Stochastic simulation

Discrete event simulation

Cathy Wu

1.041/1.200 Transportation: Foundations and Methods

Readings

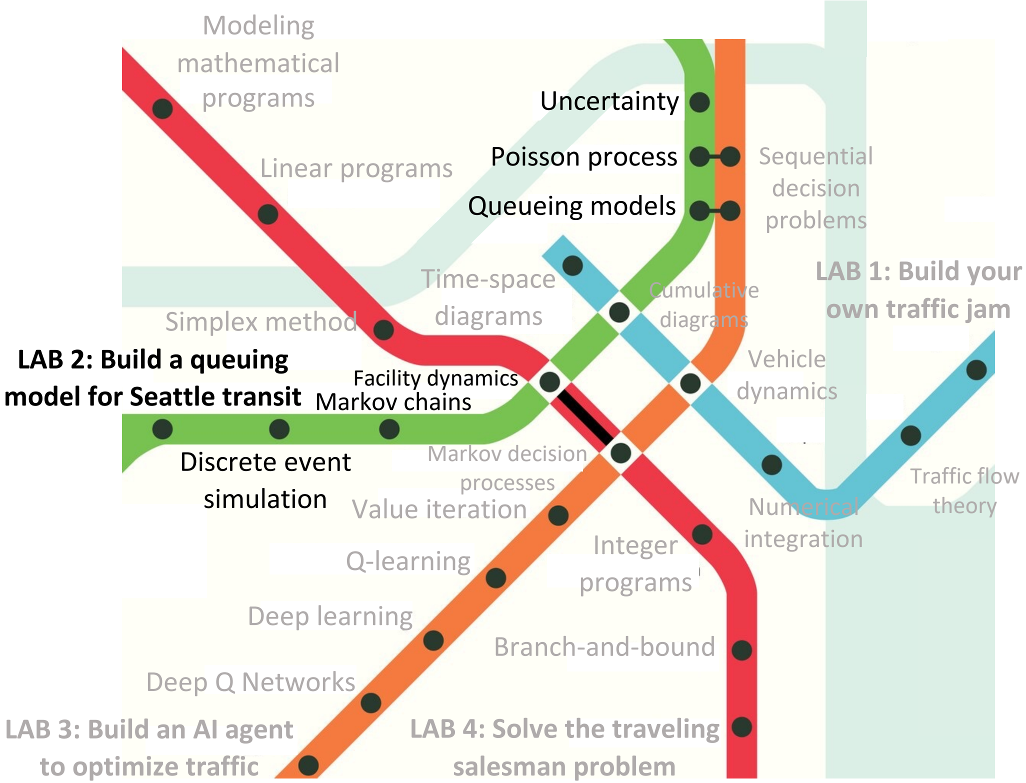
1. Larson, Richard C. and Amedeo R. Odoni. **Urban Operations Research**. Prentice-Hall (1981). Chapter 7: Simulations. URL.
2. (Optional) X. Wang *et al.*, “Traffic light optimization with low penetration rate vehicle trajectory data,” *Nat Commun*, vol. 15, no. 1, Art. no. 1, Feb. 2024, doi: 10.1038/s41467-024-45427-4.

Unit 2: Queuing systems

Unit 2

Modeling

Stochastic



Outline

1. Simulation & real-world queues
2. Discrete event simulation
3. Transient analysis

Outline

1. **Simulation & real-world queues**
2. Discrete event simulation
3. Transient analysis

Real-world queues! (Nature Communication, 2024)

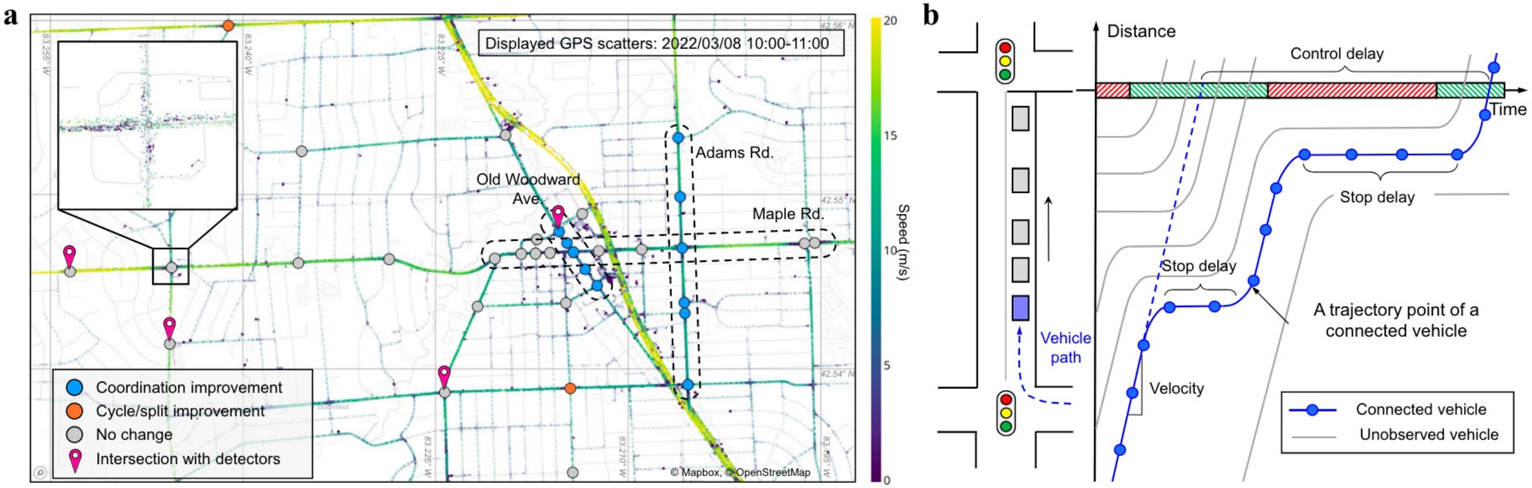


Fig. 1 | Traffic signal retiming with vehicle trajectories.

- Overall result: Decreased the delay and number of stops at signalized intersections by up to 20% and 30% with data from as low as 6% vehicle penetration.

X. Wang *et al.*, "Traffic light optimization with low penetration rate vehicle trajectory data," *Nat Commun*, vol. 15, no. 1, Art. no. 1, Feb. 2024, doi: 10.1038/s41467-024-45427-4.

Ideas from Unit 1+2

- Time-space diagram
- Newell's car following model
- Stochastic arrival process
  - Bernoulli distribution!

- Discrete event simulation
- G/D/1 queue

→ Probabilistic time-space (PTS) diagram

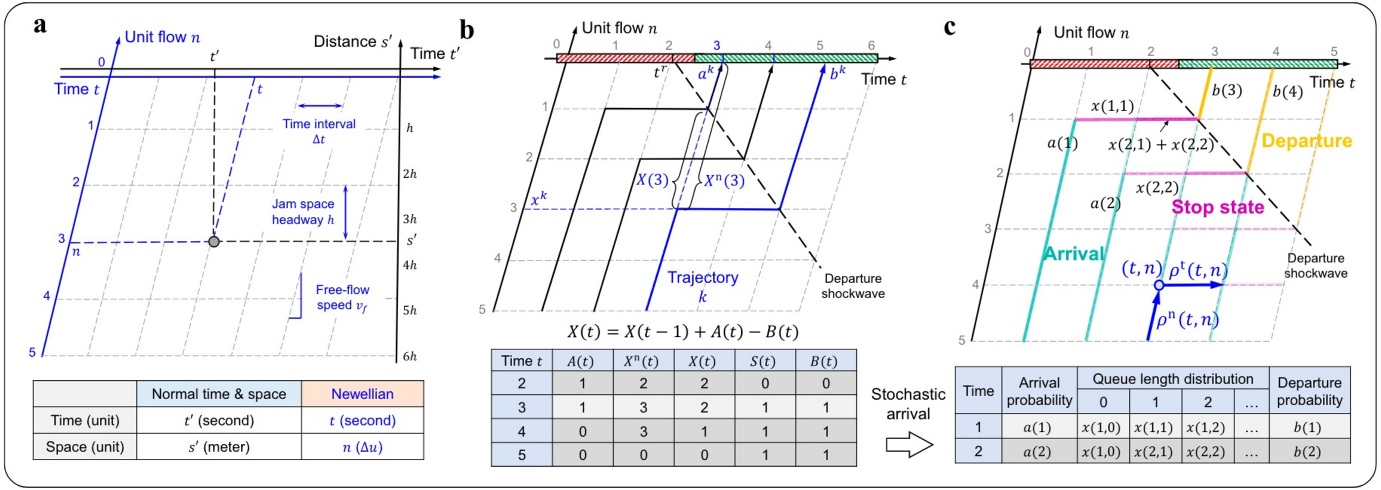
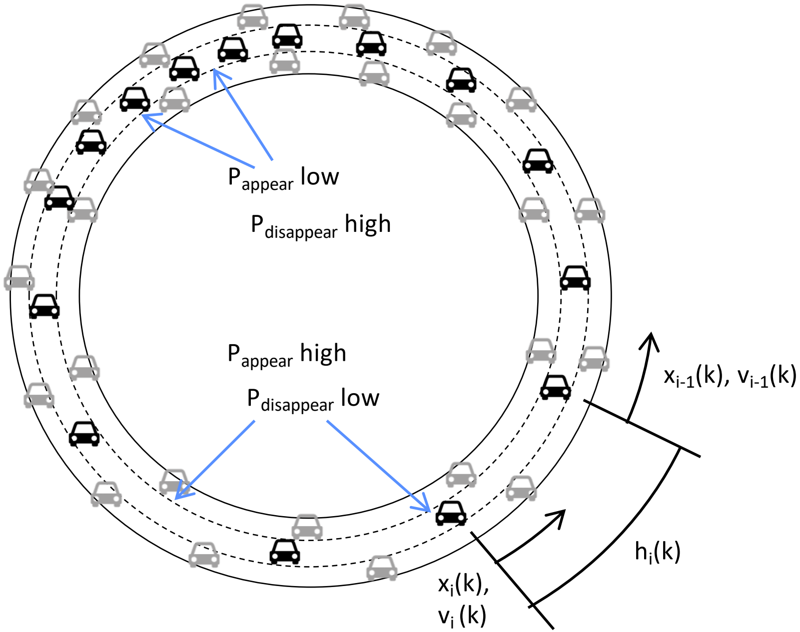


Fig. 2 | Point-queue model under Newellian coordinates and PTS diagram.

X. Wang *et al.*, "Traffic light optimization with low penetration rate vehicle trajectory data," *Nat Commun*, vol. 15, no. 1, Art. no. 1, Feb. 2024, doi: 10.1038/s41467-024-45427-4.

(Simple) multi-lane modeling

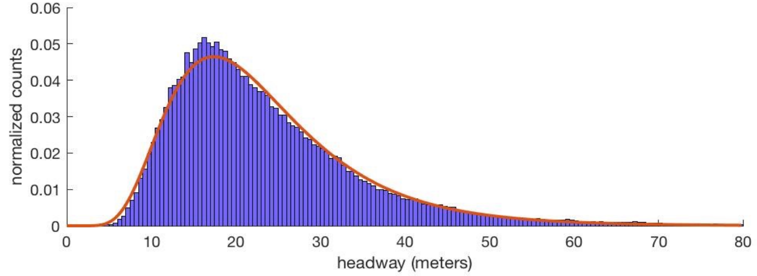
Stochastic single-lane model

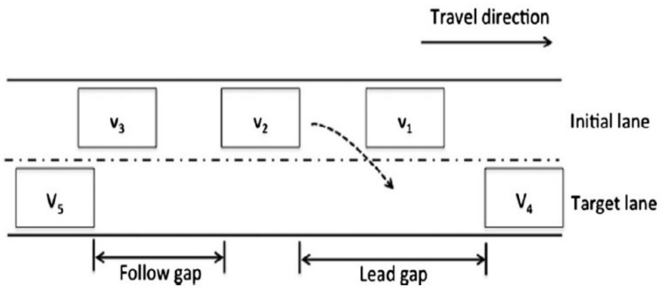


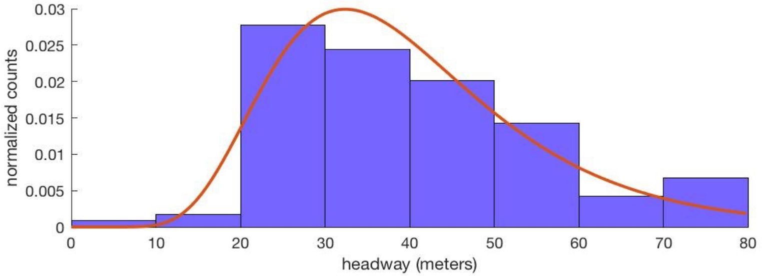
Wu, Cathy, Eugene Vinitsky, Aboudy Kreidieh, and Alexandre Bayen. "Multi-lane reduction: A stochastic single-lane model for lane changing." IEEE ITSC, 2017.

Model Calibration - Probabilities

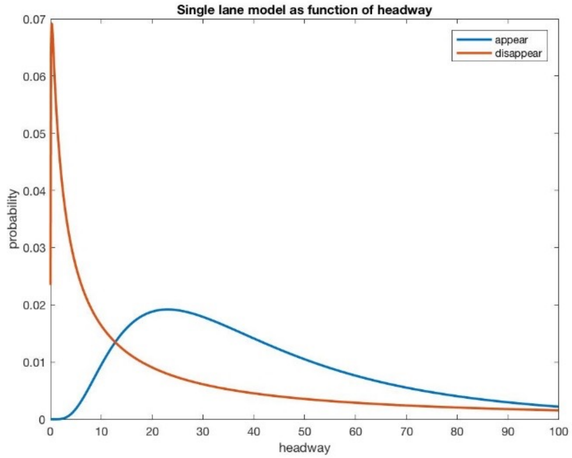
Extract headways when lane changes occur
Fit to log-normal distribution







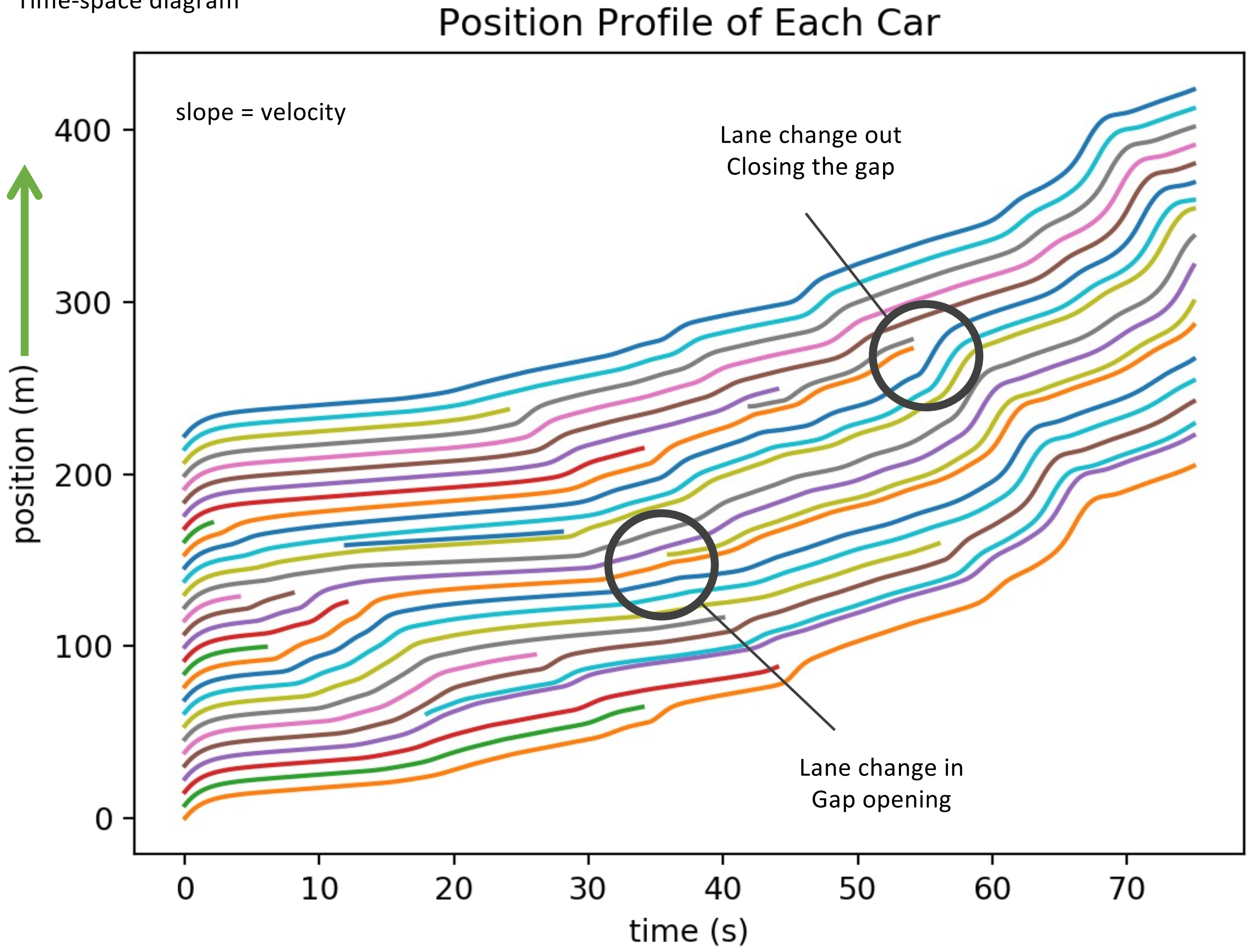
Fit of log-normal distribution for the total distribution of headways and the conditional distribution of headways when a vehicle lane changes into a lane, respectively, computed for 7:50 am on the US 101.



Wu, Cathy, Eugene Vinitsky, Aboudy Kreidieh, and Alexandre Bayen. "Multi-lane reduction: A stochastic single-lane model for lane changing." IEEE ITSC, 2017.

Slide credit: Eugene Vinitsky

Time-space diagram



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Stochastic simulation models

- **Simulation**: a computer program that represents key features of a real or abstract system over time.
  - **Stochastic simulation** incorporates randomness / uncertainty to analyze complex systems
- Simulations take data as input, executes the underlying software, and produces output data for analysis.
- Simulation approaches range from relatively simple mathematics to very complex models.

Outline

1. Simulation & real-world queues
2. **Discrete event simulation**
   a. Simulation of an $M/M/1$ queueing system
   b. Replications
3. Transient analysis

Simulation models for queuing systems

- **Discrete-event simulation (DES)**
  - Continuous-time: event driven simulation
  - The system is modeled by a set of discrete states
  - The system can change states when an **event** occurs
  - Between events the system does not change states
- **Key steps in DES**
  - Identify the events that lead to state changes
  - For each event, describe:
    - the new state
    - changes in the system attributes
    - triggered events
  - The simulator keeps an chronologically ordered *event list* of the events that are scheduled to happen with their scheduled time
  - Once an event is carried out, the simulation dock is advanced to the time the next event is scheduled to start

Discrete event simulation model: M/M/1

- State of the system: $N(t)$ = number of customers in the queueing system at time t
- Events:
  - arrival of a new customer
  - service completion for the customer currently in service
- Update system state and record information at time of each event
- State transition mechanism for event-driven simulation:

$$N(t) = \begin{cases} N(\text{preceding event}) + 1, \text{if an arrival occurs at time } t \\ N(\text{preceding event}) - 1, \text{if a service completion occurs at time } t \end{cases}$$

- Simulation end: if the simulation clock exceeds a pre-specified (simulation) time or number of customers observed reaches a specified limit
- How to obtain the time and type of next event?

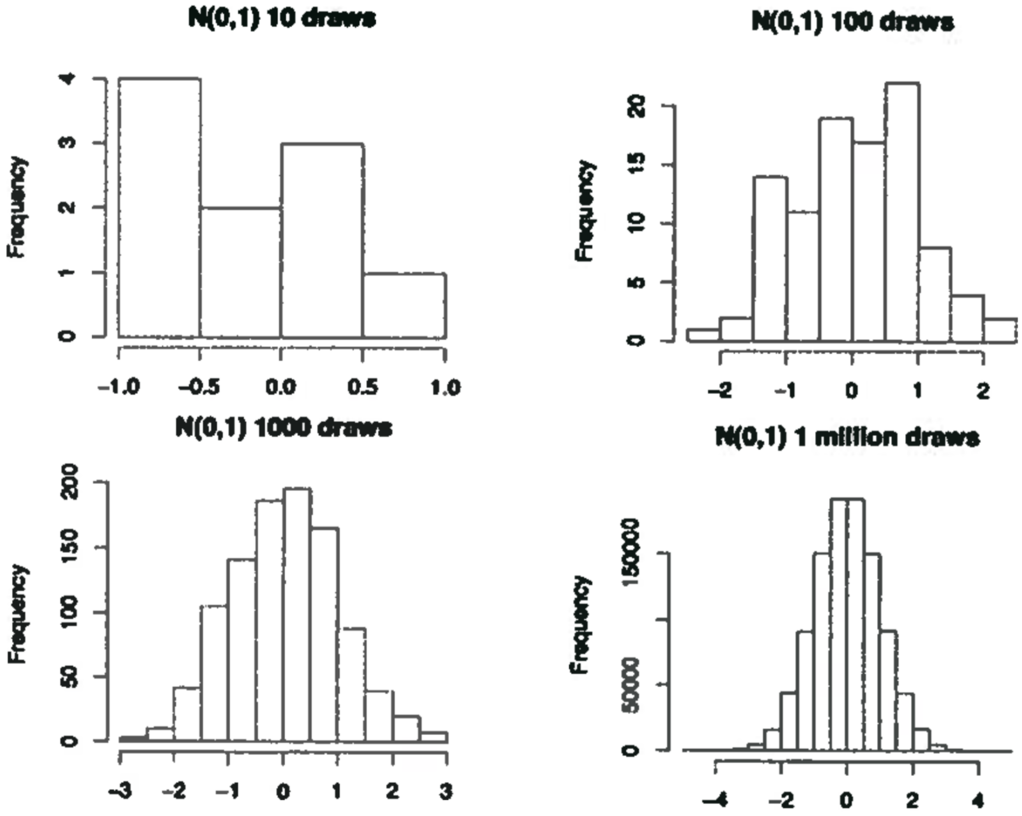
Discrete Event Simulation Model: M/M/1

- A_n: arrival time of customer n
- D_n: departure time (service completion time) of customer n
- H_n: inter-arrival (arrival headway) time of customer n
- S_n: service time of customer n

Discrete Event Simulation Model: M/M/1

- A_n: arrival time of customer n
- D_n: departure time (service completion time) of customer n
- H_n: inter-arrival (arrival headway) time of customer n
- S_n: service time of customer n
- $$\begin{cases} A_1 = 0 \\ A_n = A_{n-1} + H_n, n = 2, 3, \ldots \end{cases}$$
- $$\begin{cases} D_1 = S_1 \\ D_n = S_n + \max\{A_n, D_{n-1}\}, n = 2, 3, \ldots \end{cases}$$
  where $\max\{A_n, D_{n-1}\}$ represents the time that service starts for customer n
- H_n and S_n are generated from two exponential random variables.

Replications



Replications

- The outputs from a simulation model are random variables
- Running the simulator provides realizations of these r.v.
- Replications allow us to:
  - **Obtain independent observations**. This allows us to apply classical statistical methods to analyze the outputs: e.g. central limit theorem, confidence intervals
  - **Estimate system performance measures** (e.g. empirical cdf) that enable us to understand the "typical" behavior of the system
  - **Uncertainty quantification**: Have an idea of the underlying (unknown and often) complex distribution of the output variable
- How to obtain different replications with a simulation software?
  - For a given replication, the sequence of (pseudo-)random numbers are generated starting with an initial number, called the seed
  - Launch the simulation with the same seed → obtain identical results
  - *Saving the seed allows you to reproduce your results*
  - Different seeds → different realizations

Outline

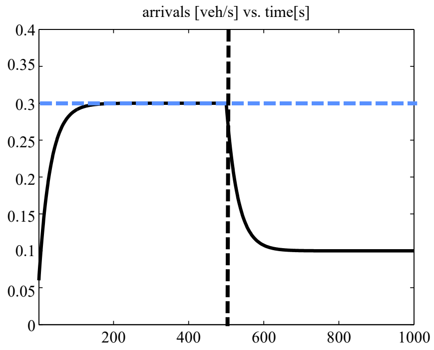
1. Simulation & real-world queues
2. Discrete event simulation
3. **Transient analysis**

Transient analysis for urban traffic

Dynamic analysis of one link over 1000 s

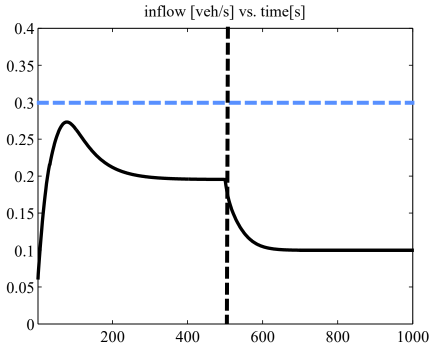
- Initially empty link, arrival rate that is 0.3 veh/s for the first 500 s and then jumps down to 0.1 veh/s, where it stays for the remaining 500 s
- The downstream flow capacity (service rate) of the link is 0.2 veh/s
  - Thus the first half of the demand exceeds the link's bottleneck capacity, whereas the second half can be served
- Dynamic analysis of queue build-up and dissipation

arrivals

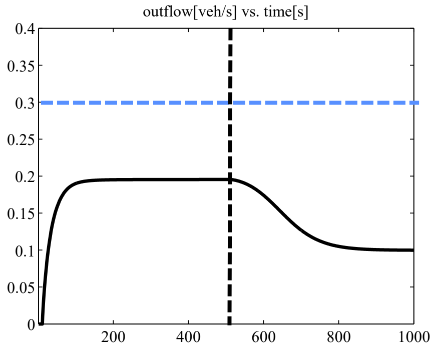


$\lambda(t)$

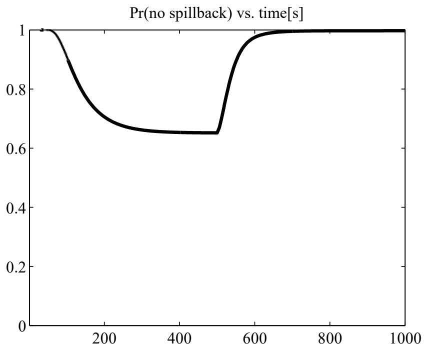
inflow

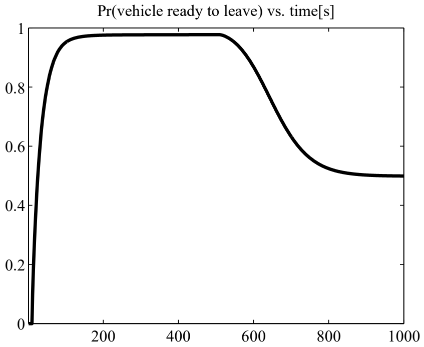


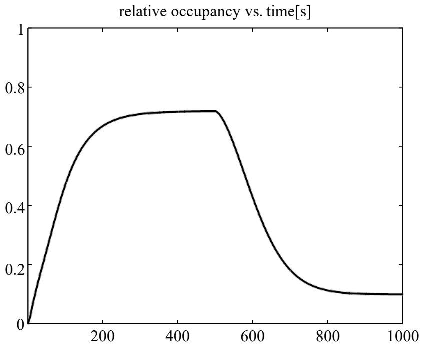
outflow



Transient analysis for urban traffic







1. Probability that there is no spillback
2. Probability that a vehicle is ready to leave the link
3. Relative occupancy (expected number of vehicles divided by maximum number of vehicles)

Transient analysis

- Relax the assumption of stationarity
- Analyze the transient (dynamic) behavior of a queue
- Transition rate (linear) differential equations for a birth-death process:

$$\begin{cases} \dfrac{dP_n(t)}{dt} = -(\lambda_n + \mu_n)P_n(t) + \mu_{n+1}P_{n+1}(t) + \lambda_{n-1}P_{n-1}(t), \forall n \geq 1 \\ \dfrac{dP_0(t)}{dt} = -\lambda_0 P_0(t) + \mu_1 P_1(t) \end{cases}$$

- Can be written as:

$$\frac{dP(t)}{dt} = P(t)Q$$

- This linear system of differential equations has general solution:

$$P(t) = P(0)e^{Qt}$$

- Numerical methods used to evaluate $P(t)$

Transient analysis

- M/M/1/K queue

$$\begin{cases} P_n(t) = s_n + \rho^{\frac{n}{2}} \sum_{j=1}^{K} C_j \left(\sin \frac{jn\pi}{K+1} - \sqrt{\rho} \sin \frac{j(n+1)\pi}{K+1} \right) e^{\tau_j t} \\ \tau_j = \lambda + \mu - 2\sqrt{\lambda\mu} \cos \frac{j\pi}{K+1} \end{cases}$$

where s is the stationary dbn, and the coefficients $\{C_j\}$ are chosen to fit the initial values of the transient distribution.

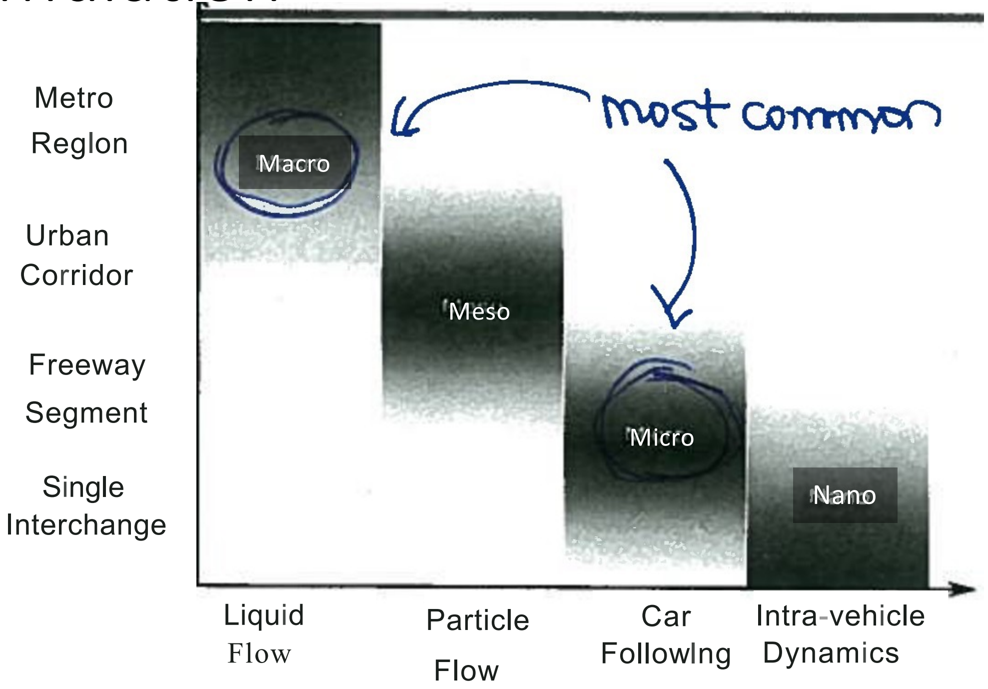
- There are few closed-form expressions for transient distributions

- Easy with stochastic simulation!

Simulation pro's and con’s

- Just as analytical models, simulation models are based on numerous assumptions and approximations, use it with caution and keep in mind that it's a simplification of reality, i.e. a MODEL!
- What is important to model depends on the use case

Traffic simulation



- Source: "Scale and Complexity Tradeoffs In Surface Transportation Modeling", Karl E. Wunderlich.

References

1. Larson, Richard C. and Amedeo R. Odoni. **Urban Operations Research**. Prentice-Hall (1981). Chapter 7: Simulations.
2. Slides adapted from Carolina Osorio