Spring 2026

Graphical analysis I

Time-space diagrams

Cathy Wu

1.041/1.200 Transportation: Foundations and Methods

Announcements

Pset Check instead of Micro-quiz (to disambiguate from Quiz)

Readings

1. C. Daganzo, *Fundamentals of transportation and traffic operations*, vol. 30. Pergamon Oxford, 1997. Chapter 1: The time-space diagram. URL.
2. (For fun) *Why There are Now So Many Shortages (It's Not COVID)*. Wendover Productions, YouTube, 2021. URL.

Outline

1. Time-space diagrams
2. From sensors to data to trajectories to time-space diagrams

Outline

1. **Time-space diagrams**
   a. Applications and traffic system design: road, air, rail, transit
   b. Exercise: Waterway capacity problem
2. From sensors to data to trajectories to time-space diagrams

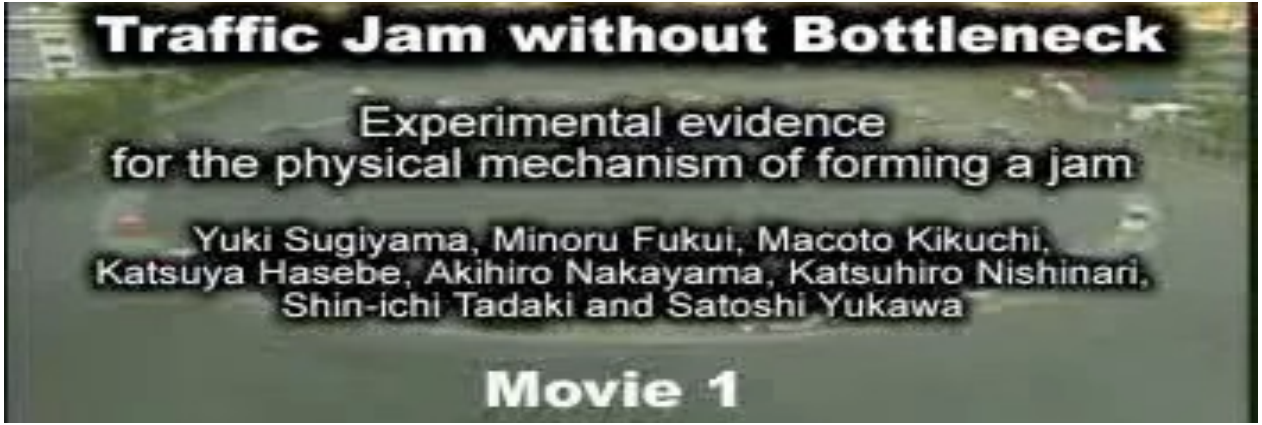
Learning objective

Time-space graphs as an analysis & design tool for transportation systems

Key idea: To holistically analyze temporal phenomena, translate time into space.

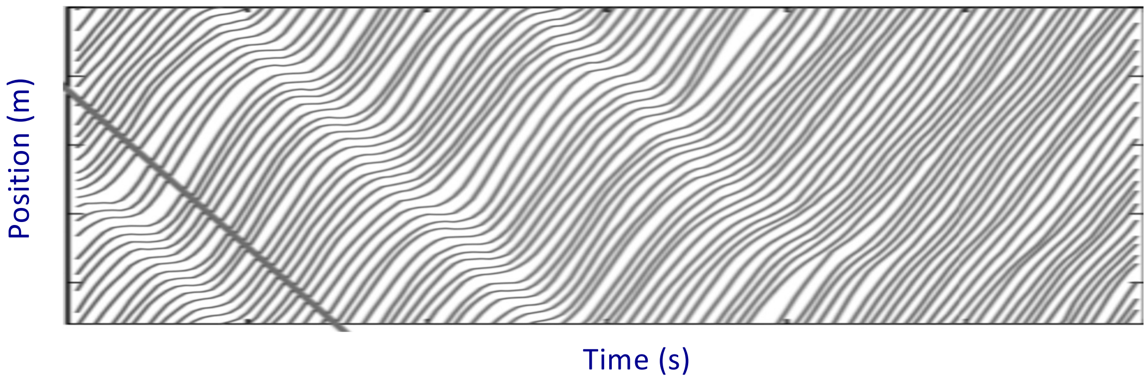
Compare two depictions of the same data

Video

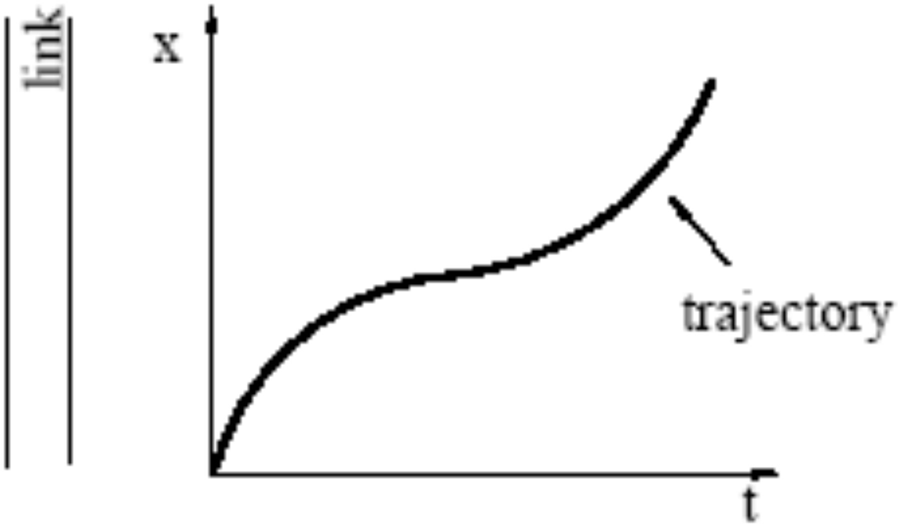


Time-space diagram

Vehicle trajectories (Sugiyama et al. 2008)

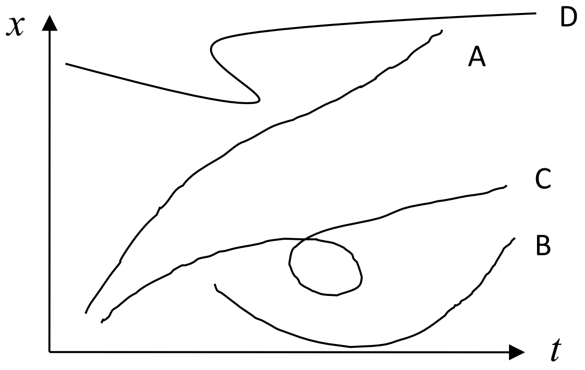


Time–Space Diagrams

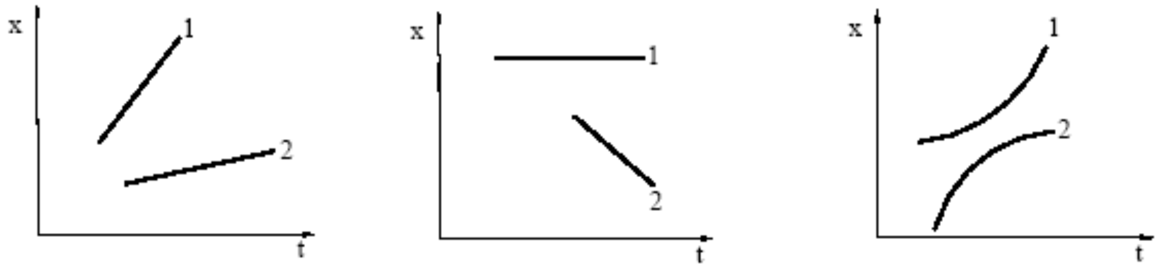


Interpreting time space diagrams

- Which are possible vehicle trajectories?



- Describe the vehicle motion



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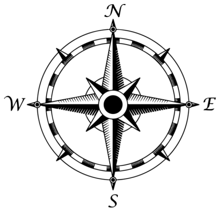
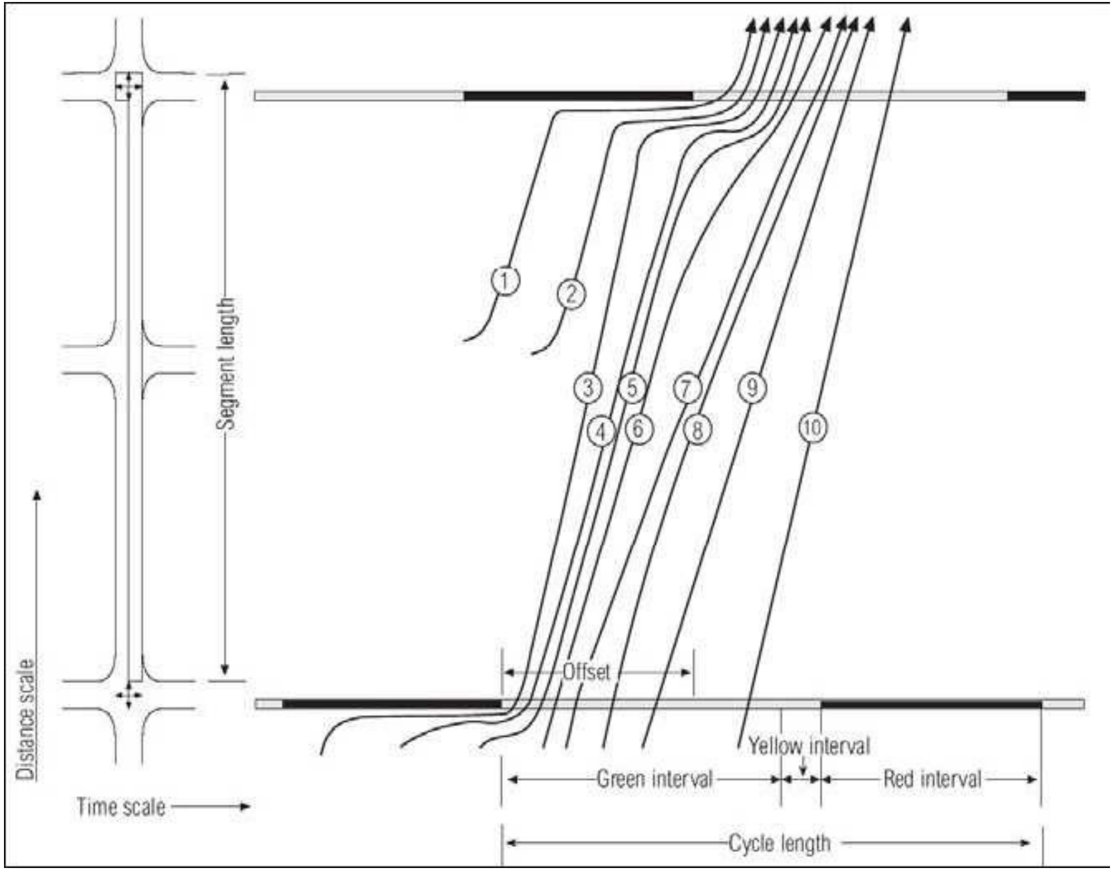
Signalized intersections

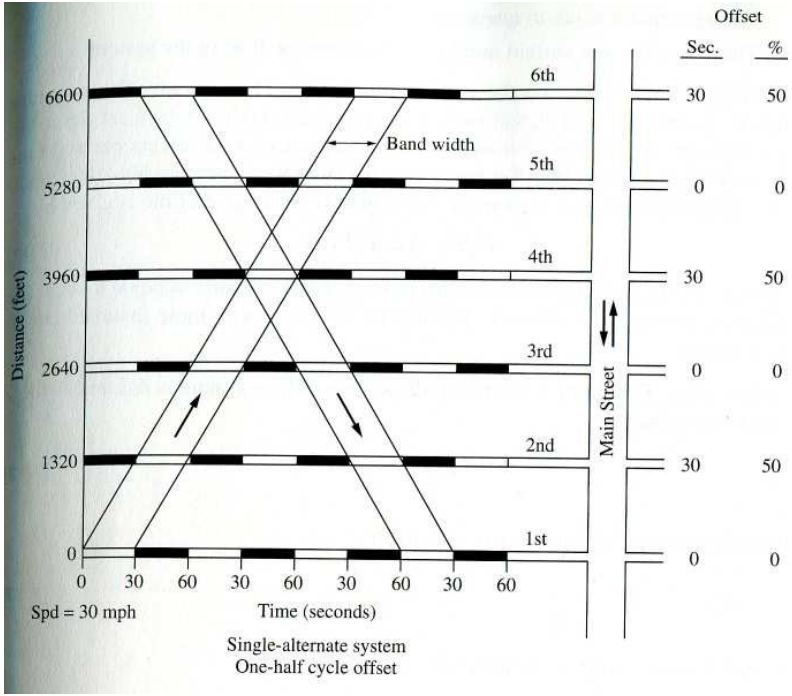
- Two intersections with signals and an intermediate cross street with a stop sign

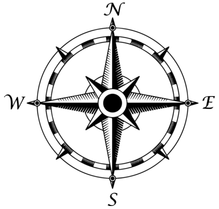




Signal timing design

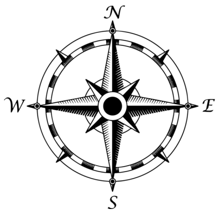
- Fixed-time control analysis

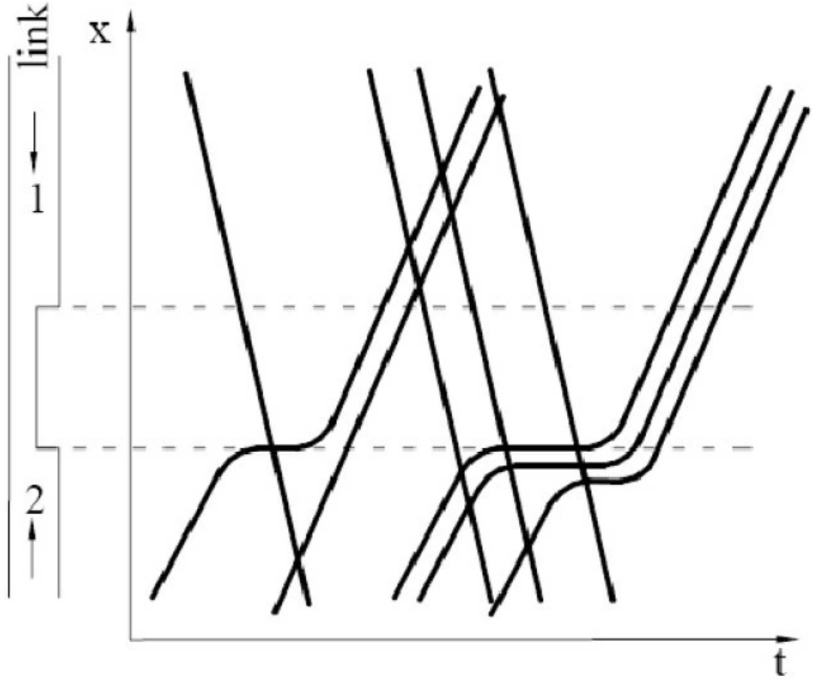




From Meyer and Miller (2001)

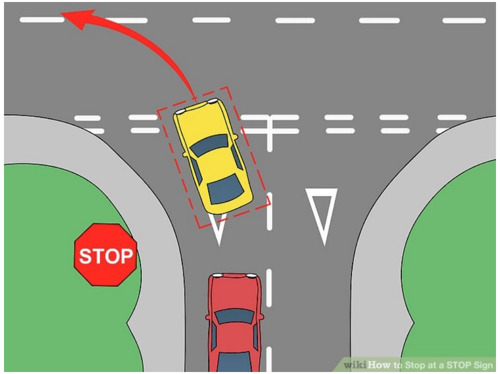
Lane reduction

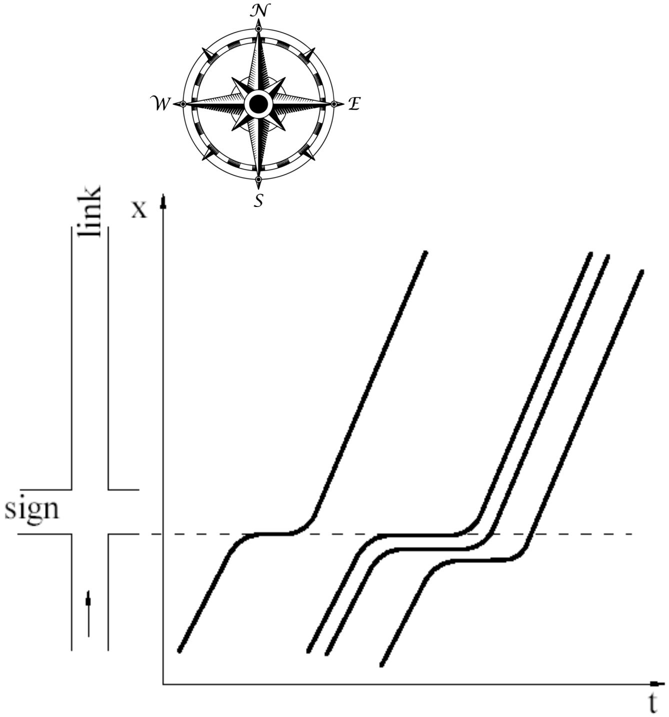




Trajectories reveal a lot of information

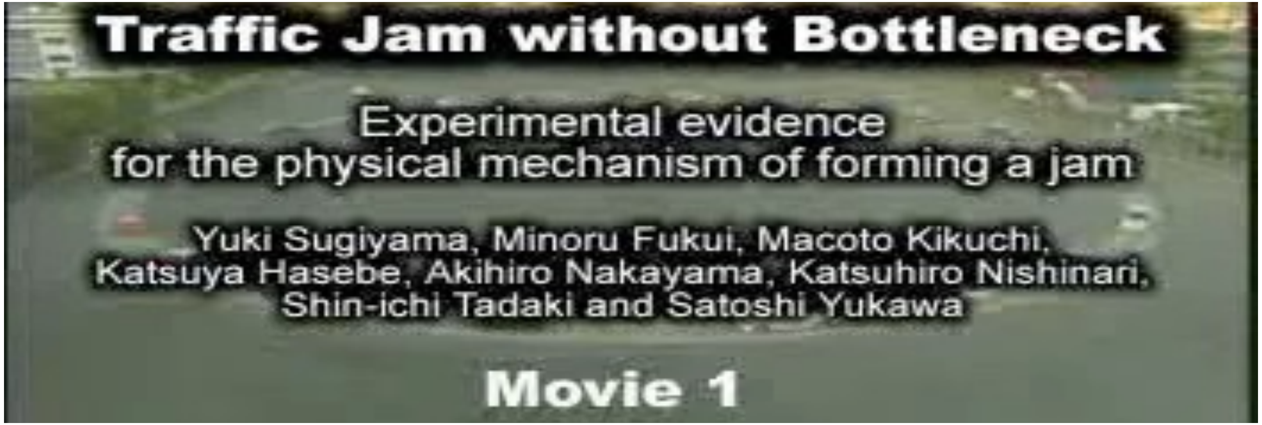
- Intersection with stop sign



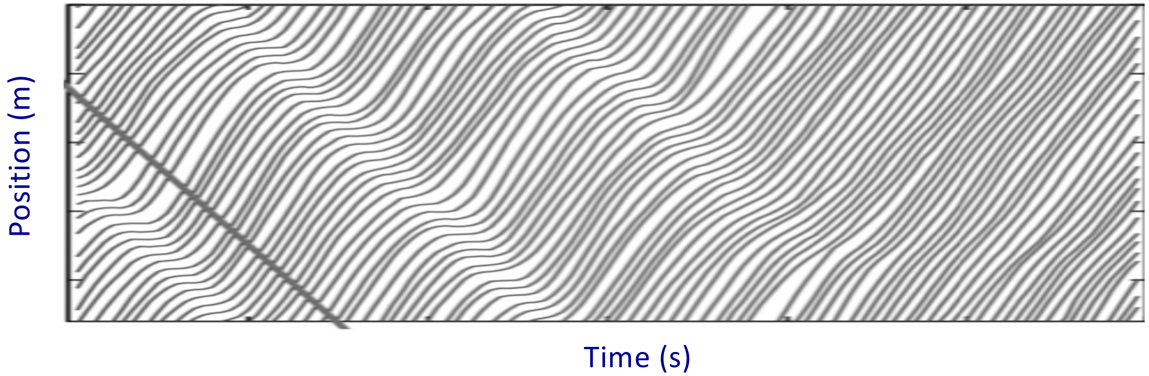


- Any violations of the law?

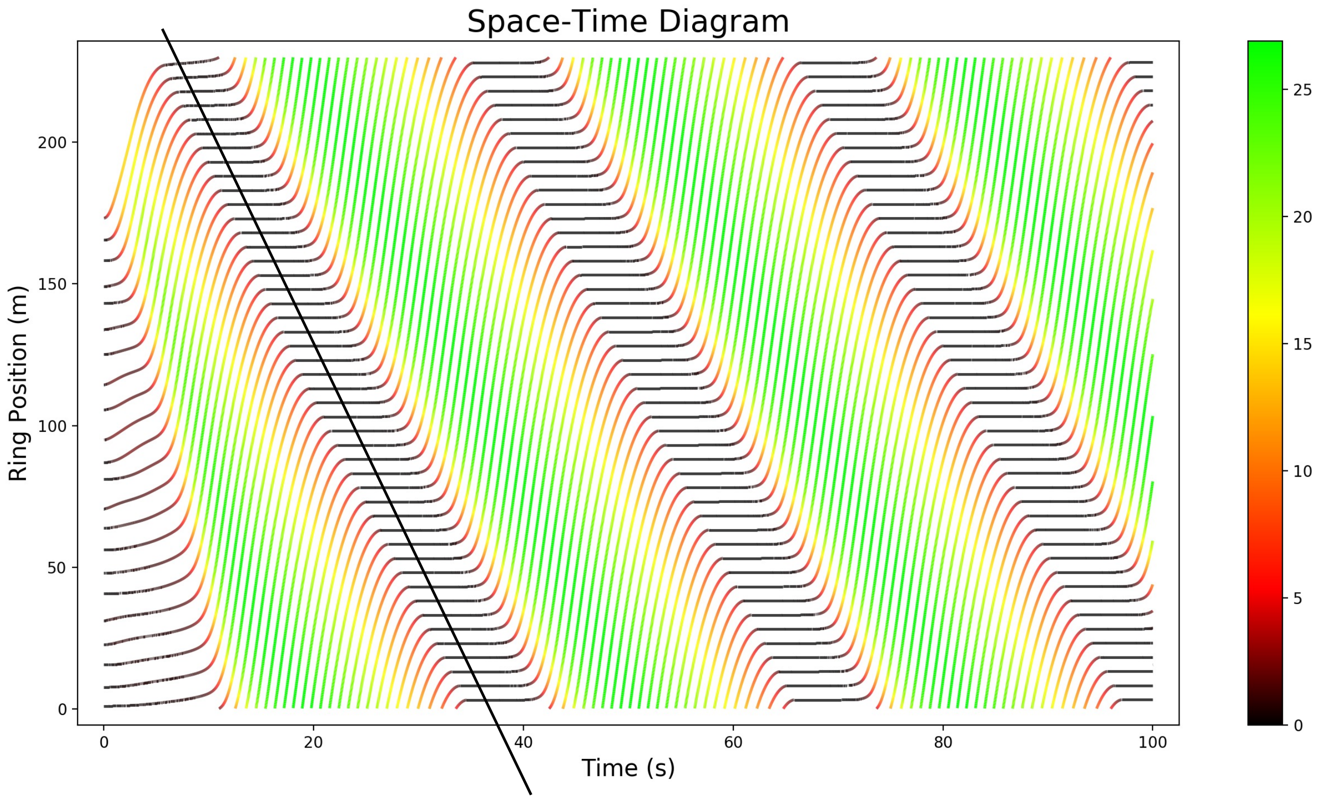
Trajectories can show traffic waves



Vehicle trajectories (Sugiyama et al. 2008)



Trajectories can show traffic waves

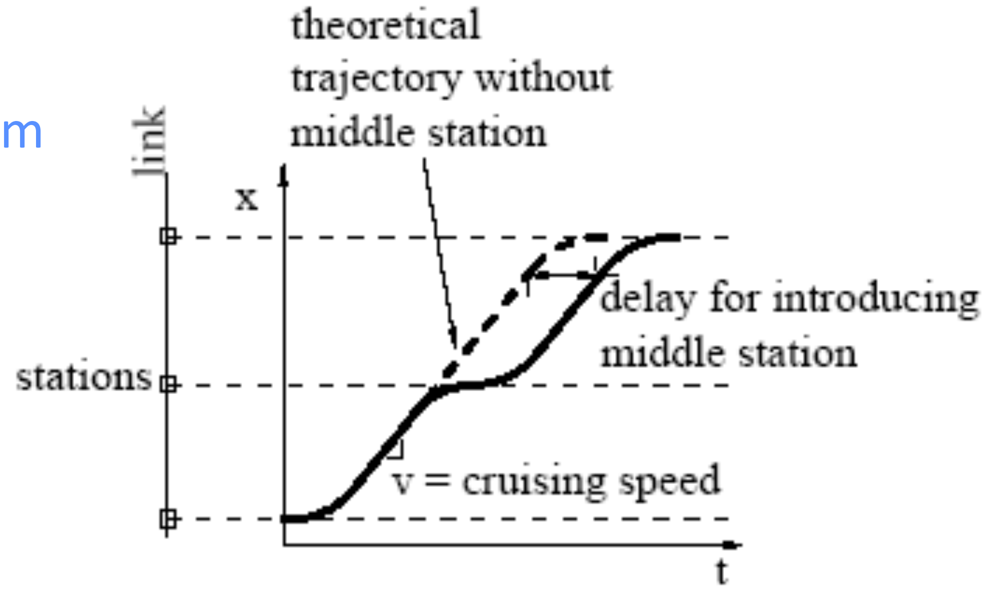


Rate of backwards wave propagation

Wu, et al. IEEE T-RO, 2021.

Transit station placement problem

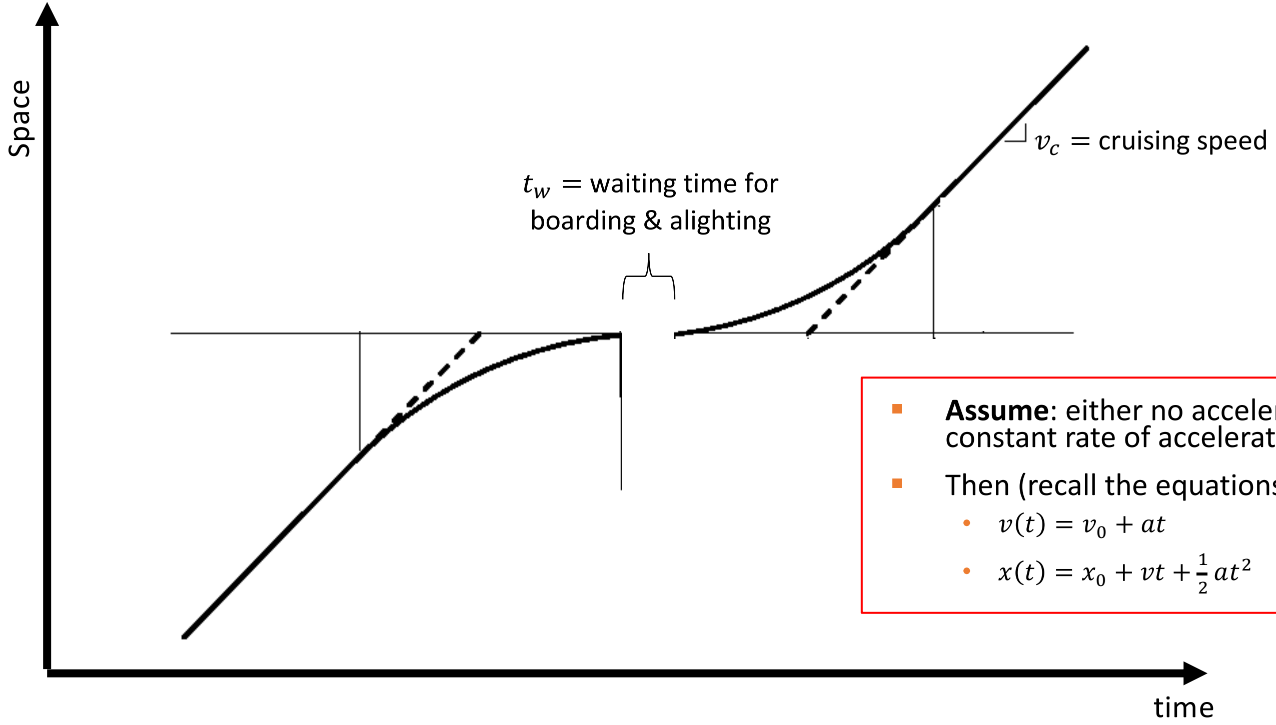
- Consider 2 adjacent transit stations.
- What's the delay incurred from introducing an intermediate station?



- Note that if the distance between stations is not long enough the vehicle can't reach its cruising speed.

Transit station placement problem

- Definition (Delay): The additional travel time experienced by a driver, passenger or pedestrian due to circumstances that impede the desirable movement of traffic.
- Delay = (actual travel time) – (free-flow travel time)



- **Assume**: either no acceleration or constant rate of acceleration $\pm a_c$
- Then (recall the equations of motion):
  - $v(t) = v_0 + at$
  - $x(t) = x_0 + vt + \frac{1}{2}at^2$

Time-space diagram – In a nutshell

- Analyze performance of multiple vehicles along a shared path
- (t, x)-diagram is useful to examine or coordinate the schedules of various vehicles that interact while traveling on the same path, to operate the system efficiently.
- Generally, they enable us to estimate/analyze:
  - Headway between operations at various transportation facilities
  - Capacity of transportation systems
  - Level of service
  - Exclusive rights-of-way, shared rights-of-way

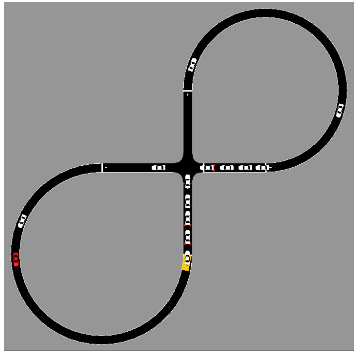
Time-space Diagrams: Advanced Use Cases

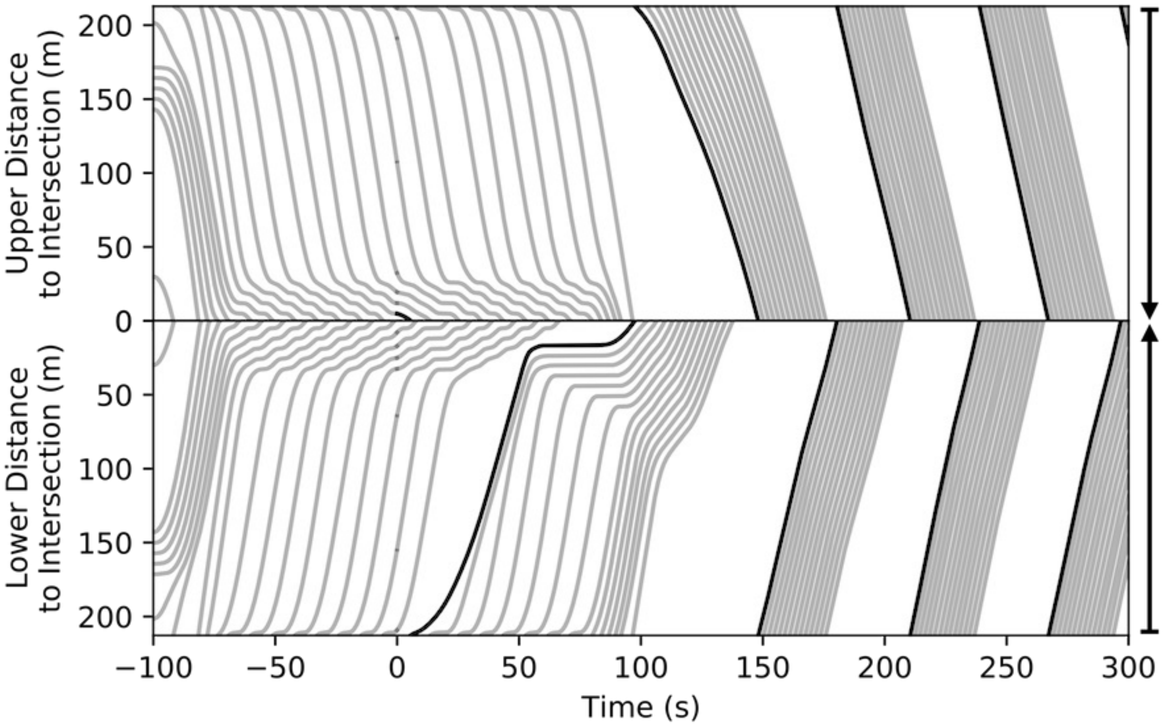
Optimizing trajectories

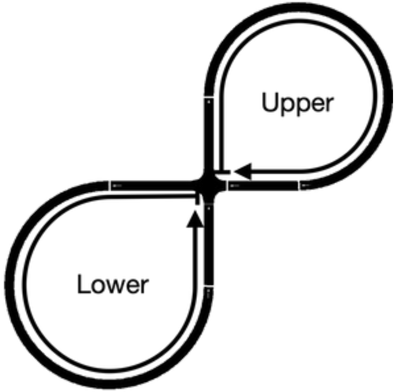
- Visualizing junctions

Uncoordinated
3.37 m/s

Coordinated
7.34 m/s



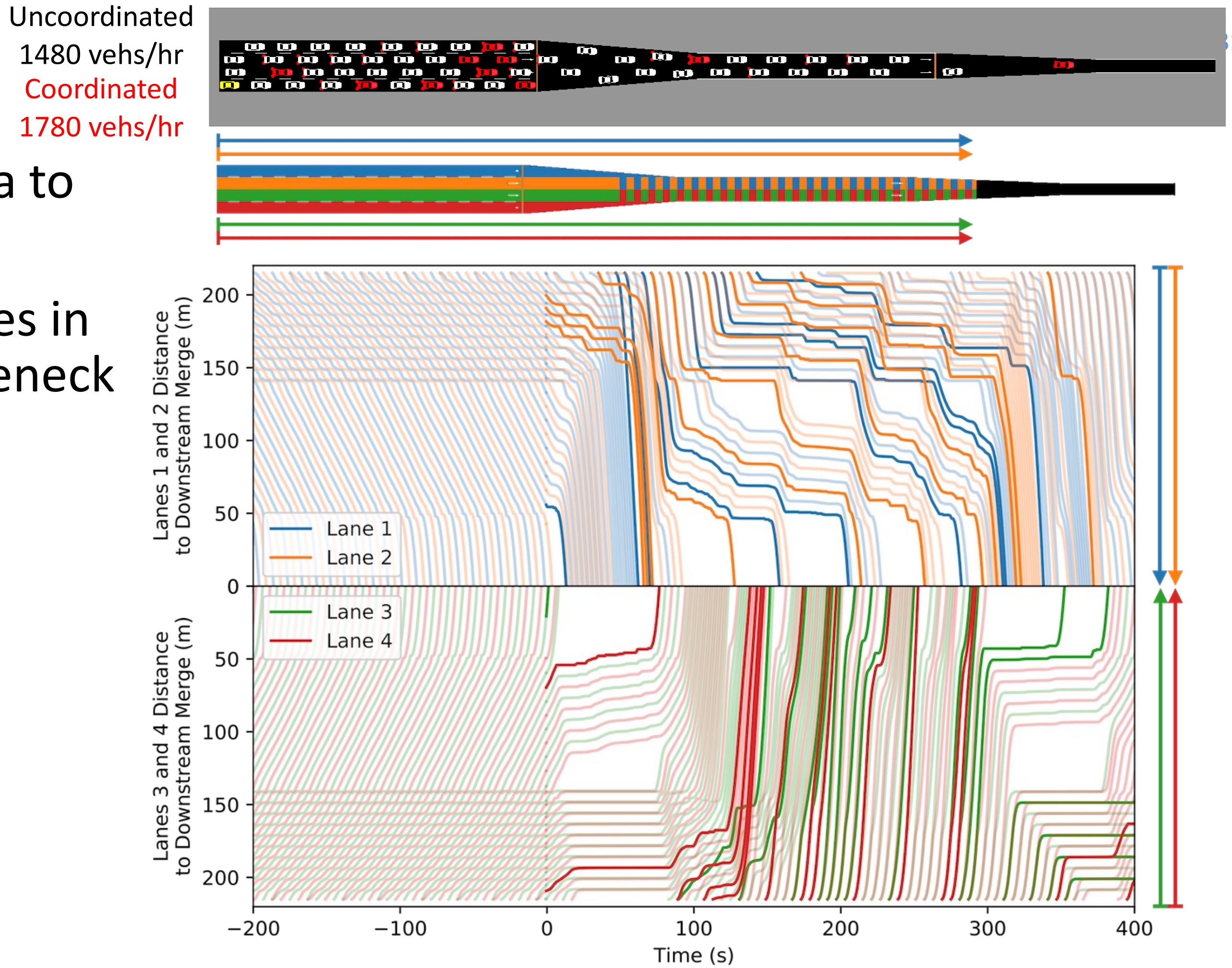




Yan, Kreidieh, Vinitsky, Bayen, **Wu**. Unified Automatic Control of Vehicular Systems with Reinforcement Learning. T-ASE (2022)

More…

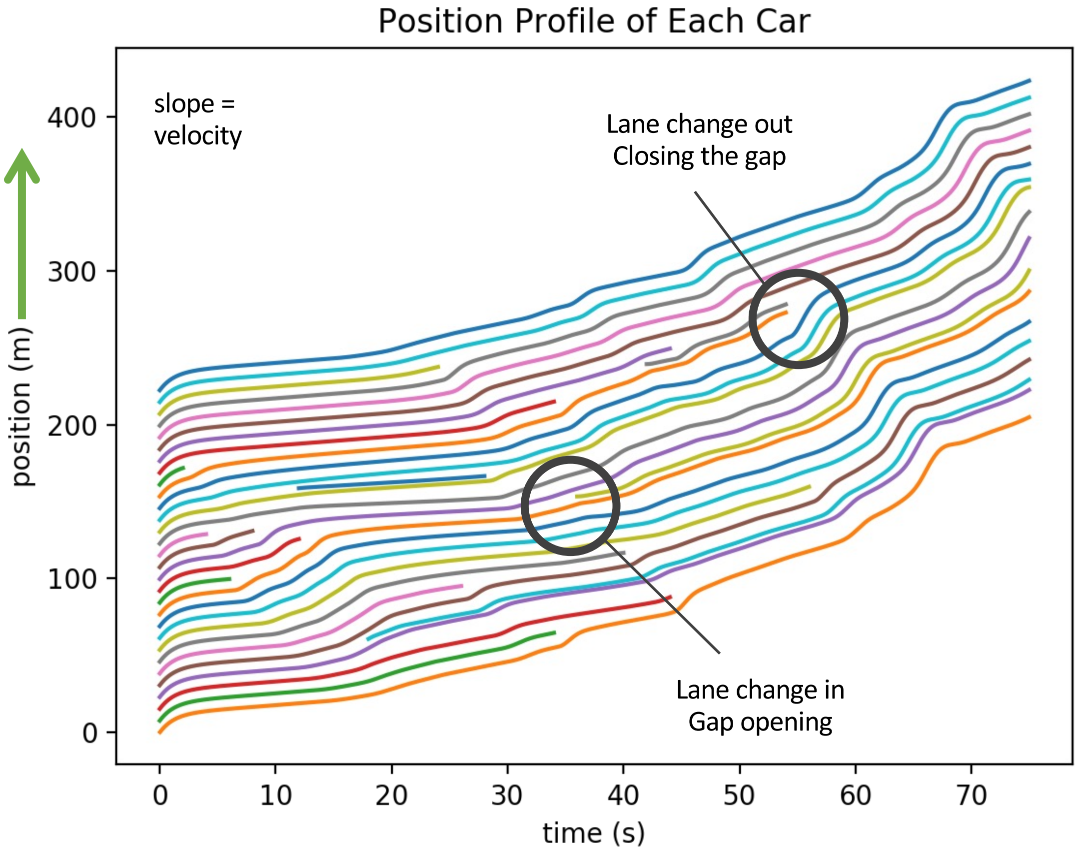
- Use colors/alpha to add dimensions
- Ex. Multiple lanes in a highway bottleneck



Yan, Kreidieh, Vinitsky, Bayen, **Wu**. Unified Automatic Control of Vehicular Systems with Reinforcement Learning. T-ASE (2022)

Modeling multi-lane phenomena

- Lane changing

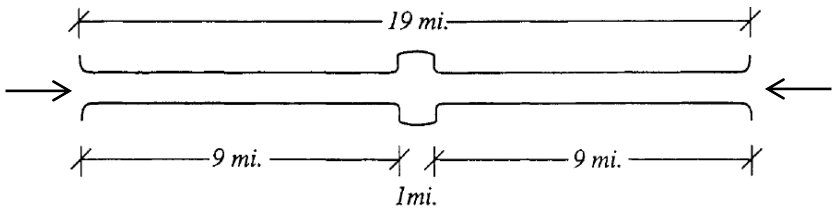


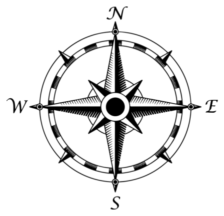
Wu, Cathy, Eugene Vinitsky, Aboudy Kreidieh, and Alexandre Bayen. "Multi-lane reduction: A stochastic single-lane model for lane changing." IEEE ITSC, 2017.
S. Li, R. Dong, and C. **Wu**, “Hybrid System Stability Analysis of Multi-Lane Mixed-Autonomy Traffic,” *IEEE T-RO*, 2024, doi: 10.1109/TRO.2024.3443504.

Outline

1. **Time-space diagrams**
   a. Applications and traffic system design: road, air, rail, transit
   b. **Exercise: Waterway capacity problem**
2. From sensors to data to trajectories to time-space diagrams

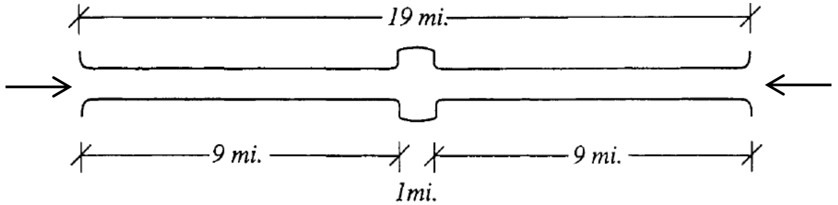
Waterway Capacity Problem

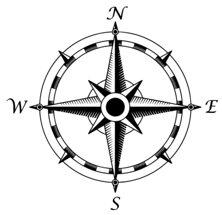




- Evaluation/design of a waterway.
- Setting: Waterway with an intermediate siding for ship crossings
  - The waterway is wide enough for 1 ship only, except in the central siding which is wide enough for 2 ships
  - Westbound ships travel full of cargo and are thus given high priority by the canal authority over the eastbound ships which travel empty

Waterway Capacity Problem





Problem:

- Ships can travel at an average speed of 6 miles/hour
- Ships must be spaced at least 0.5 miles apart while moving in the waterway and 0.25 miles apart while stopped in the siding
- Westbound ships travel in 4 ship convoys which are regularly scheduled every 3.5 hours and do not stop at the siding.
- Eastbound ships must allow a 5-minute clearance from westbound ships when using the one-way sections. We do this to take into account that ships do not accelerate instantaneously.
- Reminder: Westbound ships have priority over eastbound ships.
- For an 8-hour period, determine
  - The maximum daily traffic of eastbound ships
  - The maximum daily traffic of eastbound ships if the siding is expanded by one mile on both sides to a total of three miles.


The Cause
of 2021's
Shortages

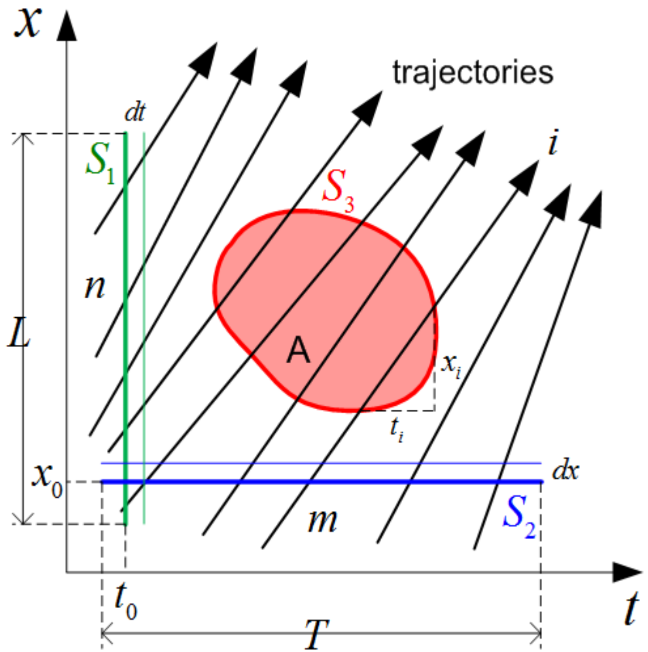
Outline

1. Time-space diagrams
    a. Applications and traffic system design: road, air, rail, transit
    b. Exercise: Waterway capacity problem
2. **From sensors to data to trajectories to time-space diagrams**

Trajectory measurements

- **Three types of measurements:**
  - **S1**-aerial surveys,
    such as aerial photograph
  - **S2**-stationary observers,
    such as loop detectors
  - **S3**-moving observers,
    such as driver logs

When trajectory data are available, they are the **most** appropriate.



Today’s sensing technologies

loops

video

GPS

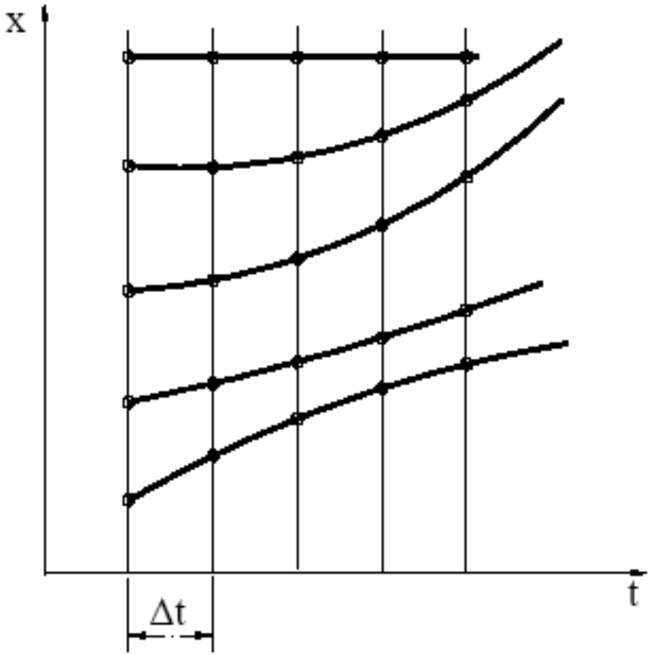
magnetometer

radar

RFID

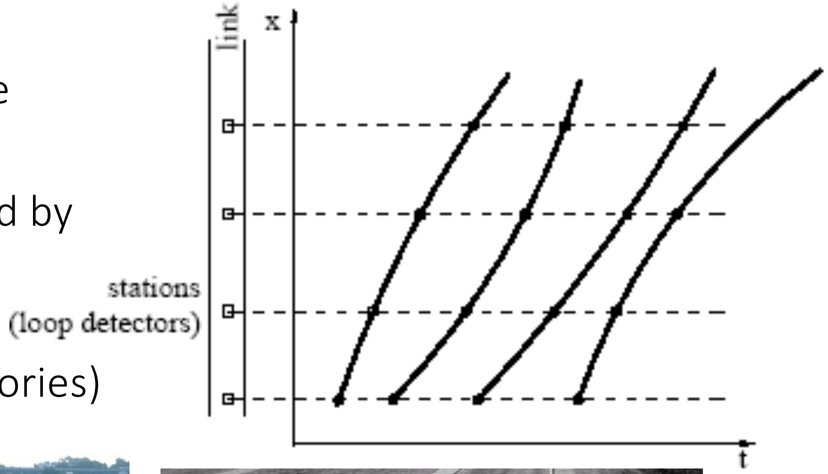
Aerial Surveys (e.g., overhead cameras)

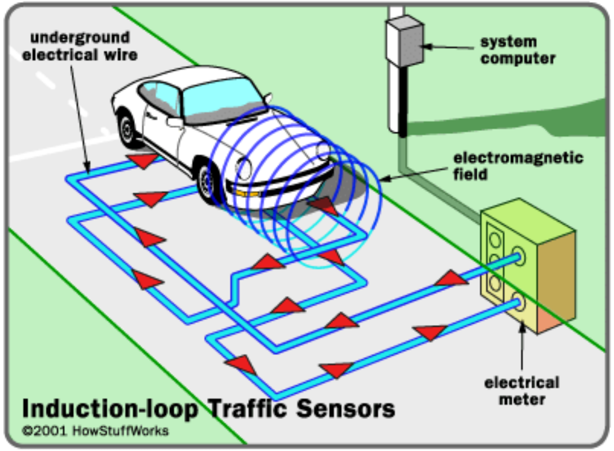
- Take consecutive photographs to the same road segment
- Place them next to each other, separated by the time interval between shots
- Draw lines across the different pictures following the location of the individual vehicles (these are the trajectories)

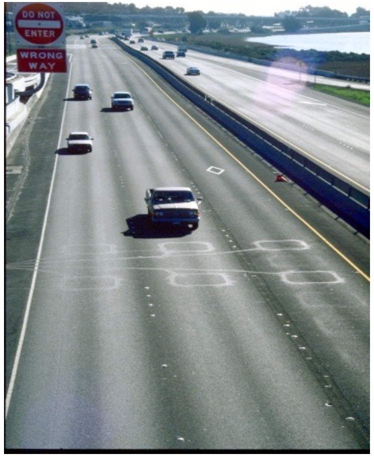


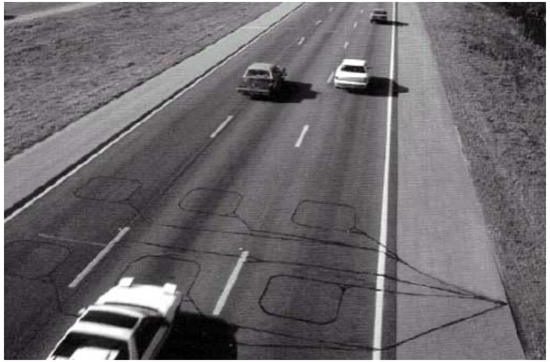
Stationary observers (e.g., loop detectors)

- Measure the time at which every vehicle passes the observers
- Place them next to each other, separated by the distance intervals
- Draw lines following the time of the individual vehicles (these are the trajectories)



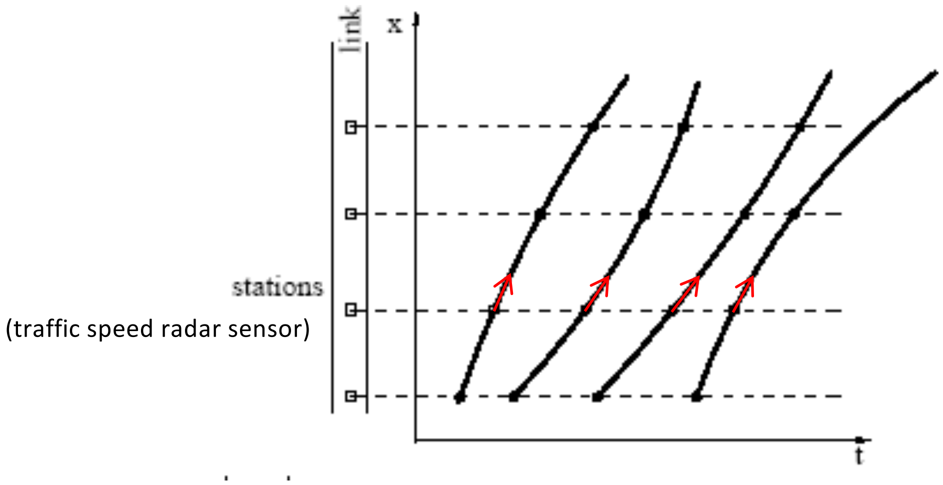






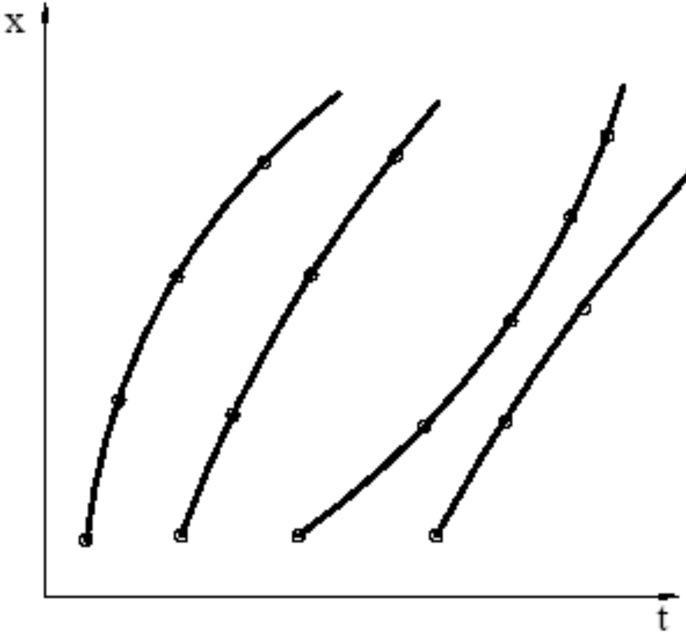
Stationary observers

- Measuring vehicle speed instead of vehicle counts



Moving observers: driver logs (e.g., GPS)

- Drivers record the time and location along their trip
- Plot the corresponding points
- Draw lines following the points corresponding to the individual vehicles (these are the trajectories)



Caveat: Sensor data can be maliciously manipulated

Artist creates traffic jams in Google Maps with a wagon full of phones

This is the kind of post-modern art we can get behind.

Igor Bonifacic, @igorbonifacic
February 3, 2020

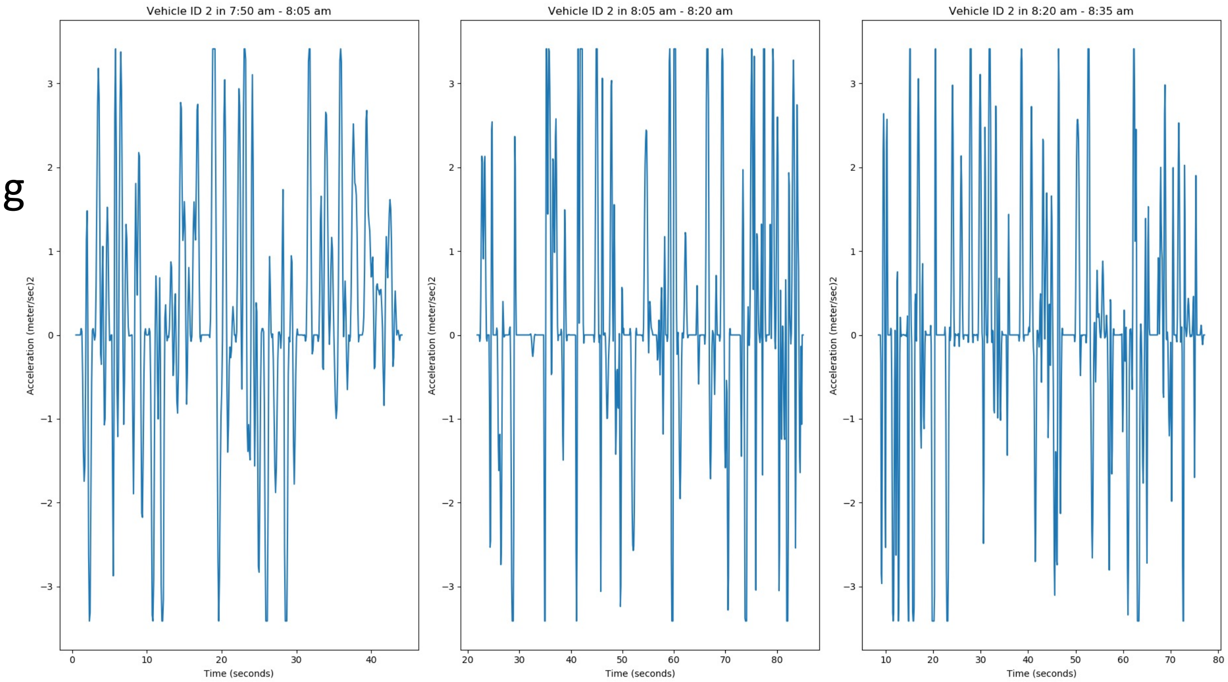
Artist: Simon Weckert | Source: Engadget

Keep these in mind for class projects!

Real trajectory datasets you can play with

NGSIM US 101 (2005)

- https://ops.fhwa.dot.gov/trafficanalysistools/ngsim.htm
- Canonical dataset for traffic modeling
- Vehicle trajectories from 8 mounted cameras
- 45 minutes of recording
- Data quality issues



NGSIM US-101 Dataset Smoothing, 2019. https://github.com/Rim-El-Ballouli/NGSIM-US-101-trajectory-dataset-smoothing

HighD dataset (2018)

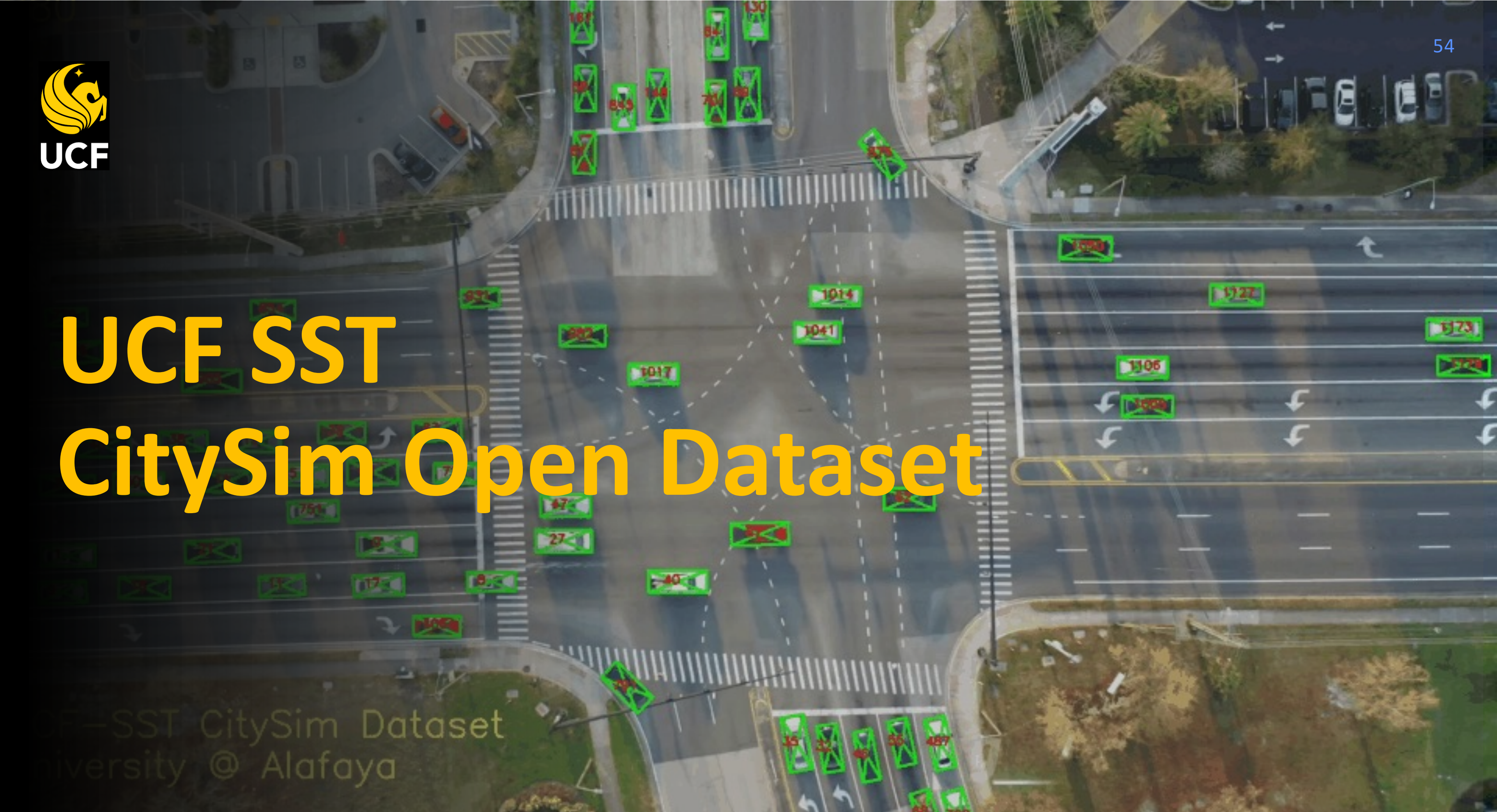
Drone-captured trajectories
16.5 hours of recording

New Request access to the dataset!

[https://www.highd-dataset.com]

Krajewski, et al. The highD Dataset: A Drone Dataset of Naturalistic Vehicle Trajectories on German Highways for Validation of Highly Automated Driving Systems. ITSC, 2018.

UCF SST CitySim Open Dataset



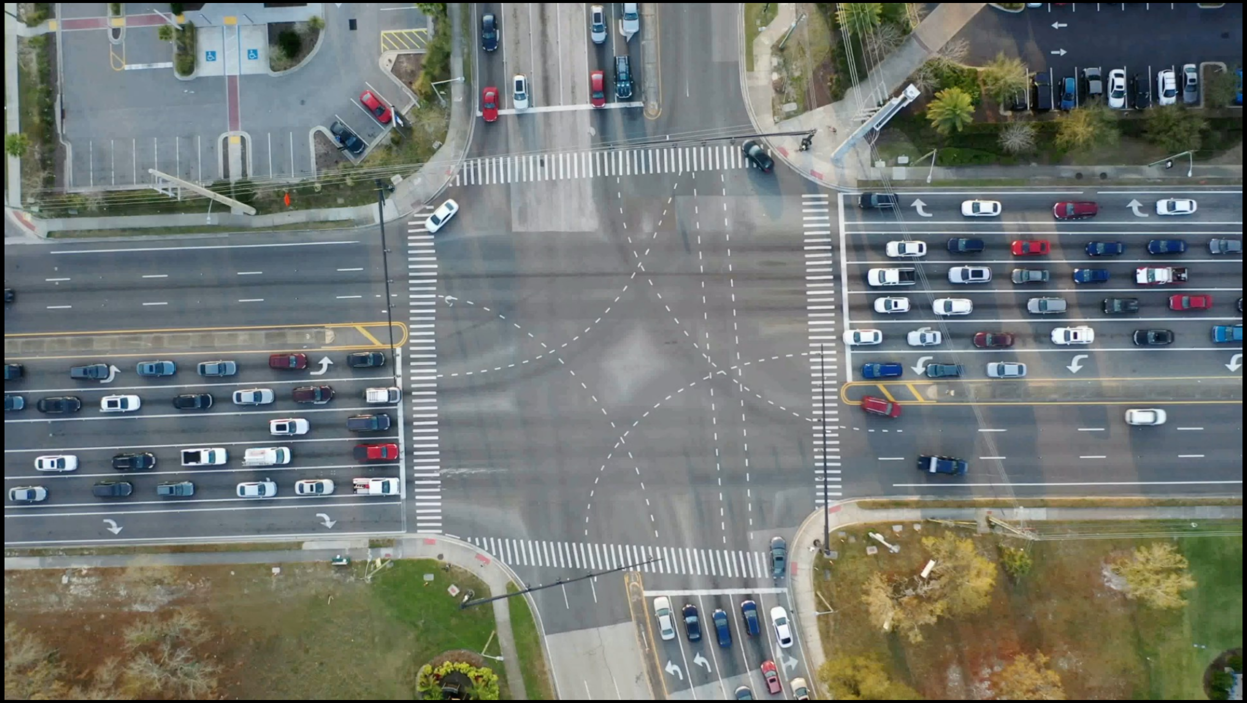
Zheng, Abdel-Aty, Yue, Abdelraouf, Wang, Mahmoud, "CitySim: A Drone-Based Vehicle Trajectory Dataset for Safety Oriented Research and Digital Twins." arXiv, 2022.

CitySim: A Drone-Based Vehicle Trajectory Dataset for Safety Oriented Research and Digital Twins

Mission: facilitating traffic safety-based research and digital twining

Meta Info:

- 12 locations:
  - freeway basic segments,
  - weaving segments,
  - merge/diverge segments,
  - signalized intersections,
  - non-signalized intersections
- 1140-minutes record duration (19 hours)
  - peak hours
  - Over 2 million frames
- First to provide vehicle rotated bounding boxes GPS trajectory
- Dense conflicts:
  - rear-end,
  - lane change,
  - merging/diverging conflicts, etc.
- High-fidelity digital twin 3D maps
- GIS road network file
- Matched signal timing, crash reports at the locations



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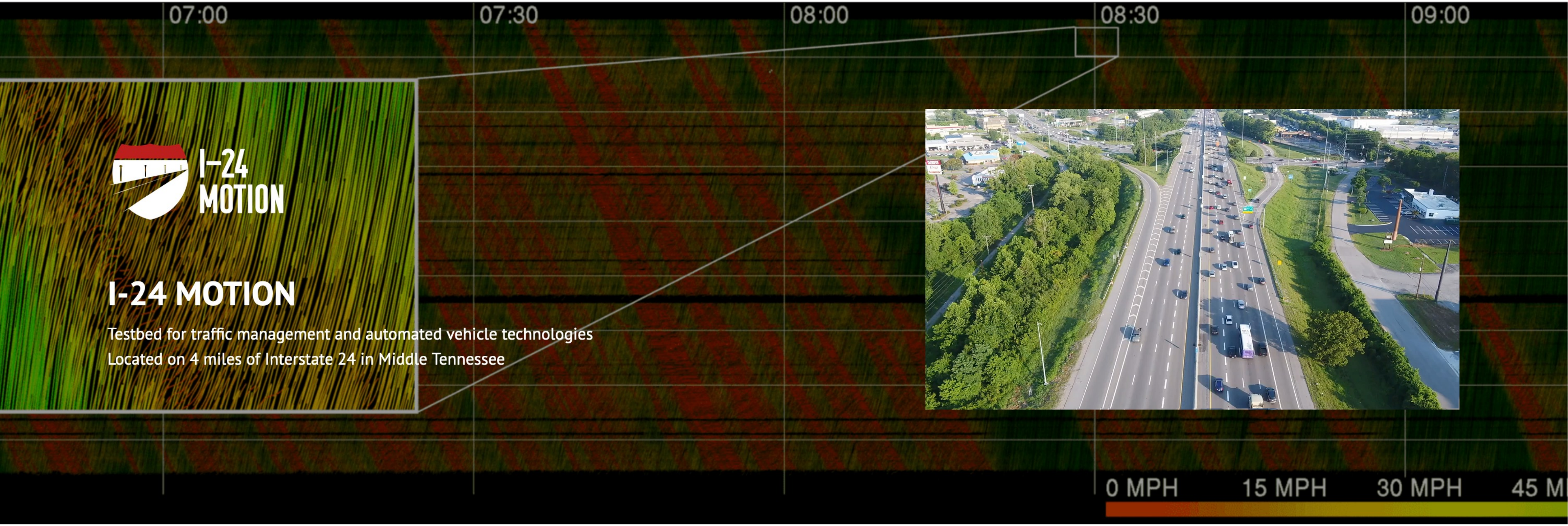
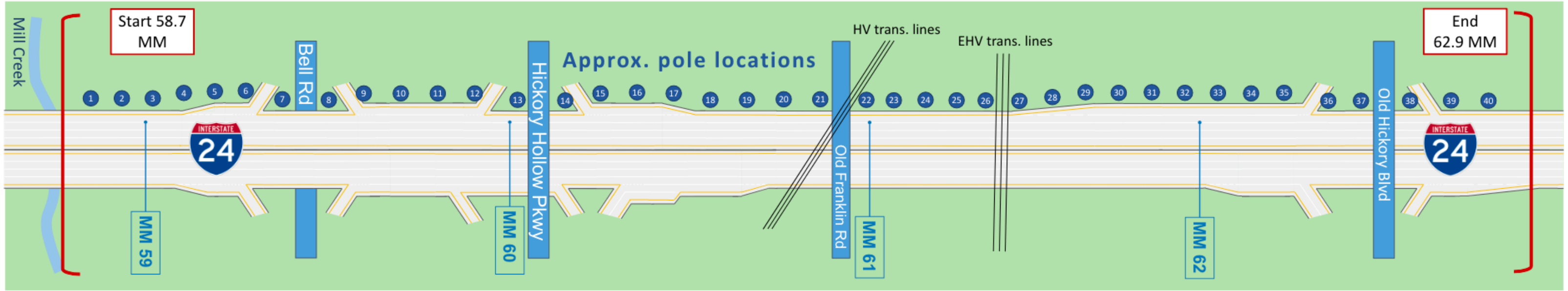
I-24 MOTION testbed

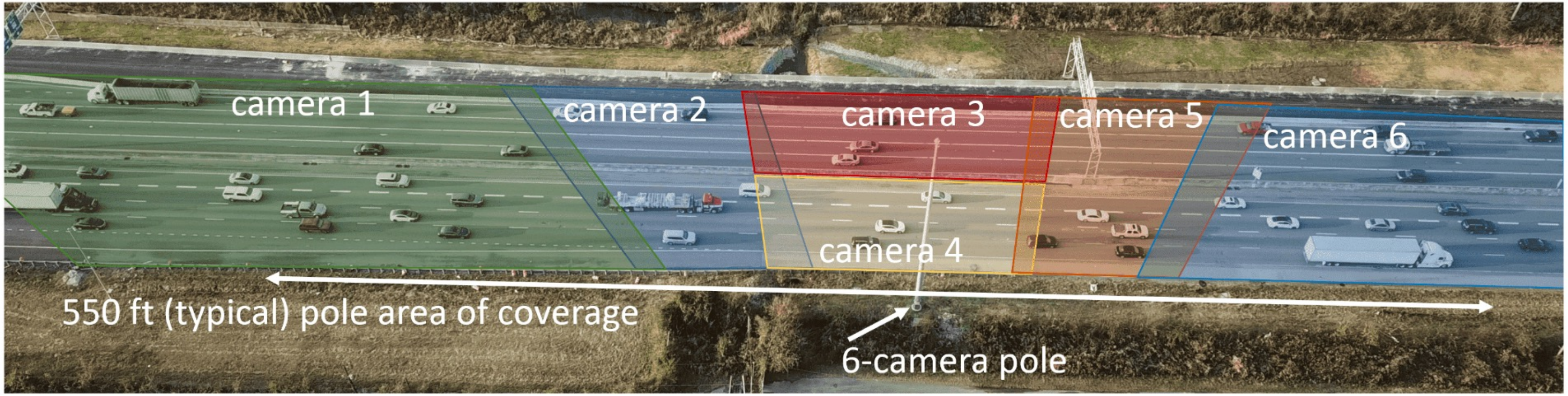


Initial INCEPTION v1.0.0 dataset: 4 hours x 10 days ≈ 1.7 million vehicle miles

D. Gloudemans *et al.*, "I-24 MOTION: An instrument for freeway traffic science," *Transportation Research Part C: Emerging Technologies*, vol. 155, p. 104311, Oct. 2023, doi: 10.1016/j.trc.2023.104311.

I-24 MOTION testbed





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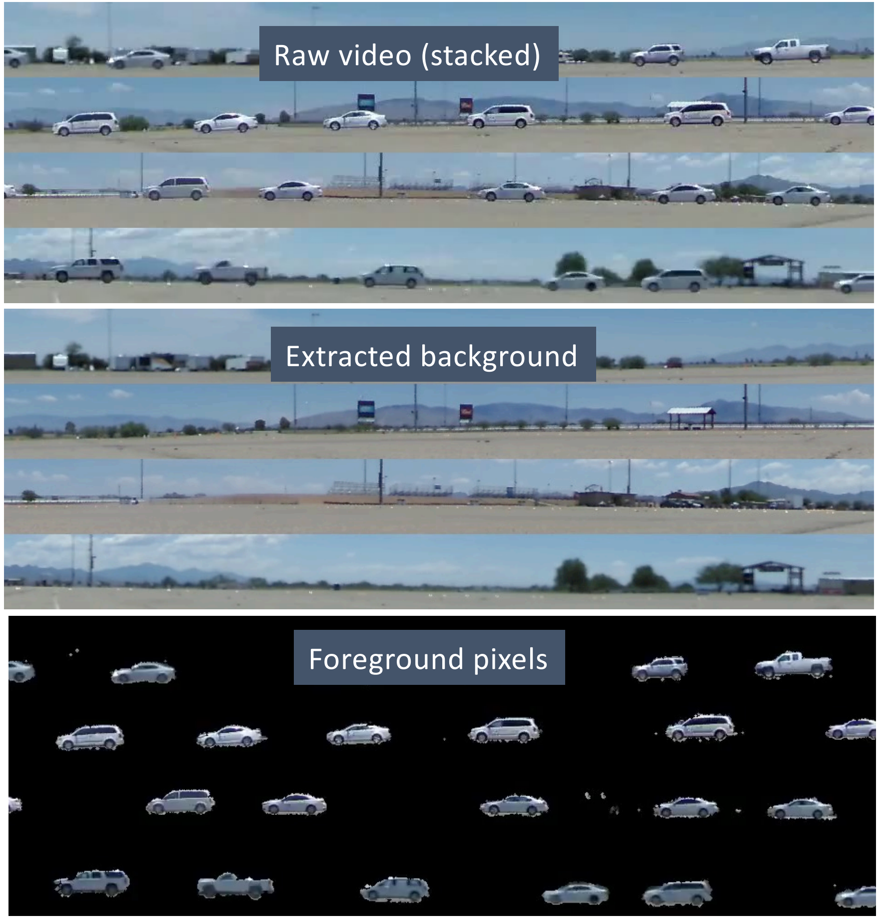
Conceptual overview of camera based tracking

- Step 1. Identify Foreground
  - Filter moving pixels
  - estimate static background image
  - Subtract background to find vehicle pixels



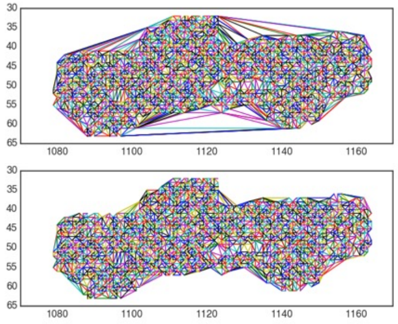
[Stern, et al 2017]

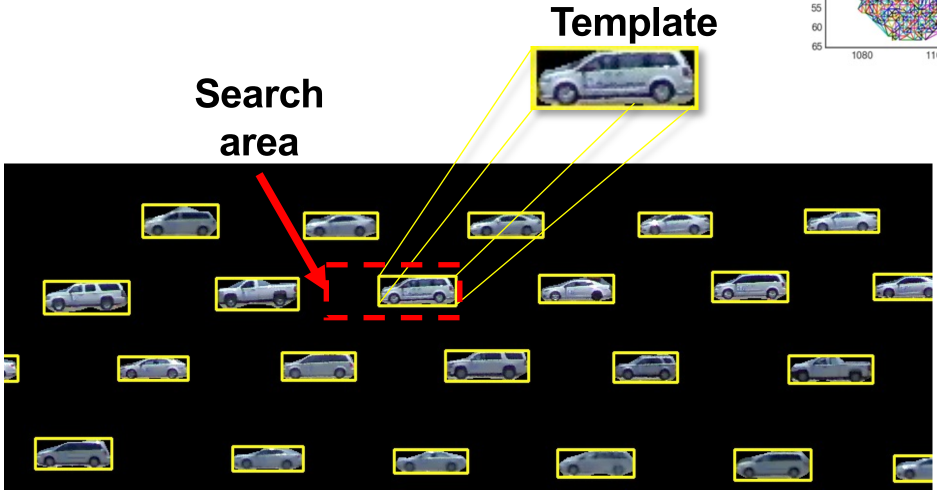
Conceptual overview of camera based tracking

- Step 2. Cluster foreground pixels
  - Construct a template for each vehicle

- Step 3: Tracking
  - Match template frame by frame

Template refinement from pixel cluster





[Stern, et al 2017]

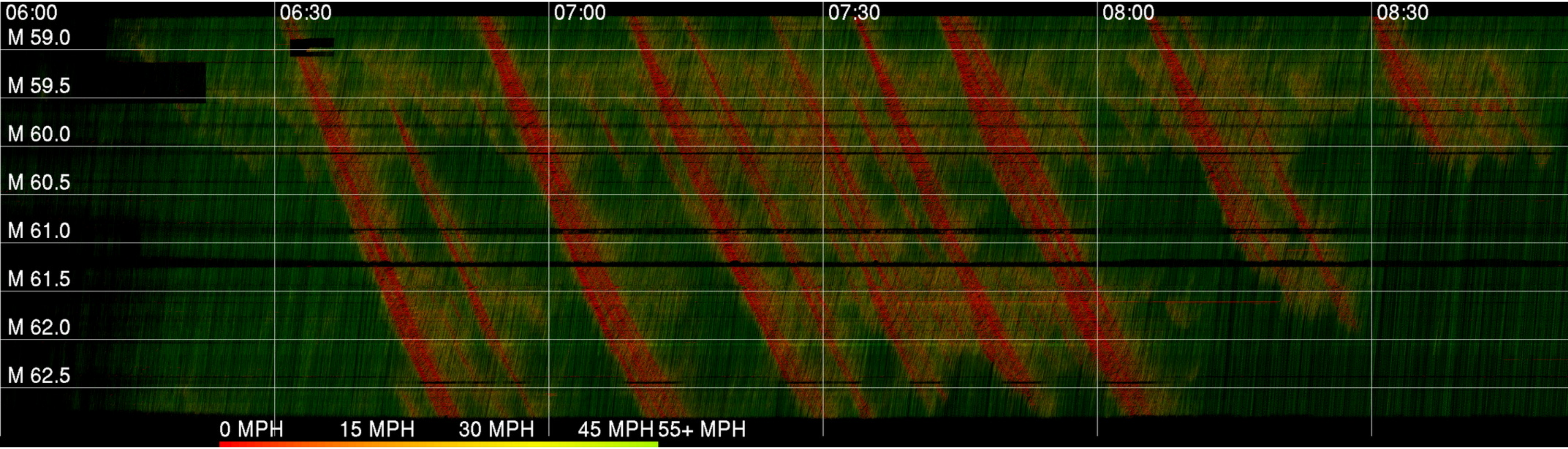
Conceptual overview of camera based tracking

- Position Accuracy: 10 cm error; matched with human annotated data
- Velocity Accuracy: 0.14 m/s error; matched with Odometer data

[Stern, et al 2017]

Final product

- See anything interesting?



D. Gloudemans et al., "I-24 MOTION: An instrument for freeway traffic science," Transportation Research Part C: Emerging Technologies, vol. 155, p. 104311, Oct. 2023, doi: 10.1016/j.trc.2023.104311.

References

1. C. Daganzo, *Fundamentals of transportation and traffic operations*, vol. 30. Pergamon Oxford, 1997. Chapter 1: The time-space diagram.
2. Prof. Nikolas Geroliminis' lecture Fundamentals of Traffic Operations and Control, Spring 2010 EPFL
3. Chap 7 of Prof. Michael Meyer and Prof. Eric Miller's book Urban Transportation Planning (2001)
4. Some slides adapted from Profs. Carolina Osorio and Dan Work.