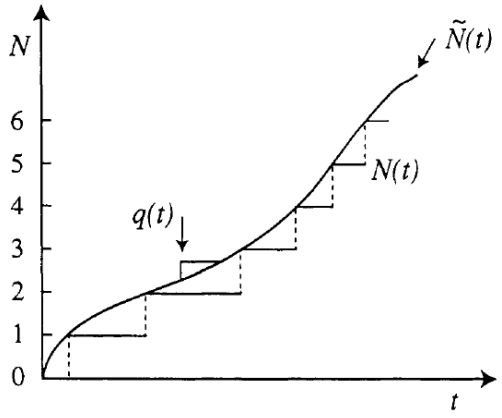
Slide adapted from Prof. Zhengbing He

Outline

1. **Cumulative diagrams**
    a. Visualizing queues and delays
    b. Cumulative diagrams vs. time-space diagrams
    c. Working with cumulative diagrams
    d. Analysis & interpretation
    e. Little's Law
    f. Application: Signalized intersections
2. Reconstructing cumulative diagrams
3. Ramp metering problem

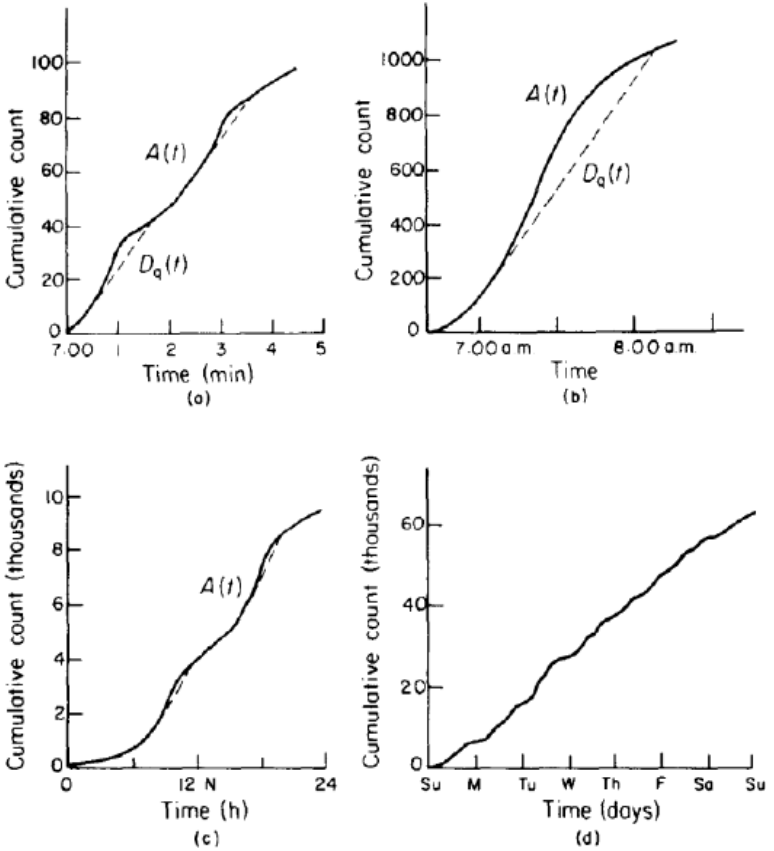
Discrete → continuous cumulative diagrams

- The main advantage for continuous analysis is that differential calculus can be used:
  - $q(t) \approx \frac{d\tilde{N}(t)}{dt}$
- In most transportation applications cumulative count is made up of discrete objects (e.g. passengers, buses, cars, etc...), so $N(t)$ is a step function
- However, in applications dealing with many moving objects, where predicting the exact number to the nearest integer is not important, it is preferable to work with a smooth approximation of $N(t)$: $\tilde{N}(t)$, e.g., an interpolation
- Typical applications of cumulative plots involve long observation periods where many observations are collected. In these cases, the individual steps of the curve cannot be seen.



Scales of counts and time

- **Seconds or minutes**:
  queue (a)
- **Hours**:
  peak demand or rush hours (b,c);
- **Days**:
  different patterns on different days (d);



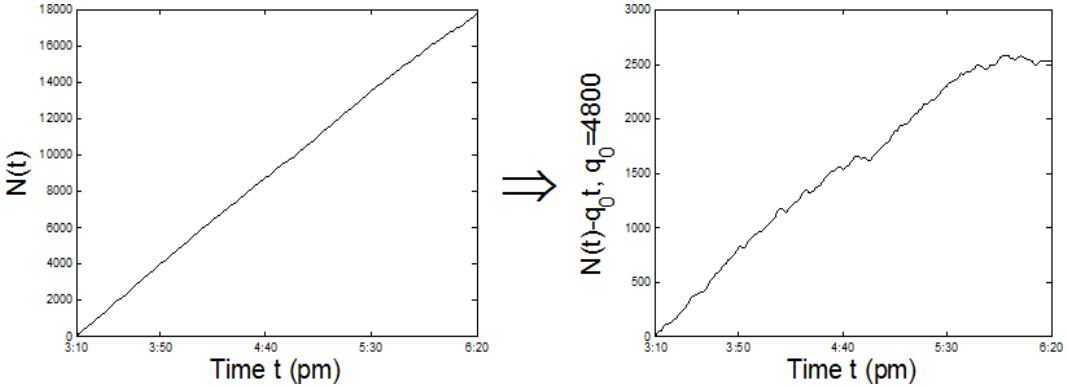
Slide adapted from Prof. Zhengbing He

Oblique cumulative diagram

- In freeways accumulation of arrival counts is **very fast** since the flow is large. The detail is hard to observe in the regular cumulative diagram.
- **An oblique coordinate system** is used to show arrival changes, through a linear transformation, which keeps the **relative relationship** (See, for example, [1, 2]):

$$(N, t) \Longrightarrow (N - q_0 t, t)$$

where q_0 is a coefficient.



1 Cassidy M. Bivariate relations in nearly stationary highway traffic. Transportation Research Part B. 1998.
2 Cassidy M. Some traffic features at freeway bottlenecks. Transportation Research Part B. 1999;33:25-42.

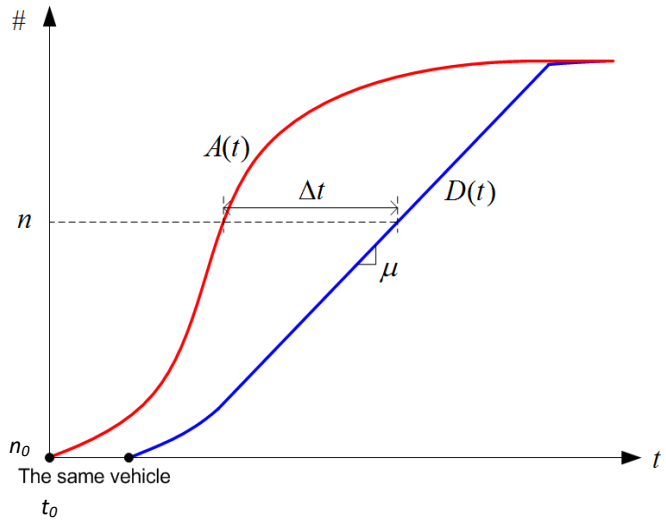
Slide adapted from Prof. Zhengbing He

Outline

1. **Cumulative diagrams**
    a. Visualizing queues and delays
    b. Cumulative diagrams vs. time-space diagrams
    c. Working with cumulative diagrams
    d. Analysis & interpretation
    e. Little's Law
    f. Application: Signalized intersections
2. Reconstructing cumulative diagrams
3. Ramp metering problem

Continuous cumulative arrivals and departures

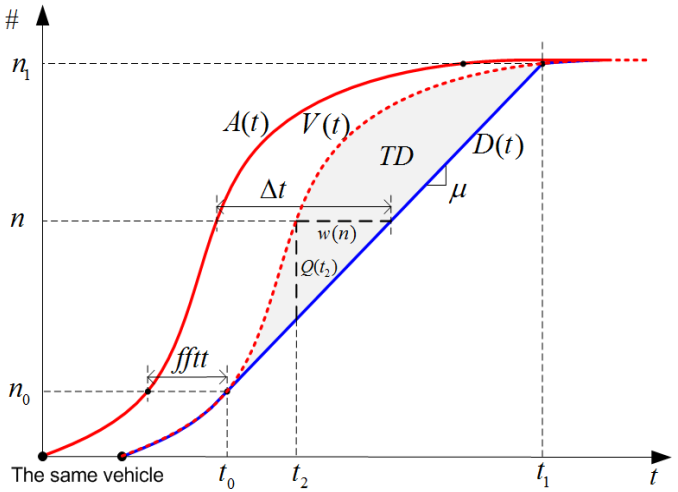
- Arrivals at x_0; departures at x_1
- Starting counting with a reference customer / vehicle;
  - Let $A(t)$ denote the arrivals
  - Let $D(t)$ denote the departures
  - Let μ denote the service rate (customers / time)



Slide adapted from Prof. Zhengbing He

Virtual arrivals

- Total time in the system is composed of
  - "Service time": time to go through the system independently of traffic conditions
    - E.g. travel time along a link in uncongested conditions: free flow travel time (fftt)
  - Delay: additional time in system due to congestion
- To better analyze and interpret the queueing system, we shift $A(t)$ right by fftt, and obtain the virtual arrival count curve $V(t) = A(t - \text{fftt})$.
- $V(t)$ represents the departure time under no delay

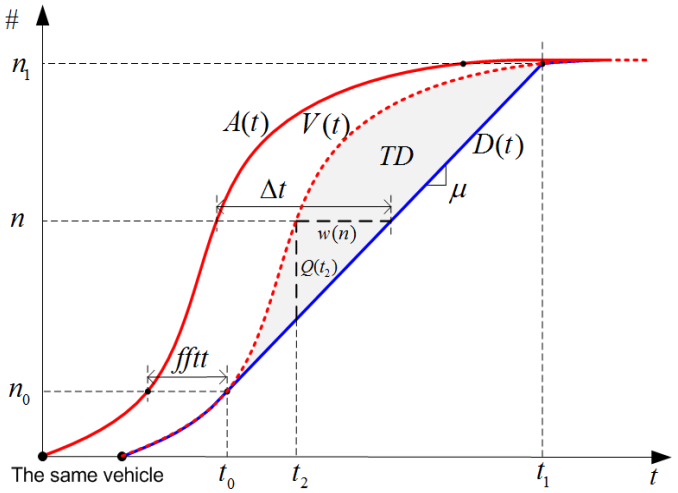


- The delay of vehicle n: $w(n)$
- Queue at $t_2 \approx$ excess vehicle accumulation: $Q(t_2)$
- Total Delay:

$$TD = \int_{t_0}^{t_1} [V(t) - D(t)]dt$$
$$= \int_{t_0}^{t_1} Q(t)dt$$

Slide adapted from Prof. Zhengbing He

Little's Law (1961, deterministic case)



- The delay of vehicle n: $w(n)$
- Queue at t_2 $\approx$ excess vehicle accumulation: $Q(t_2)$
- Total Delay:

$$TD = \int_{t_0}^{t_1} [V(t) - D(t)]dt$$
$$= \int_{t_0}^{t_1} Q(t)dt$$

Little's Law (1961)

Updated 2023-02-16

- Beauty is in its simplicity:

$$\bar{Q} = \lambda \bar{w}$$

  - avg queue length = (avg arrival rate) x (avg waiting time)
- Previous slide: deterministic proof.
- Also holds for probabilistic settings. We'll revisit in Unit 2.
- Assumptions:
  - System stability

- Preview: Little's Law is independent of:
  - Arrival process distribution
  - Service distribution
  - Service order
  - Structure of queue(s)
  - Number of servers
  - Etc.
- Applications:
  - Highway traffic, transit, airports
  - Shops
  - Manufacturing plants
  - Bank tellers
  - Emergency rooms
  - Operations management
  - Computer architecture (webserver requests, CPU, DRAM, RAM, HDD)

1961, John Little
MIT Institute Professor
See: "Little's Law as Viewed on its 50th Anniversary" (INFORMS)

Outline

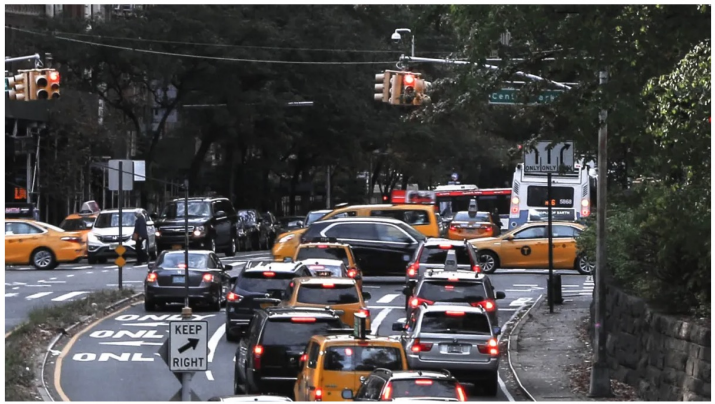
1. **Cumulative diagrams**
    a. Visualizing queues and delays
    b. Cumulative diagrams vs. time-space diagrams
    c. Working with cumulative diagrams
    d. Analysis & interpretation
    e. Little's Law
    f. **Application: Signalized intersections**
2. Reconstructing cumulative diagrams
3. Ramp metering problem

Example: Traffic signals

How poorly timed traffic lights can make climate change worse

Pointless delays result in unnecessary idling.

by YCC TEAM
JUNE 21, 2021



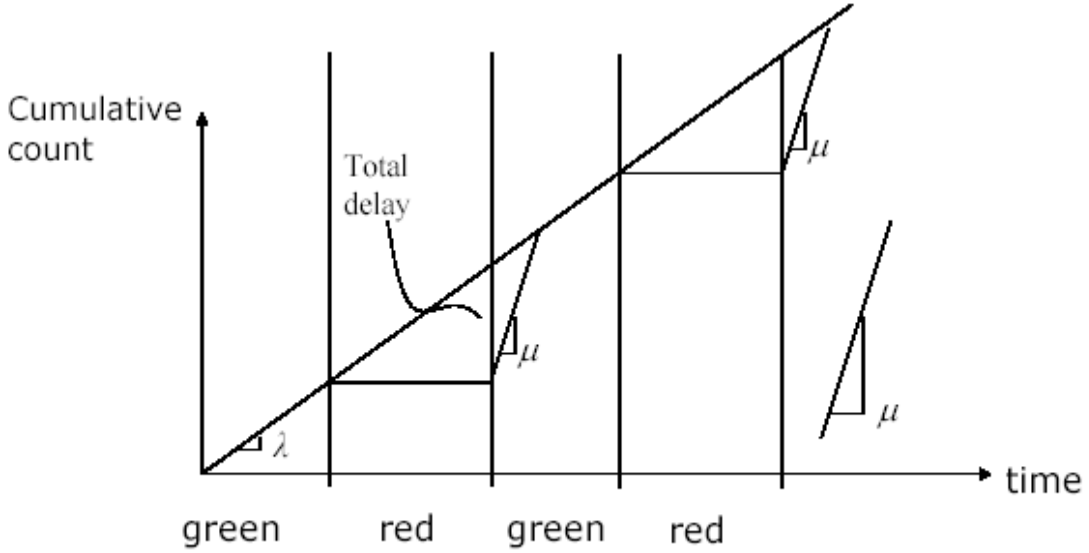
The average signal caused more than 80 hours of delay each day in October, 2020

Source: INRIX

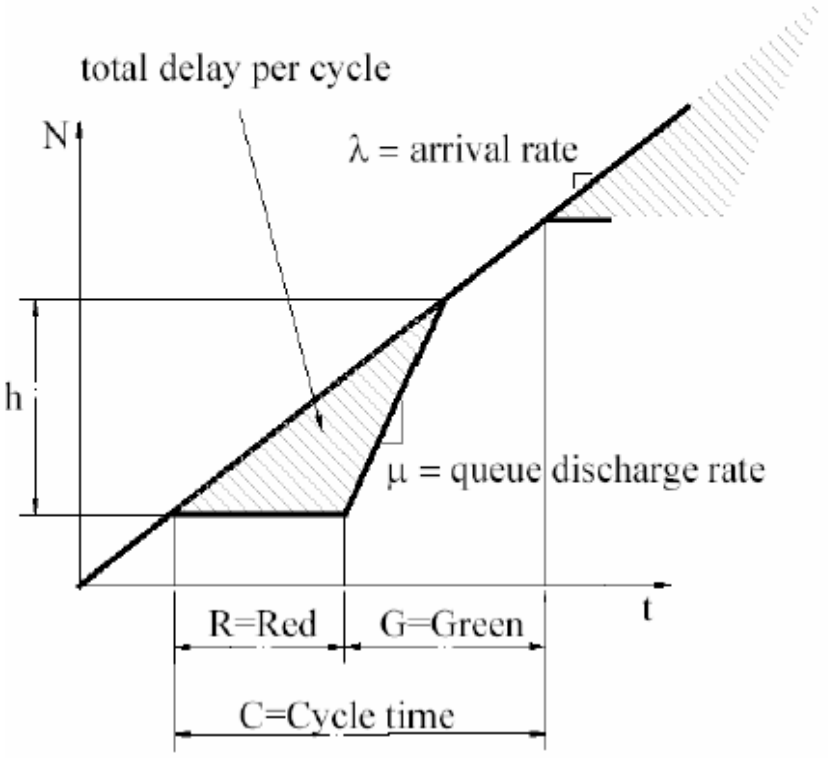
Slide adapted from Dajiang Suo

Queuing diagram application

- Example: On-off service (traffic signal basics)
  - Given: Arrival flow (rate) at a one way signalized intersection is constant λ; we have information about the green/red timing plan of the signal (server), and the service rate during green μ
  - Find: The departure curve, total delay, average delay



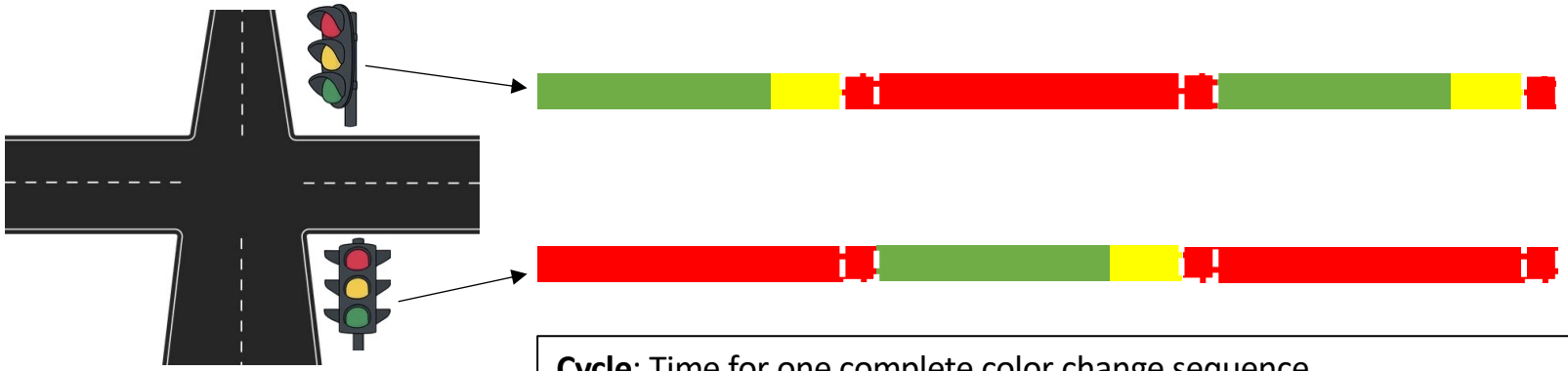
Deterministic intersection delay



Reality check

Real traffic signals are more complex

Traffic control fundamentals



Cycle: Time for one complete color change sequence

Phase: The part of a cycle for one or more movements (left/through/right)

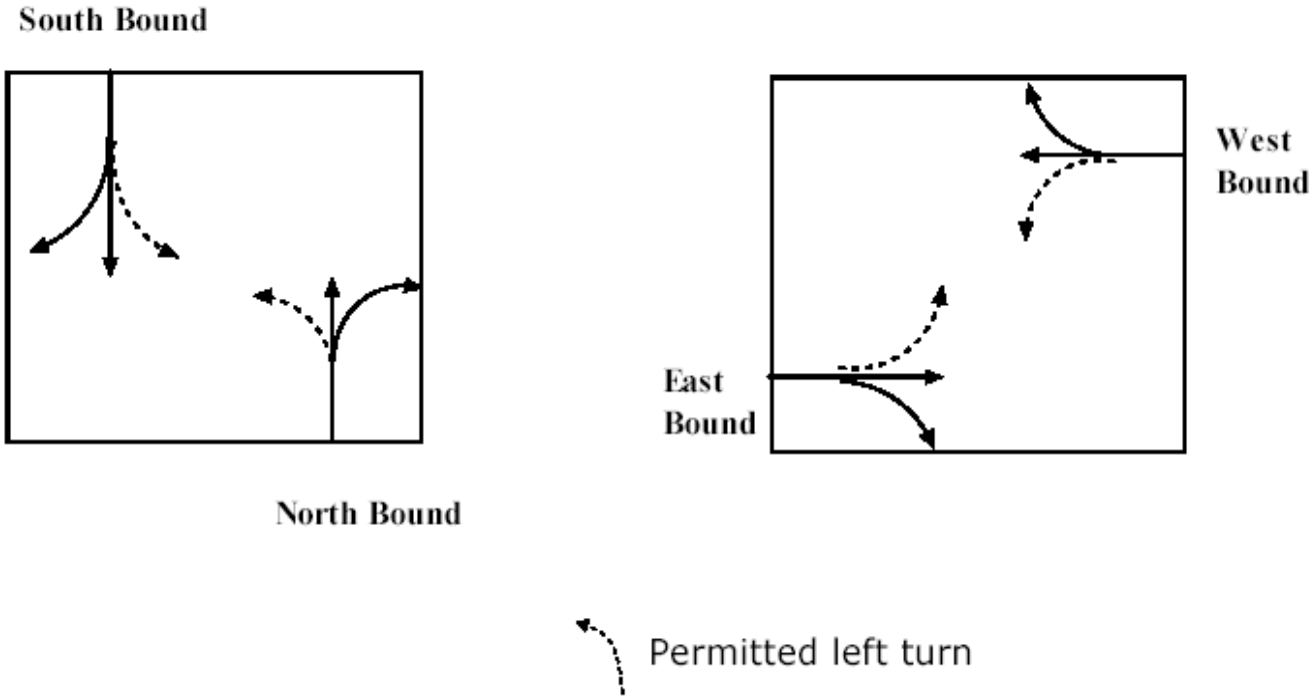
Green/Red/Yellow time

All-red time (lost time)

Slide adapted from Dajiang Suo

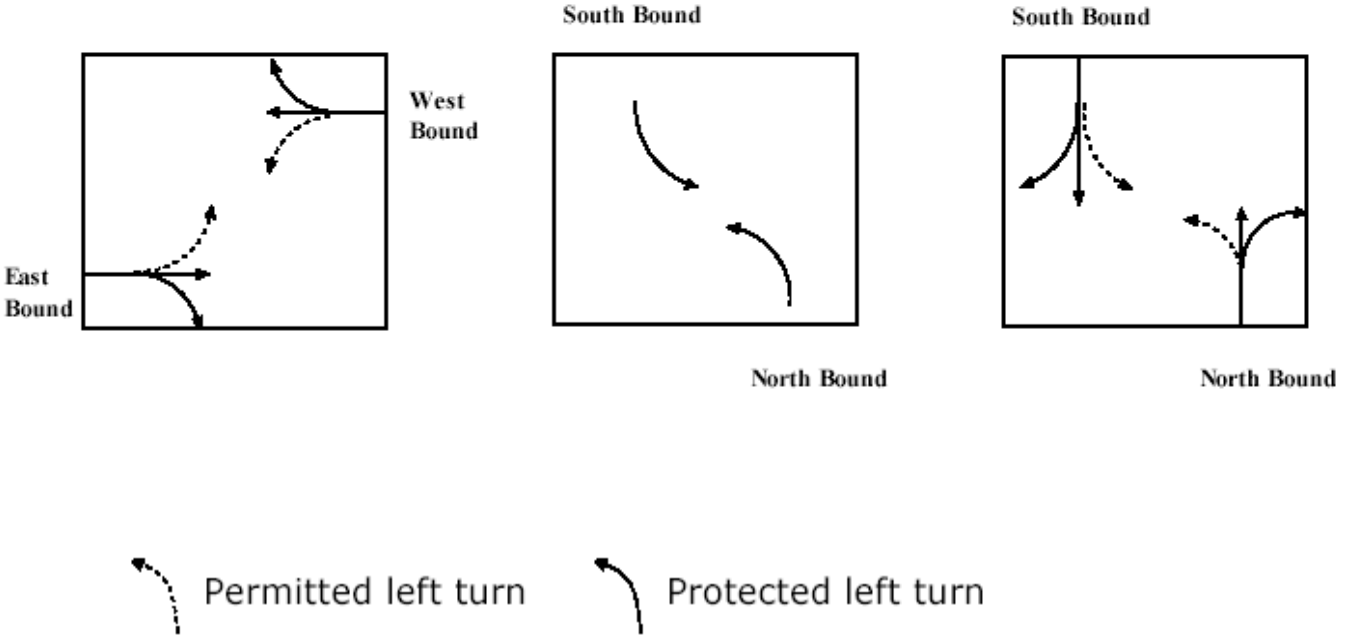
Some typical timing plans

- Simple two phase timing plan



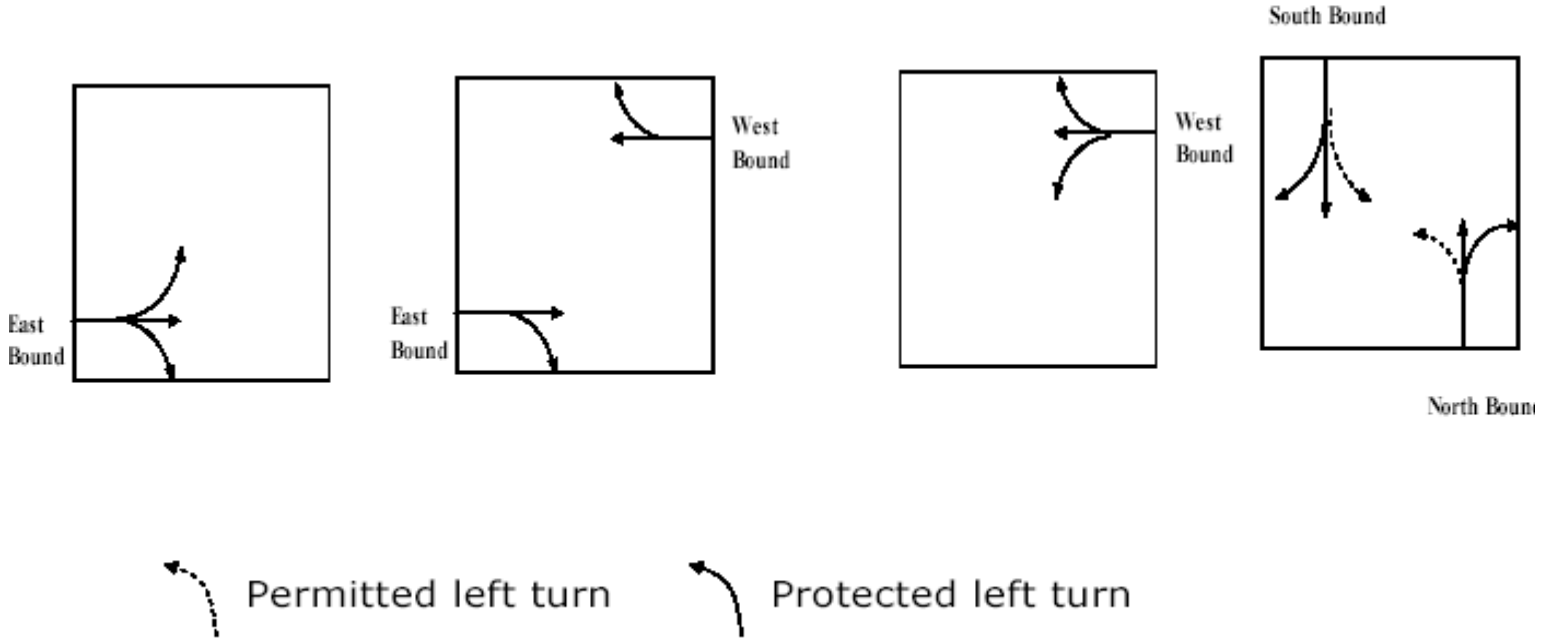
Some typical timing plans

- Three phase timing plan (dual left on N-S)



Some typical timing plans

- Four phase timing plan (lead-lag on E-W)



Strategies for traffic signal control

Pretimed

Semi-actuated

Fully-actuated



Slide adapted from Dajiang Suo

Strategies for traffic signal control

Pretimed | **Semi-actuated** | Fully-actuated

- Detectors only placed on minor approach
- Major road is interrupted only if vehicle (or other participants) present at the minor approach



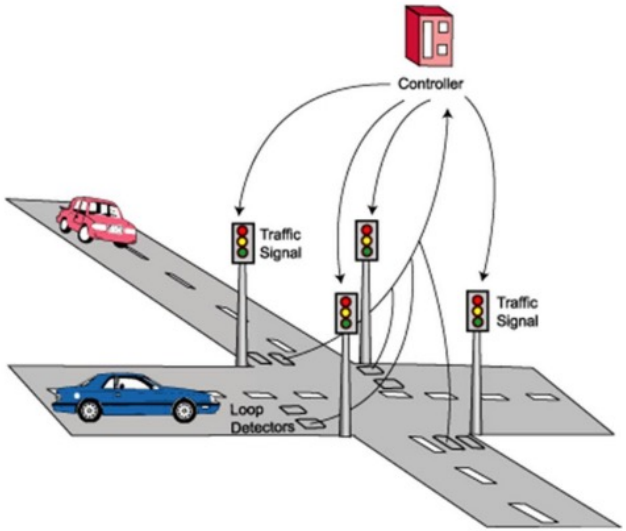
Slide adapted from Dajiang Suo

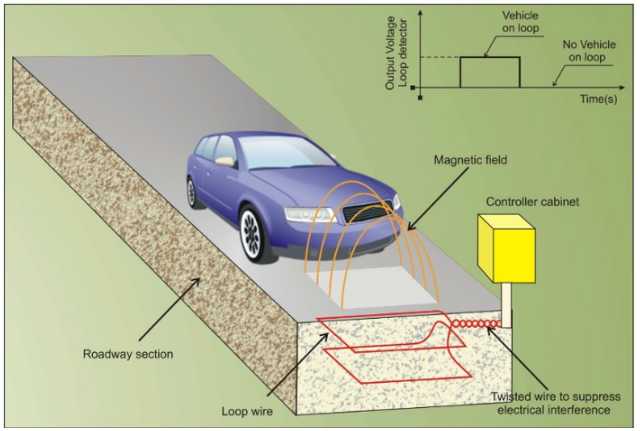
Strategies for traffic signal control

Pretimed

Semi-actuated

Fully-actuated

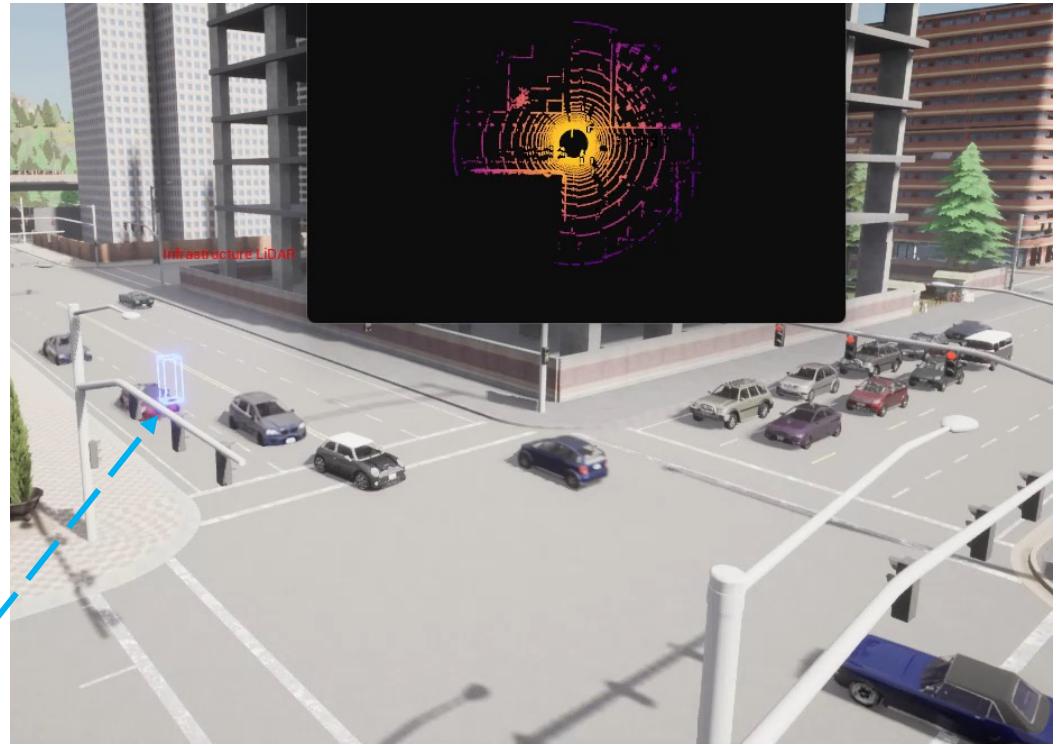




[Mircea, 2014]

Slide adapted from Dajiang Suo

Sensing solutions for fully-actuated control: roadside sensors

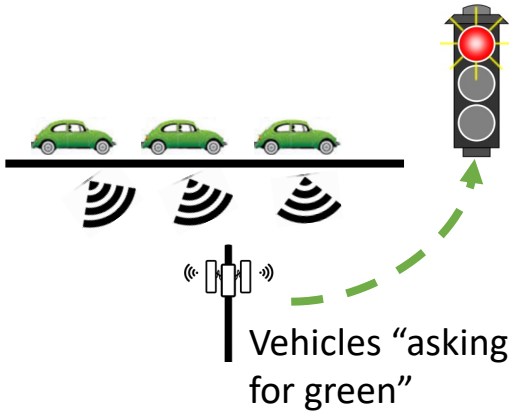


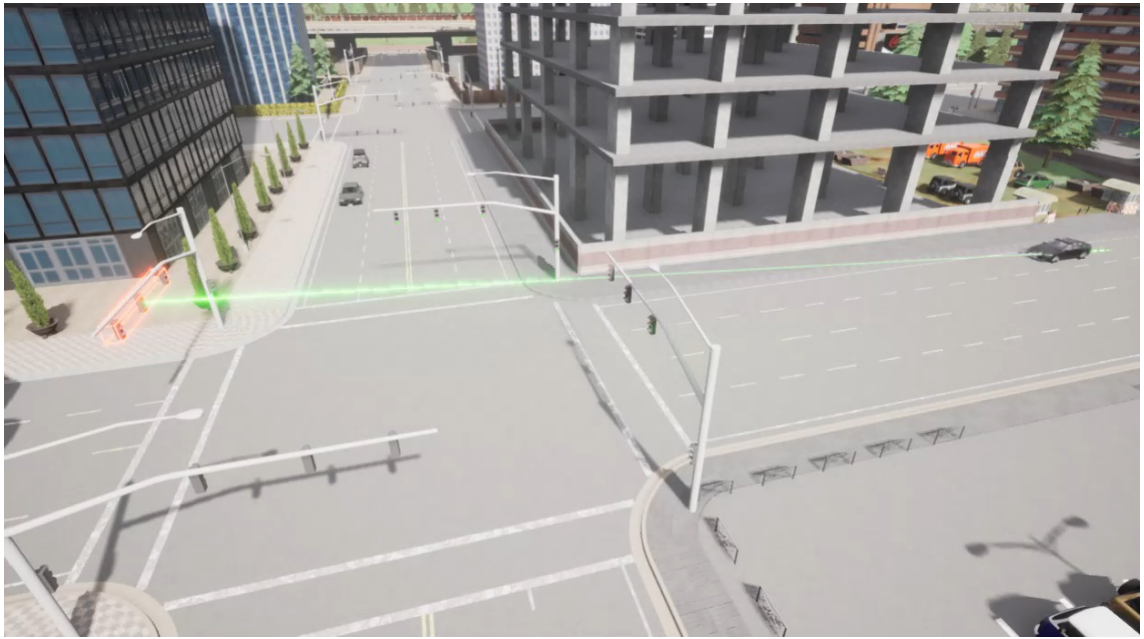
Infrastructure sensors



Slide adapted from Dajiang Suo

Sensing solutions for fully-actuated control: vehicle-to-infrastructure communication

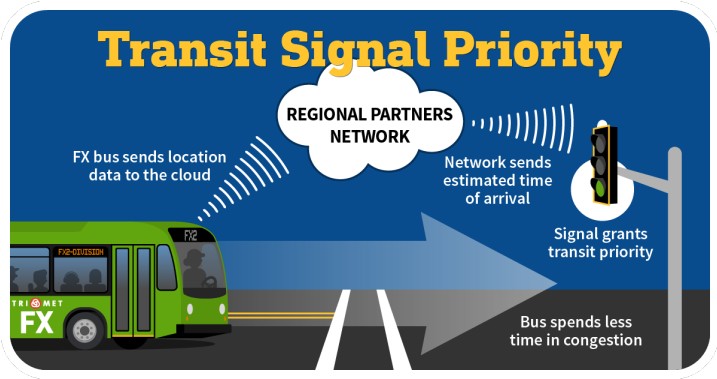




Slide adapted from Dajiang Suo

Exercise: Which type of control strategy is used here?

Source: TriMet.org



TriMet News

(VIDEO) Green lights ahead for TriMet's Division Transit Project, with new advances to speed up buses

July 15, 2022 by Tyler Graf

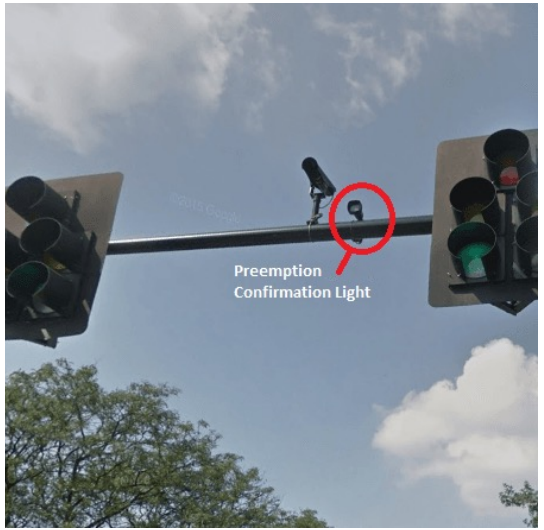


Slide adapted from Dajiang Suo

Advanced signal timing concepts: Traffic signal preemption



FHWA



Ohio Hudson

Slide adapted from Dajiang Suo

More considerations in fully-actuated control

Technologies to use for detecting pedestrians, bicyclists, or scooters?

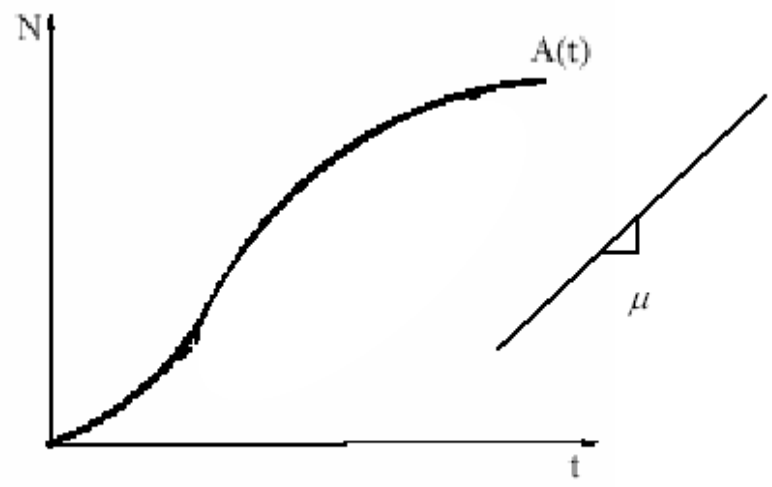
Slide adapted from Dajiang Suo

Outline

1. Cumulative diagrams
2. **Reconstructing cumulative diagrams**
3. Ramp metering problem

Reconstructing the departure curve

- We often have incomplete information.
  - We might have $V(t)$ or $A(t)$ and the operating features of the server (e.g., constant service rate μ), but we need $D(t)$
- Consider:



Outline

1. Cumulative diagrams
2. Reconstructing cumulative diagrams
3. **Ramp metering problem**

Ramp metering problem

- A ramp meter is being considered at an entrance to a freeway.
- Currently, rush hour traffic arrives at the on-ramp at a rate q_1 from time $t = 0$ to time $t = t^*$. After $t = t^*$, vehicles arrive at a (lower) rate q_2.

1. Assuming that drivers will not change their trips, draw and label a cumulative plot showing a metering (i.e. departure) rate of $\mu (q_2 < \mu < q_1)$.
   - Label the maximum delay experienced by any vehicle, w_{max}.
   - What is w_{max} as a function of q_1, q_2, μ, and t^*?
2. If an alternate route is available to drivers, and it is known that they will take this route if their expected delay at the ramp meter is greater than $\frac{w_{max}}{2}$, add this new scenario to your diagram. Now, show graphically the following:
   a) The number of vehicles which will divert.
   b) How much earlier the queue will dissipate (compared to part 1)?

Ramp metering problem

- A ramp meter is being considered at an entrance to a freeway.
- Currently, rush hour traffic arrives at the on-ramp at a rate q_1 from time $t = 0$ to time $t = t^*$. After $t = t^*$, vehicles arrive at a (lower) rate q_2.

1. Assuming that drivers will not change their trips, draw and label a cumulative plot showing a metering (i.e. departure) rate of $\mu(q_2 < \mu < q_1)$.
   - Label the maximum delay experienced by any vehicle, w_{max}.
   - What is w_{max} as a function of q_1, q_2, μ, and t^*?
2. If an alternate route is available to drivers, and it is known that they will take this route if their expected delay at the ramp meter is greater than $\frac{w_{max}}{2}$, add this new scenario to your diagram. Now, show graphically the following:
   a) The number of vehicles which will divert.
   b) How much earlier the queue will dissipate (compared to part 1)?

