Graphical analysis II

Cumulative diagrams

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1.041/1.200 Transportation: Foundations and Methods

Readings

1. C. Daganzo, *Fundamentals of transportation and traffic operations*, vol. 30. Pergamon Oxford, 1997. Chapter 2: Cumulative plots. URL.
2. John D.C. Little and Stephen C. Graves, Chapter 5: Little's Law from *Building Intuition: Insights From Basic Operations Management Models and Principles*, 2008. doi: 10.1007/978-0-387-73699-0.
3. (Optional) *How Do Traffic Signals Work?* Practical Engineering, YouTube, 2019. URL.

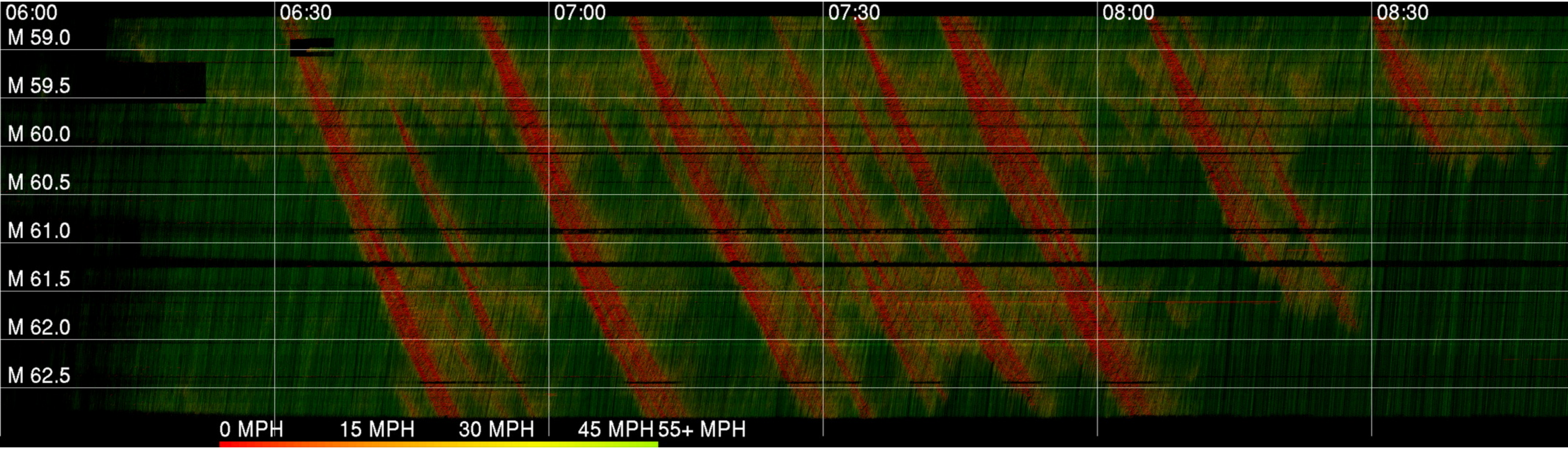
Outline

1. Cumulative diagrams
2. Ramp metering problem

Outline

1. **Cumulative diagrams**
    a. Application: Signalized intersections
2. Ramp metering problem

Limitations of time-space diagrams

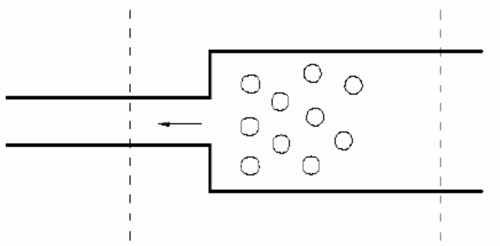


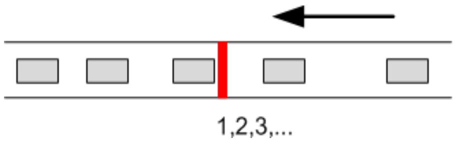
- Delay: How long did drivers wait?
- Issue: Too many trajectories!

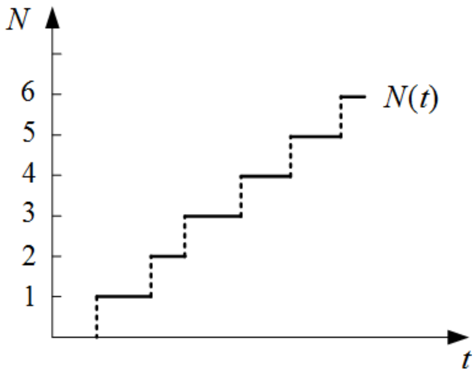
D. Gloudemans et al., "I-24 MOTION: An instrument for freeway traffic science," Transportation Research Part C: Emerging Technologies, vol. 155, p. 104311, Oct. 2023, doi: 10.1016/j.trc.2023.104311.

Cumulative diagram – Moskowitz (1954)

- Cumulative diagrams $N(t)$ are for analyzing delay in transportation systems.
- Represents the cumulative number (count) of arrivals at a fixed location



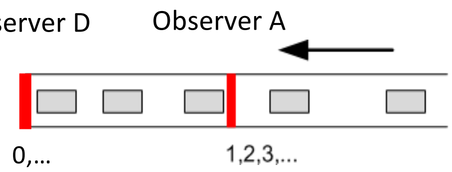




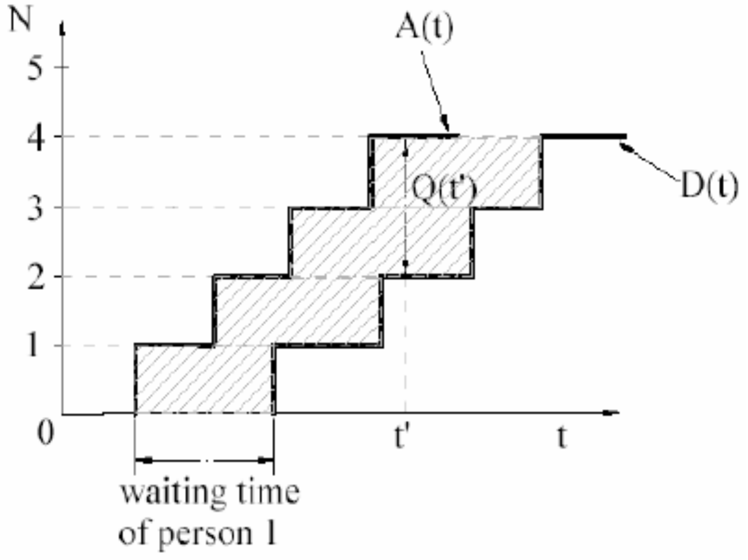
Slide adapted from Prof. Zhengbing He

Moskowitz, K. Waiting for a gap in a traffic stream. Proc. Highway Res. Board, 33, 385-395, 1954.

Cumulative diagram: illustrating system delays

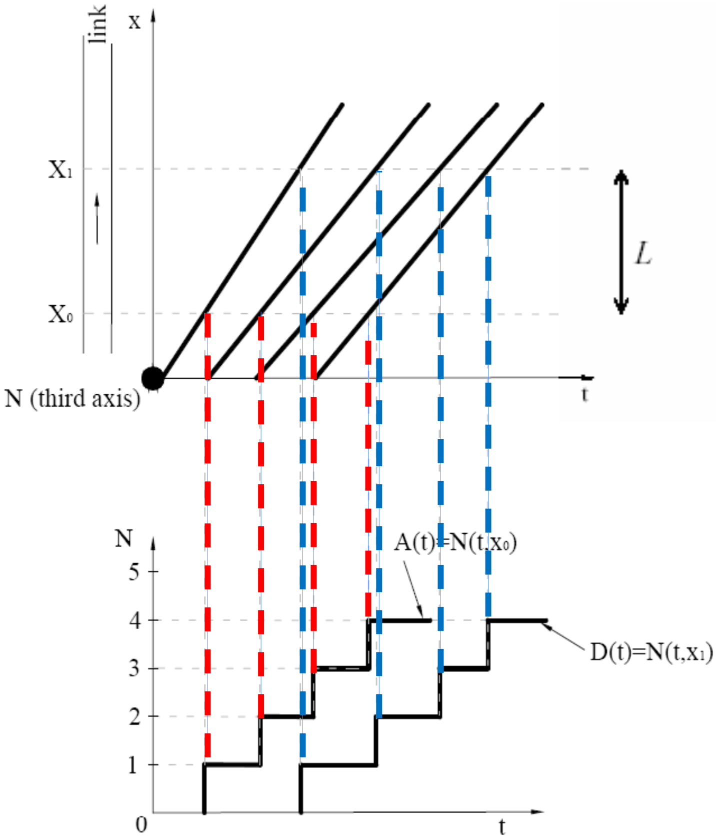


- We can use two observers:
  - Observer A looks at the arrivals to the system
  - Observer D looks at the departures from the system
- The following diagram represents the cumulative number of **arrivals and departures**
- Assumptions: FIFO, no passing / reordering



Cumulative diagram vs. time space diagram

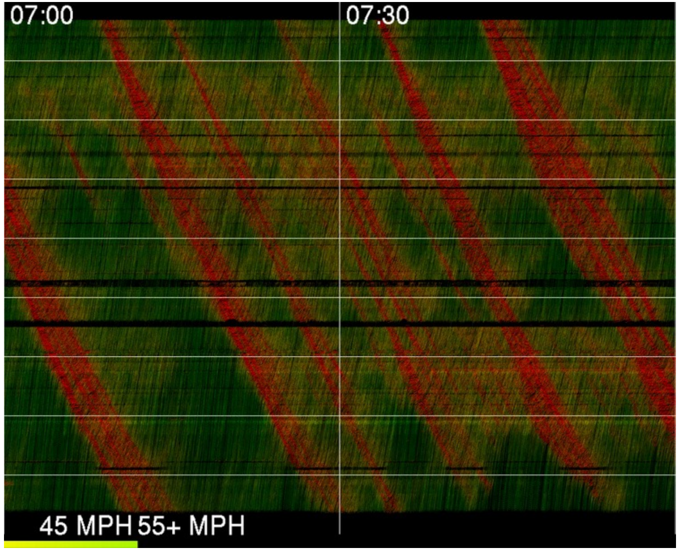
- Cumulative diagram can be obtained from the time-space diagram
  - Time space diagram has complete info
  - Cumulative diagram has a subset of the info



- Remark: Slope of cumulative diagram = flow (veh/time)

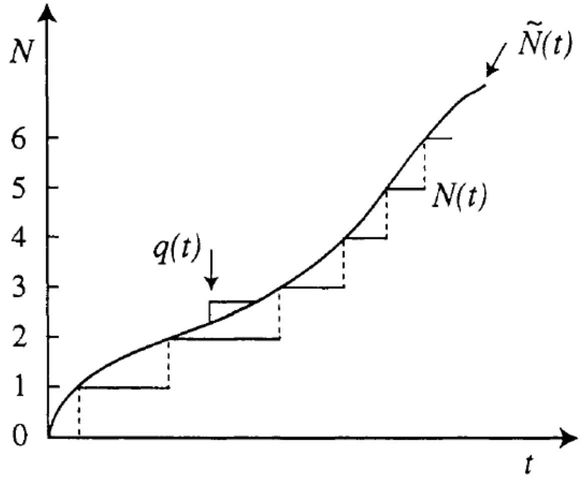
Use cases – rules of thumb

- Time-space diagram $x(t)$
  - Identifying patterns in trajectories
- Cumulative diagram $N(t)$
  - Identifying properties of a single bottleneck (arrivals and/or departures)

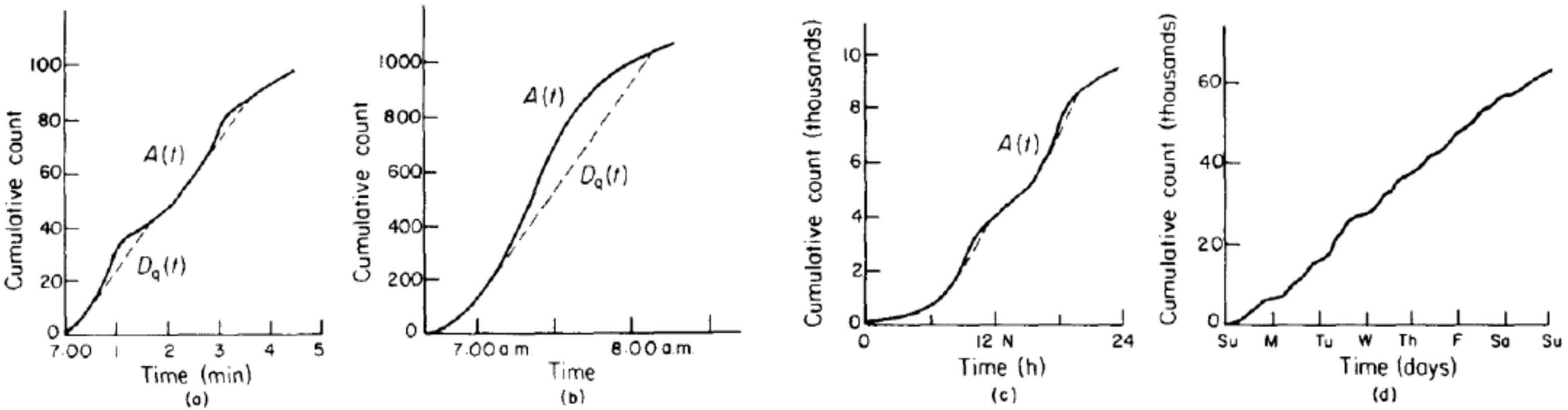


Continuous cumulative diagrams

- Cumulative count are typically discrete in transportation (e.g. passengers, buses, cars), so $N(t)$ is a step function.
- When the exact count is not important, can leverage continuous analysis.
- Advantage for continuous analysis: differential calculus can be used, i.e. $q(t) \approx \frac{d\tilde{N}(t)}{dt}$, where $\tilde{N}(t)$ is a smooth approximation of $N(t)$



Cumulative diagrams apply across scales



Seconds or minutes (queue)

Hours (peak demand or rush hours)

Days (different patterns on different days)

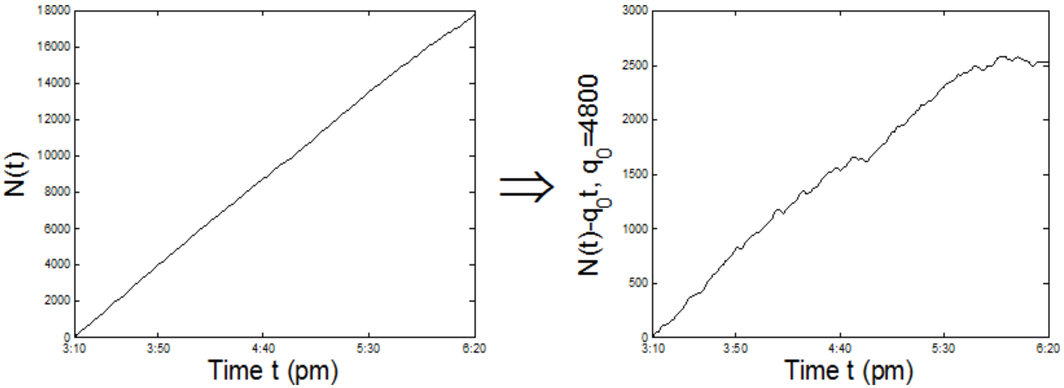
Slide adapted from Prof. Zhengbing He

Oblique cumulative diagram

- In freeways, **large flows** obscure details.
- An **oblique coordinate system** (linear transformation), isolates changes in arrivals by retaining the **relative relationship** [1, 2]:

$$(N, t) \Longrightarrow (N - q_0 t, t)$$

where q_0 is a coefficient.

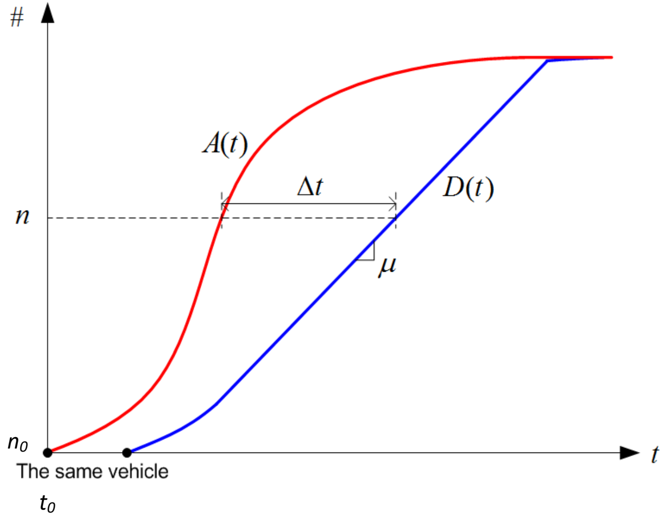


1 Cassidy M. Bivariate relations in nearly stationary highway traffic. Transportation Research Part B. 1998.
2 Cassidy M. Some traffic features at freeway bottlenecks. Transportation Research Part B. 1999;33:25-42.

Slide adapted from Prof. Zhengbing He

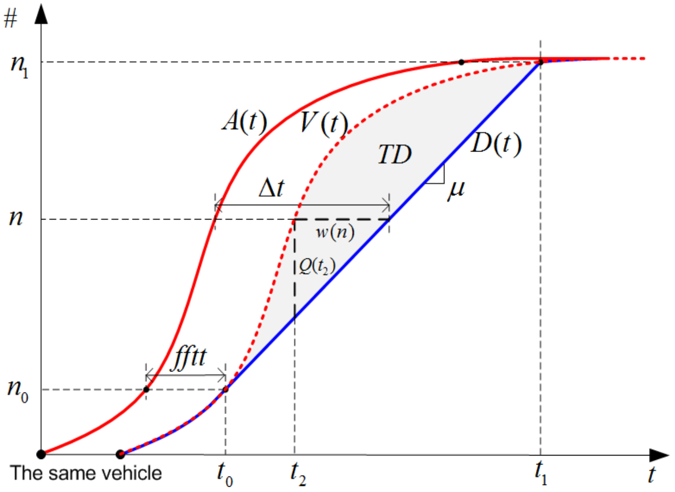
Cumulative arrivals and departures

- Arrivals at x_0; departures at x_1 (not shown)
- Starting counting with a reference customer / vehicle;
  - Let $A(t)$ denote the arrivals
  - Let $D(t)$ denote the departures
  - Let μ denote the service rate (customers / time)



Virtual arrivals

- Total time in the system is composed of
  - "Service time": time to go through the system independently of traffic conditions
    - E.g. travel time along a link in uncongested conditions: free flow travel time (fftt)
  - Delay: additional time in system due to congestion
- Virtual arrivals $V(t) = A(t - \text{fftt})$ isolate the delay in the system, obtained by shifting $A(t)$ right by fftt.
  - $V(t)$ represents the departure time under no delay



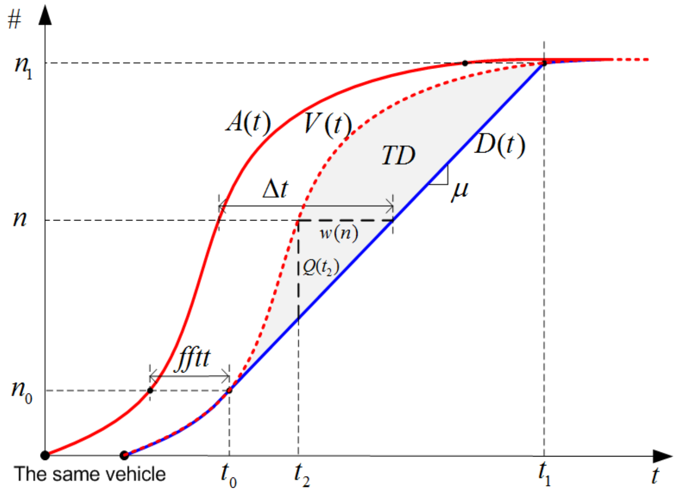
- The delay of vehicle n: $w(n)$
- Queue at $t_2 \approx$ excess vehicle accumulation: $Q(t_2)$
- Total Delay:

$$TD = \int_{t_0}^{t_1} [V(t) - D(t)]dt$$
$$= \int_{t_0}^{t_1} Q(t)dt$$

Slide adapted from Prof. Zhengbing He

Little's Law (1961) – deterministic version

- Simple relationship between arrival rate, average queue length, and average delay (waiting time).
  - Definition (Average arrival rate): $\lambda = \frac{n_1 - n_0}{t_1 - t_0}$
  - The delay of vehicle n: $w(n)$
  - Queue at t_2: $Q(t_2)$
  - Total Delay: $TD = \int_{t_0}^{t_1} [V(t) - D(t)]dt = \int_{t_0}^{t_1} Q(t)dt$



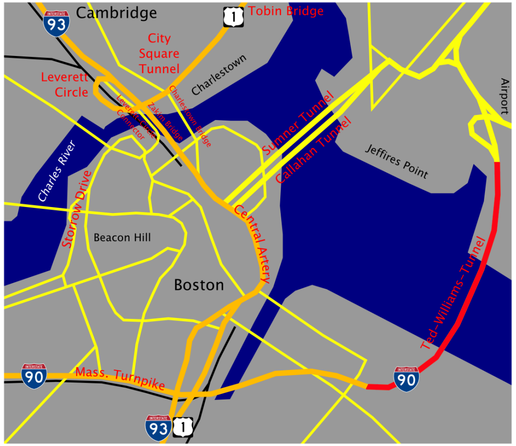
- Assumption 1: Finite time window & vehicles
- Assumption 2: Conservation of vehicles (all arriving vehicles eventually depart)
- Then: $\bar{Q} = \lambda \bar{w}$

Proof:

1961, John Little, MIT Institute Professor; See "Little's Law as Viewed on its 50th Anniversary" (INFORMS)

Example: Toll booths for East Boston Tunnel (I-90)

- Ted Williams Tunnel connecting East Boston to South Boston
  - Massachusetts Transit Authority (MTA) modulates the number of open toll booths (up to 6 booths) such that on average there are no more than 20 vehicles waiting.
  - Tunnel handles up to 3600 vehicles/h during morning rush hour (with all 6 booths).
  - The tunnel sees a total of 50,000 vehicles/day.



- Little's law for quick approximation of quality of service
  - Arrival rate to toll booth: $\lambda = 3600$ veh/h (1 veh/sec)
  - Expected number of vehicles in the system: $\bar{Q} = 20$
  - Average time spent at toll booth: $\bar{w} = 20/3600 \text{ h} = 20 \text{ s}$

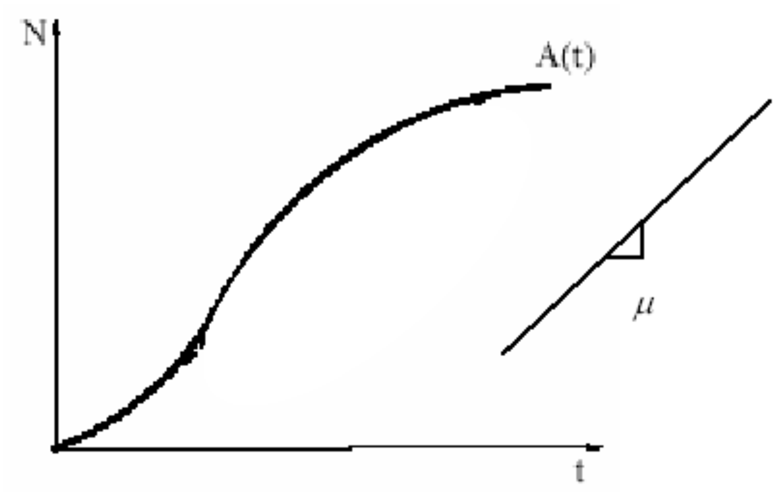
John D.C. Little and Stephen C. Graves, Chapter 5: Little's Law from *Building Intuition: Insights From Basic Operations Management Models and Principles*, 2008. doi: 10.1007/978-0-387-73699-0.

Outline

1. **Cumulative diagrams**
   a. **Application: Signalized intersections**
2. Reconstructing cumulative diagrams
3. Ramp metering problem

Reconstructing the departure curve

- We often have incomplete information.
  - We might have $V(t)$ or $A(t)$ and the operating features of the server (e.g., constant service rate μ), but we need $D(t)$
- Consider:



Example: Traffic signals

How poorly timed traffic lights can make climate change worse

Pointless delays result in unnecessary idling.

by YCC TEAM
JUNE 21, 2021

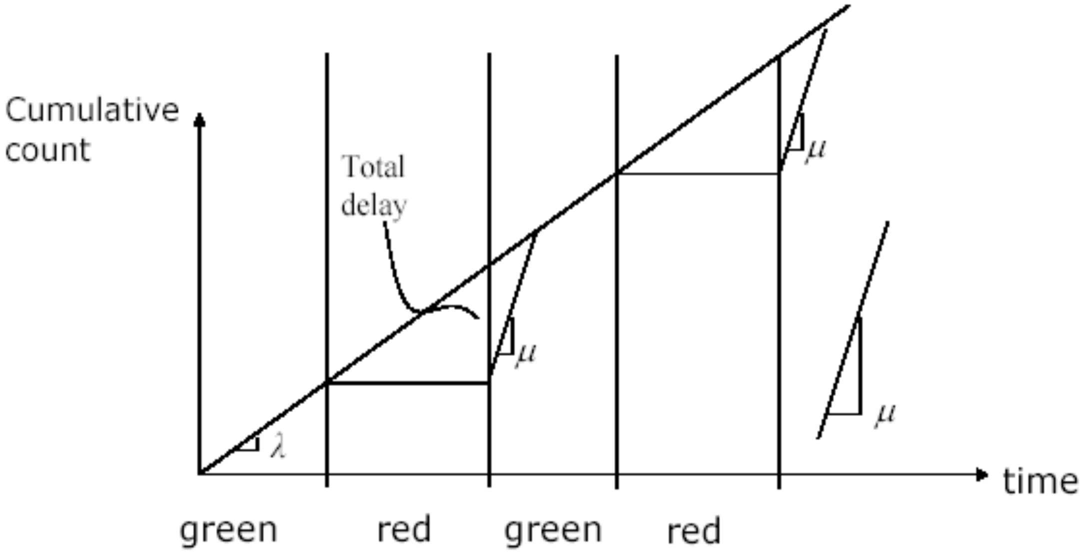
The average signal caused more than 80 hours of delay each day in October, 2020

Source: INRIX

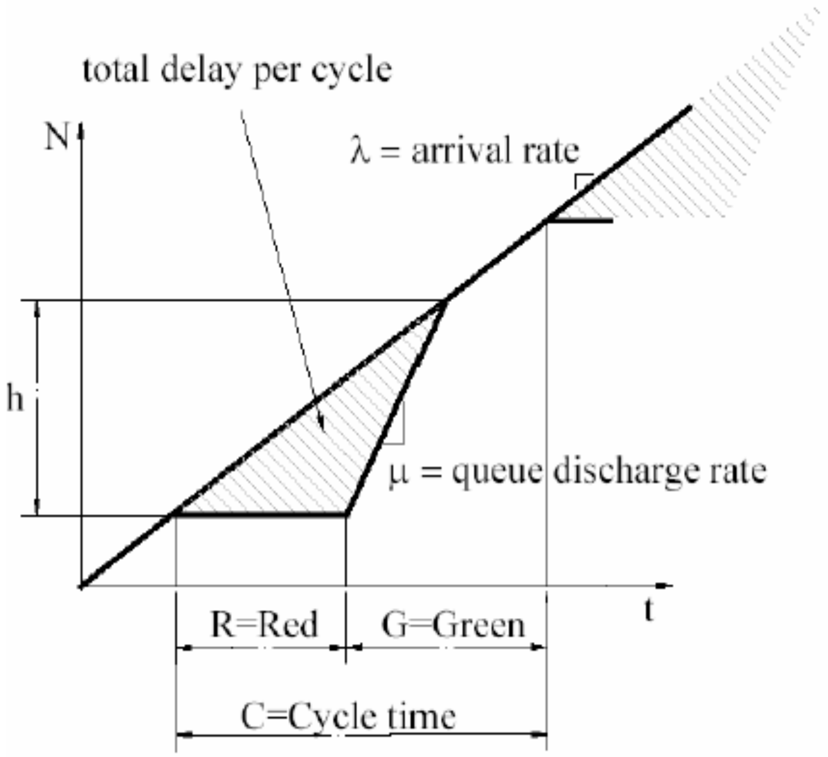
Slide adapted from Dajiang Suo

Queuing diagram application

- Example: On-off service (traffic signal basics)
  - Given: Arrival flow (rate) at a one way signalized intersection is constant λ; we have information about the green/red timing plan of the signal (server), and the service rate during green μ
  - Find: The departure curve, total delay, average delay



Deterministic intersection delay

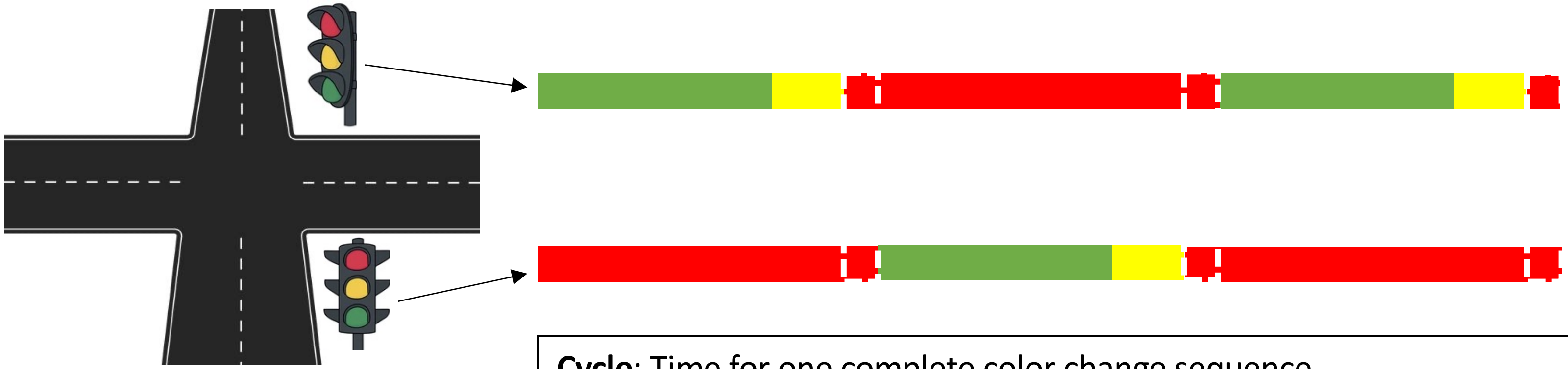


Reality check

Real traffic signals are more complex


TRAFFIC SIGNALS

Traffic control fundamentals



Cycle: Time for one complete color change sequence

Phase: The part of a cycle for one or more movements (left/through/right)

Green/Red/Yellow time

All-red time (lost time)

Outline

1. Cumulative diagrams
2. **Ramp metering problem**

Ramp metering problem

- A ramp meter is being considered at an entrance to a freeway.
- Currently, rush hour traffic arrives at the on-ramp at a rate q_1 from time $t = 0$ to time $t = t^*$. After $t = t^*$, vehicles arrive at a (lower) rate q_2.

1. Assuming that drivers will not change their trips, draw and label a cumulative plot showing a metering (i.e. departure) rate of μ, s.t. $q_2 < \mu < q_1$.
   - Label the maximum delay experienced by any vehicle, w_{max}.
   - What is w_{max} as a function of q_1, q_2, μ, and t^*?
2. If an alternate route is available to drivers, and it is known that they will take this route if their expected delay at the ramp meter is greater than $\frac{w_{max}}{2}$, add this new scenario to your diagram. Now, show graphically the following:
   a) The number of vehicles which will divert.
   b) How much earlier the queue will dissipate (compared to part 1)?

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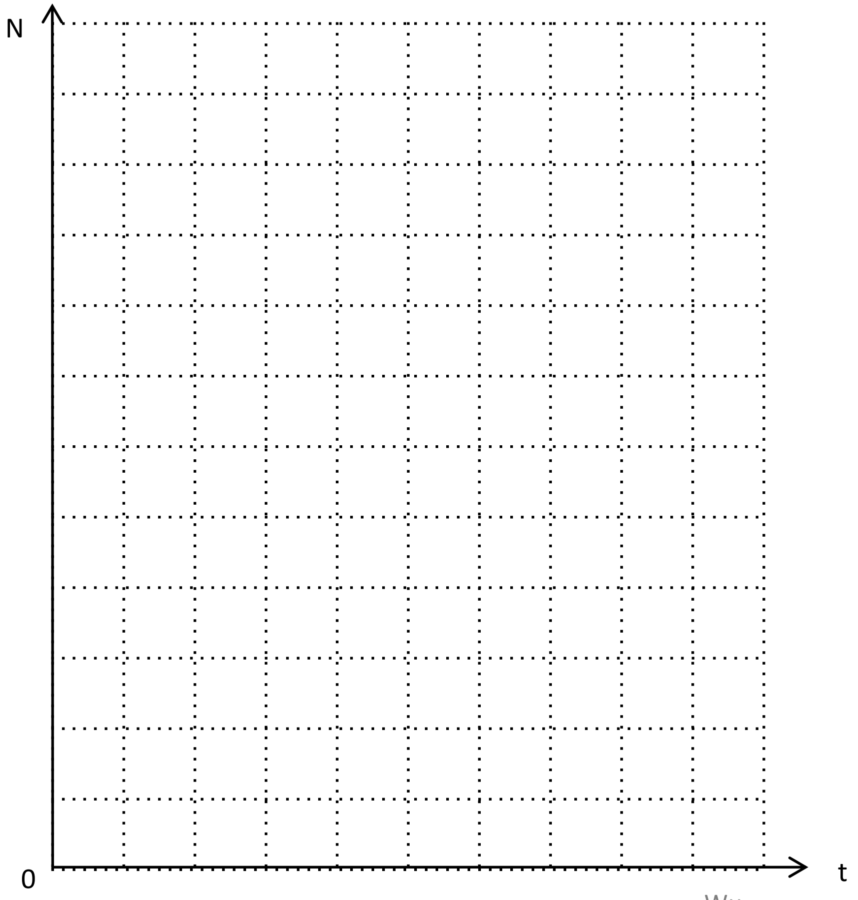

Wu

References

1. C. Daganzo, *Fundamentals of transportation and traffic operations*, vol. 30. Pergamon Oxford, 1997. Chapter 2: Cumulative plots

2. John D.C. Little and Stephen C. Graves, Chapter 5: Little's Law from *Building Intuition: Insights From Basic Operations Management Models and Principles*, 2008. doi: 10.1007/978-0-387-73699-0.

3. Prof. Nikolas Geroliminis' lecture Fundamentals of Traffic Operations and Control, Spring 2010 EPFL

4. Some slides adapted from Profs. Zhengbing He, Carolina Osorio, and Dan Work.