Spring 2023

Microscopic traffic models

Vehicle motion, behavior, and simulation

Cathy Wu

1.041/1.200 Transportation: Foundations and Methods

Readings

1. M. Treiber and A. Kesting, "Chapter 10: Elementary Car-Following Models," *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

2. M. Treiber and A. Kesting, "Chapter 11: Car-following Models Based on Driving Strategies," *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

3. (For fun) Phil Koopman, "L126 AV Trajectories: Newtonian Mechanics vs. the Real World," YouTube, 2022. URL. (Start until 8:22)

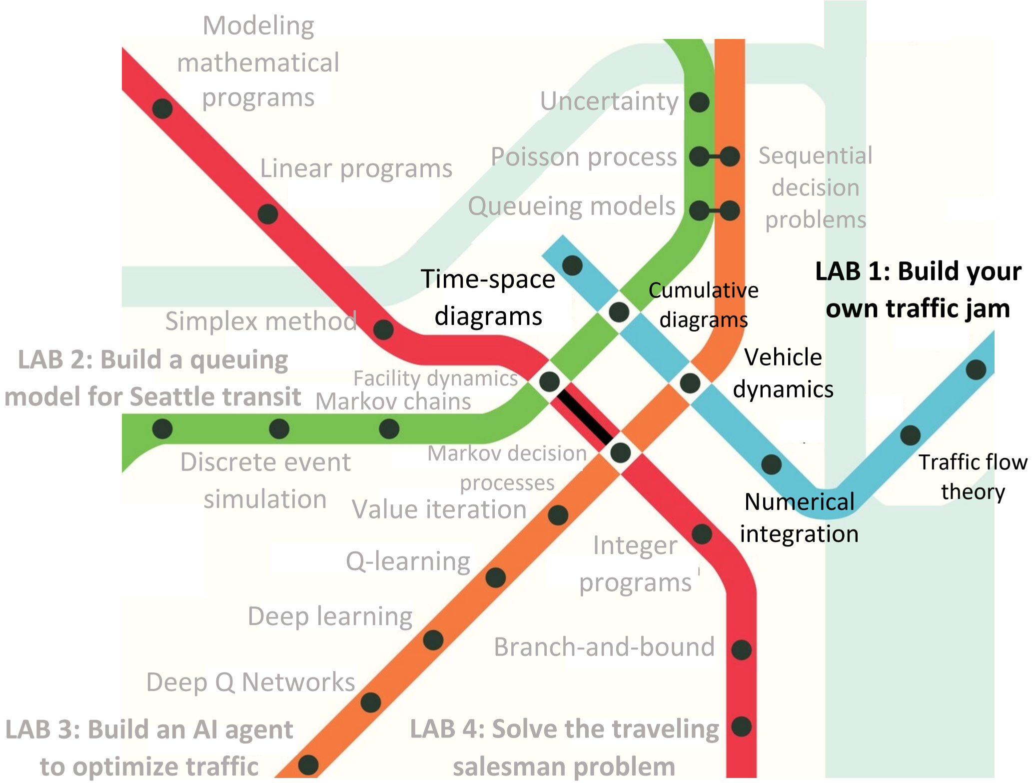
Outline

Unit 1: Traffic flow fundamentals

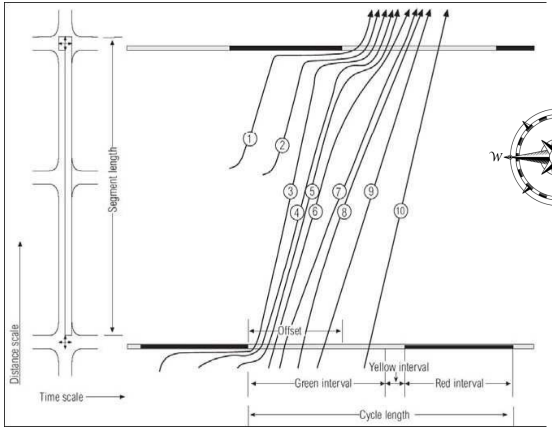
Unit 1

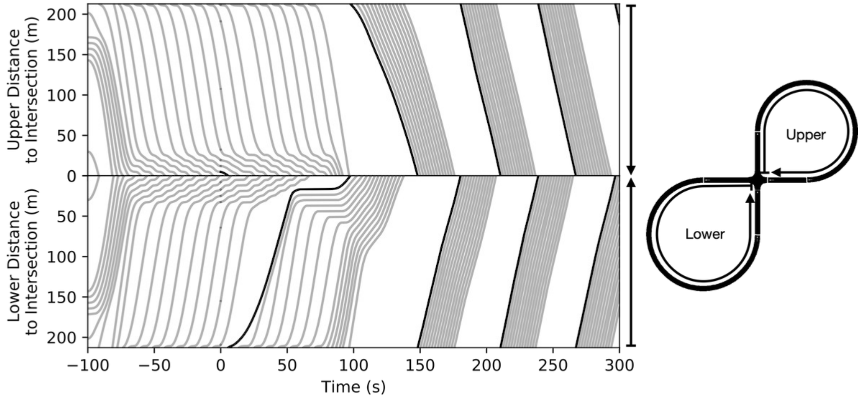
Modeling

Deterministic

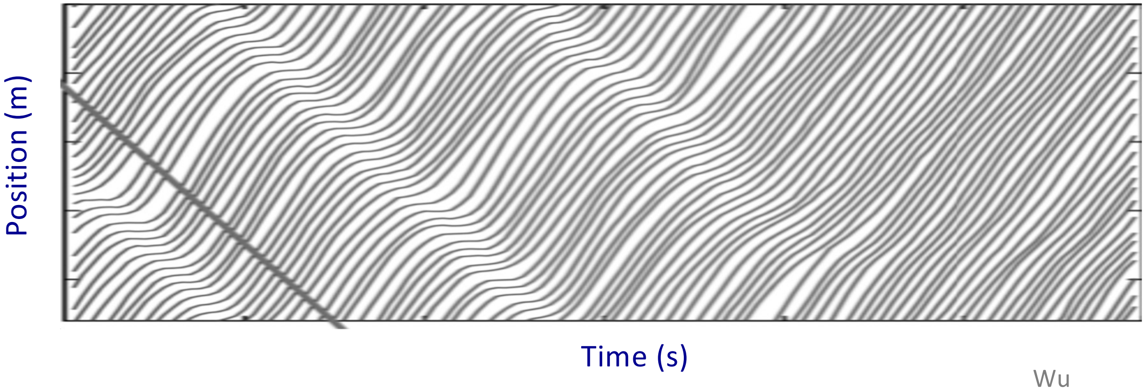


Today: ”Under the hood” of vehicle trajectories





Vehicle trajectories (Sugiyama et al. 2008)



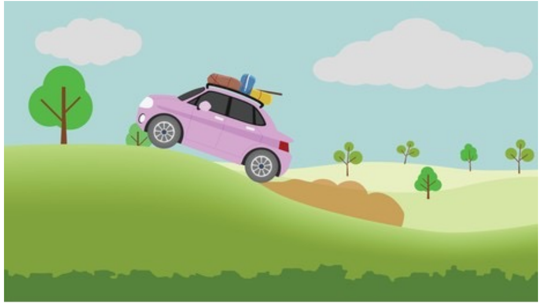
Daganzo (2007); Sugiyama (2008); Yan, et al. (2022)

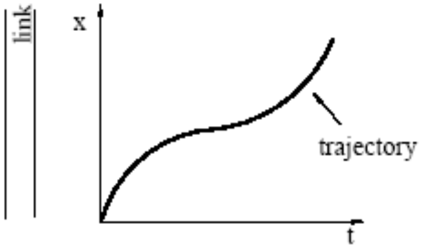
Outline

1. **Vehicle motion**
2. Driving behavior and car following models
3. Numerical integration
4. Advanced driving behavior models

Vehicle dynamics

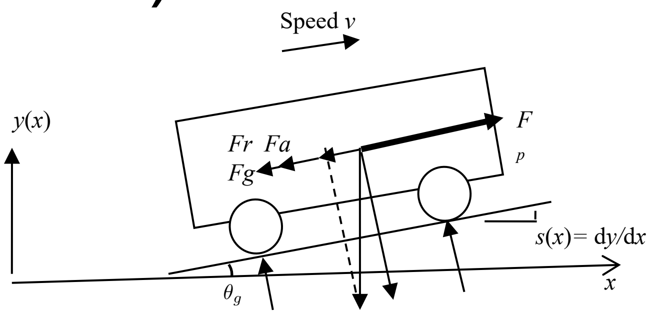
- **Mechanics** — Objects that move
  - **Dynamics** — Body in acceleration (forces on the motion)
    - **Kinetics** — Newton’s laws of motion (without consideration of physics)
    - **Kinematics** — Pure motion, e.g. behavior (without consideration of physics)
  - **Statics** — Body in equilibrium (at rest or constant velocity)





Basic resistance forces (kinetics)

$$F_p = ma + F_a + F_g + F_r$$



- **Aerodynamic resistance; F_a**
  - $F_a = -\frac{\rho}{2} DAv^2$
    - ρ = air density
    - D = drag coefficient
    - A = frontal cross-sectional area of the vehicle
    - v = vehicle speed

- **Grade resistance (from gravity); F_g**
  - $F_g = -mg \sin\theta_g$
    - m = mass
    - θ_g = angle of the slope/grade
    - g = gravity acceleration

- **Rolling resistance; F_r**
  - $F_r = -\beta mv \quad (\ll F_a)$
    - $\beta \approx$ tire rigidity and road surface

Prof. Philip Koopman

Carnegie Mellon University

AV Trajectories: Newtonian Mechanics vs. The Real World

@PhilKoopman

Vehicle Kinematics

- The branch of mechanics that studies the motion of a body or a system of bodies without consideration given to its mass or the forces acting on it.

- $a(v, x, t)$ describes driving behavior.
  - Longitudinal motion
  - Lateral motion

- Note that from basic kinematics:

$$a(v, x, t) = \dot{v} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

  - These are *ordinary differential equations* (ODEs) of $x(t)$ and $v(t)$. We can solve them to obtain the full motion of a vehicle.

Outline

1. Vehicle motion
2. **Driving behavior and car following models**
3. Numerical integration
4. Advanced driving behavior models

What influences driving behavior?

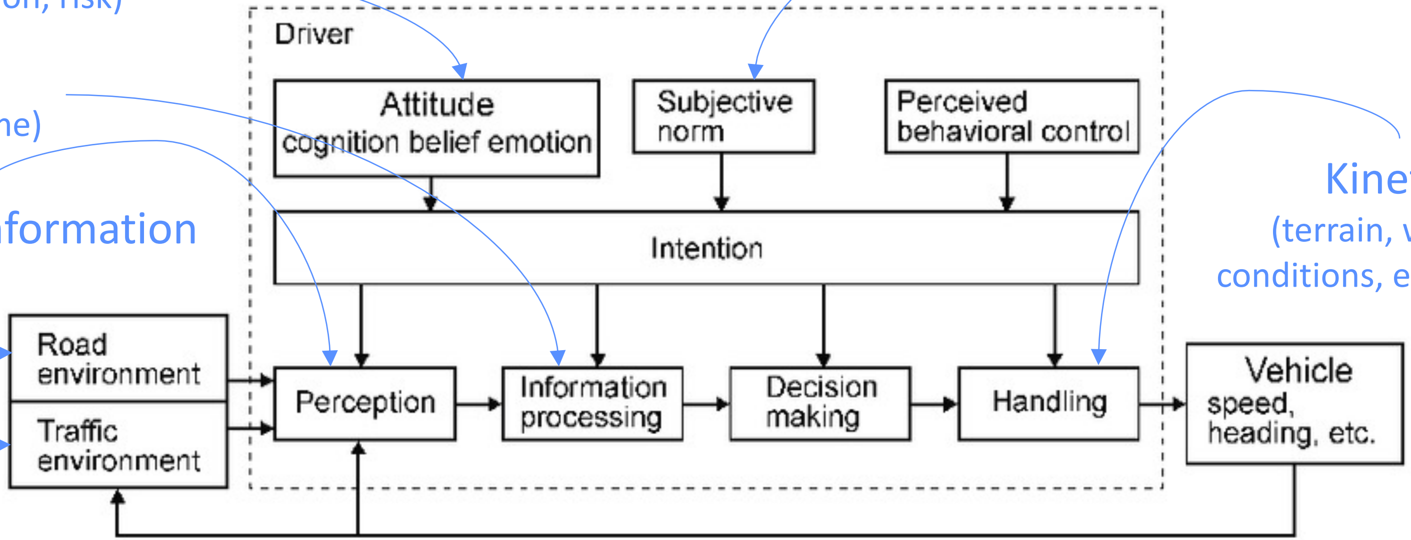
Driving style
(aggression, risk)

Delays
(reaction time)

Imperfect information

Driving culture
(Pittsburgh left, drink-driving, phone use while driving)

Kinetics
(terrain, weather conditions, engine type)



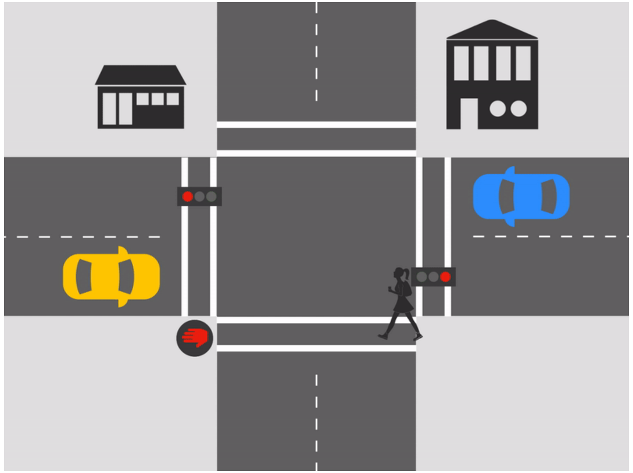
Other road users
(vehicles, pedestrians)

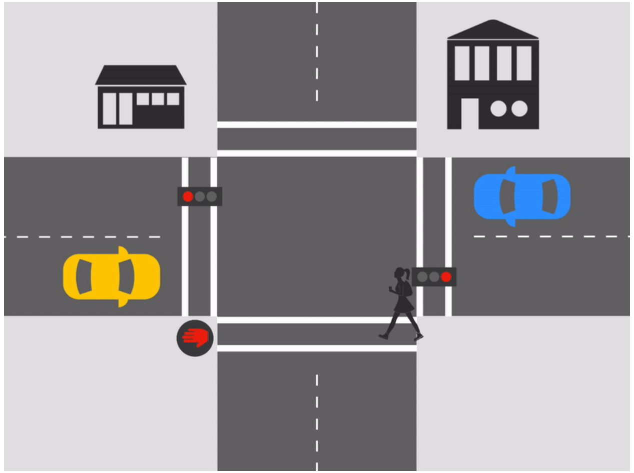
Rules & regulations
(traffic signals, speed limits)

Combined behavioural model to indicate factors that influence driver behaviour (after Van der Horst, 1998)

N. Schaap, "Driving behaviour in unexpected situations. A study into drivers' compensation behaviour to safety-ciritical situations and the effects of mental workload, event urgency and task prioritization," 2012.

Pittsburgh left

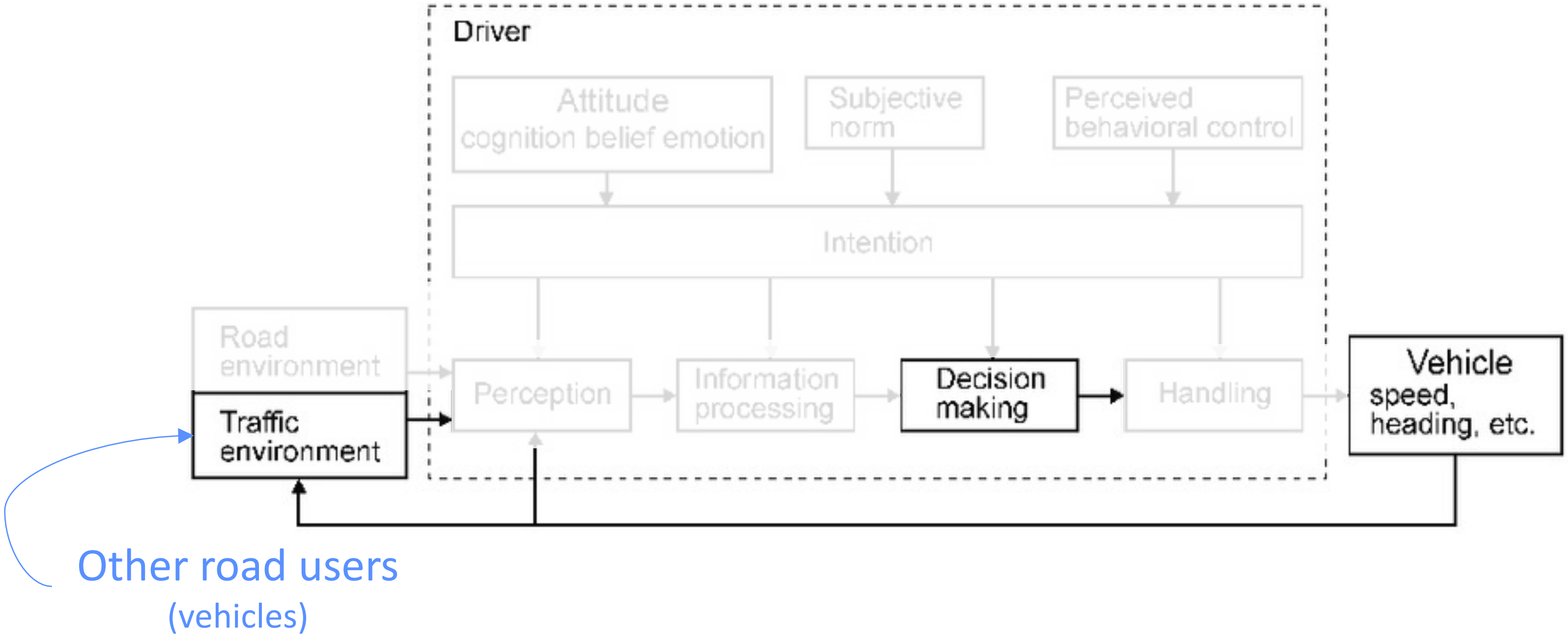




The Pittsburgh left is imposing potential danger to the pedestrians crossing in the same direction.

Source: Wikipedia

What influences driving behavior? (Simplified)

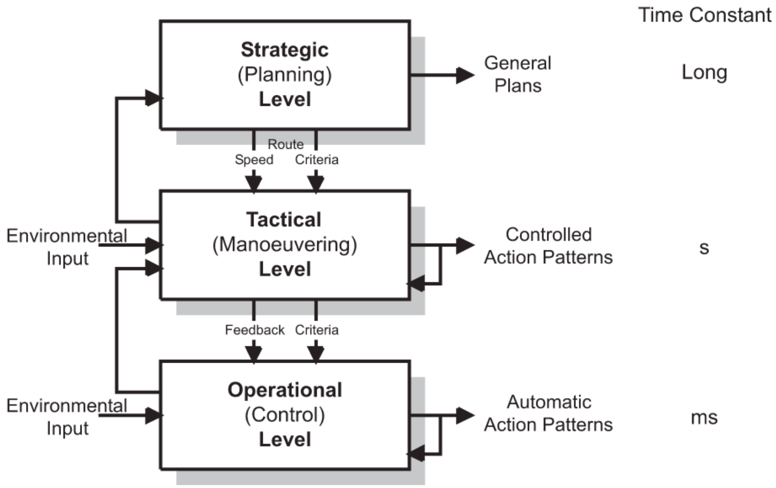


N. Schaap, "Driving behaviour in unexpected situations. A study into drivers' compensation behaviour to safety-ciritical situations and the effects of mental workload, event urgency and task prioritization," 2012.

[Michon, 1985]

Levels of the driving task

- Strategic
  - E.g., choice of turn / route
- Tactical
  - E.g., car following, overtaking, lane changing
- Operational / Control
  - Car operations (i.e., steering, operating throttle)
  - Lane keeping



Driver task hierarchy (after Michon 1985).

C. Lim, T. Sayed, and F. Navin, “A driver visual attention model. Part 1. Conceptual framework,” *Canadian Journal of Civil Engineering*, vol. 31, pp. 463–472, Feb. 2011, doi: 10.1139/l04-020.

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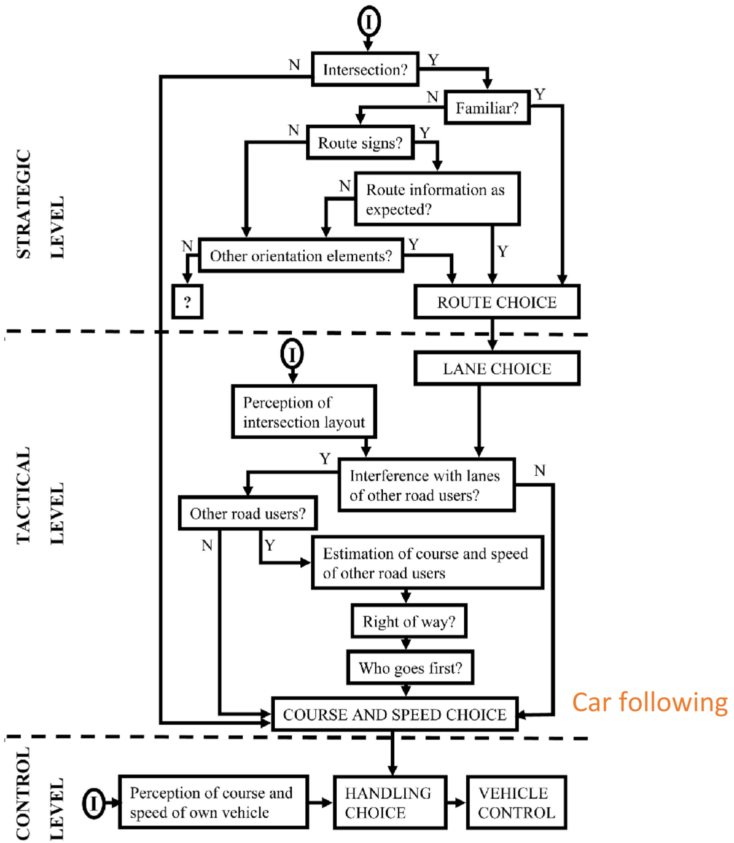


N. Schaap, "Driving behaviour in unexpected situations. A study into drivers' compensation behaviour to safety-ciritical situations and the effects of mental workload, event urgency and task prioritization," 2012.

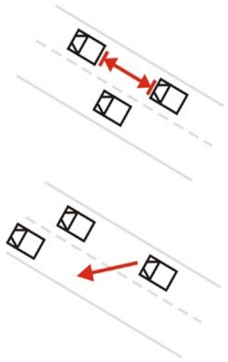
Introduction to inter-vehicle interactions

How do drivers interact with other (nearby) vehicles?

Summary of basic driving subtasks

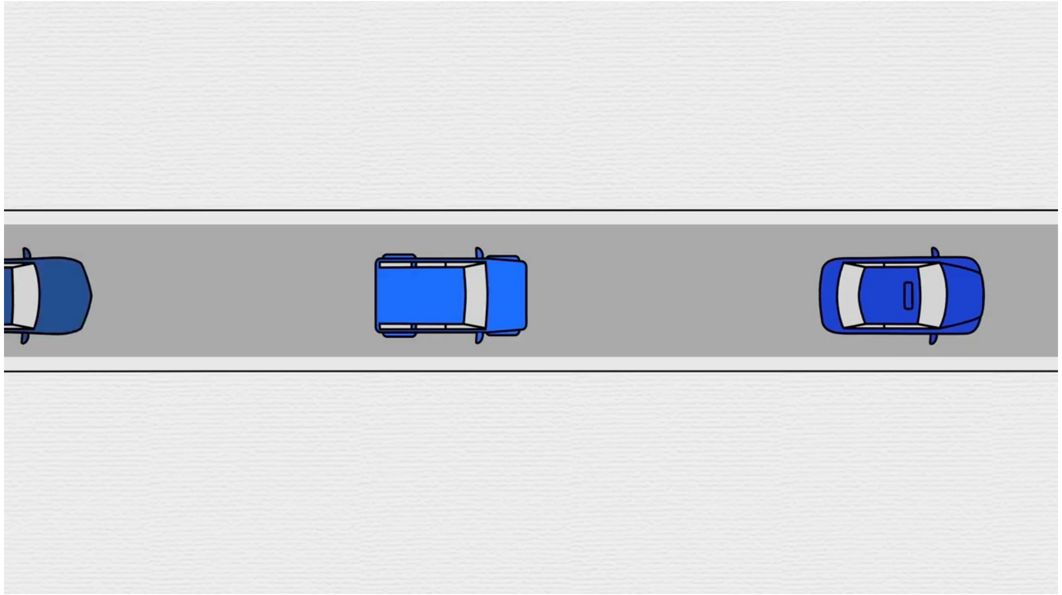
	Roadway interaction subtask	Vehicle interaction subtasks
Longitudinal	Speed choices (free speed)	Car-following
Lateral	Lane choice	Lane changing, merging, overtaking

How do drivers accelerate to their desired speed?



A basic traffic system

- Consider a single lane road with multiple vehicles
  - Vehicles exhibit car following behavior
  - No traffic signals, no lane changes, no merging, no stop signs



Video source: CGP Grey, 2016

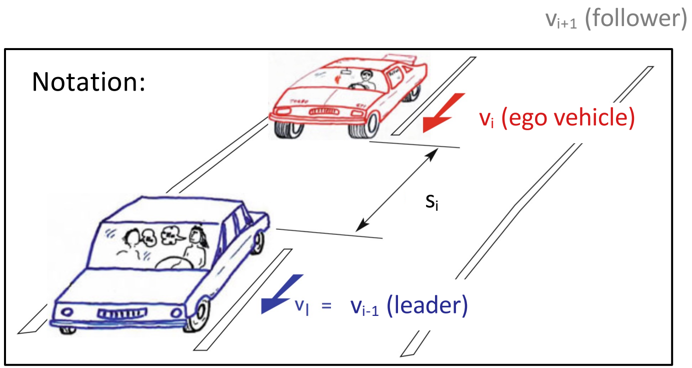
Continuous-time car-following model

Mathematical model that describe how a vehicle moves on a road longitudinally.

Defined by an acceleration function a:

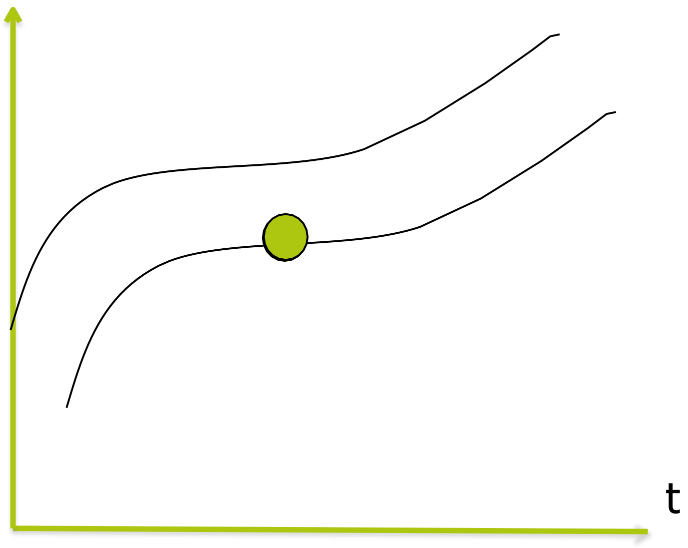
$$\dot{v}_t(t) = a\big(x_i(t), x_{i-1}(t), v_i(t), v_{i-1}(t)\big)$$

- Vehicle $(i \; - 1)$ is the leader of vehicle i
- $x_i(t)$ and $v_i(t)$ are the position and speed of vehicle i at time t, respectively
- Input: position and speed of vehicles i and $(i \; - 1)$ at time t
- Output: changes in speed



Newell simple car-following model (2002)

1. Translate the trajectory in time
2. Translate the trajectory in space



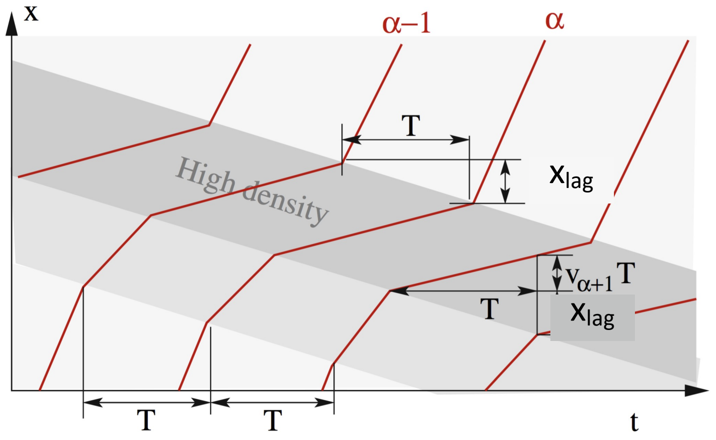
Newell G.F. (2002) A simplified car-following theory: a lower order model. ITS, UC Berkeley.
M. Treiber and A. Kesting, “Chapter 10: Elementary Car-Following Models,” *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

Newell's car following model (2002)

- The simplest car following model
  - $x_\alpha(t+T) = x_{\alpha-1}(t) - x_{lag}$
  - $v_\alpha(t+T) = v_{\alpha-1}(t)$

  Time lag

  Distance lag



- Not well-defined
  - E.g., needs a first trajectory
  - Cannot reproduce traffic phenomena by its
- Can produce any shifted pattern, even if not realistic

Parameter	Typical value highway	Typical value city traffic
Desired speed v_0	120 km/h	54 km/h
Time gap T	1.4 s	1.2 s

Newell G.F. (2002) A simplified car-following theory: a lower order model. ITS, UC Berkeley.
M. Treiber and A. Kesting, "Chapter 10: Elementary Car-Following Models," *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

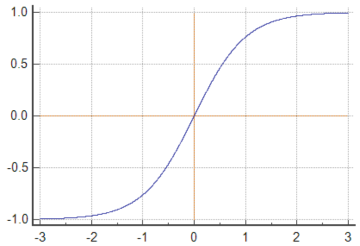
Optimal velocity model (OVM) (Bando, 1995)

- Intuition: Relax speed towards an optimal velocity v_{opt}.

$$a(s,v) = \frac{v_{opt}(s) - v}{\tau}$$

- s: spacing between ego and leading vehicle (headway)
- v: velocity of ego vehicle
- $v_{opt}(s)$ is a target velocity function

tanh(s)



$$v_{opt}(s) = v_0 \frac{\tanh\left(\frac{s}{\Delta s} - \beta\right) + \tanh(\beta)}{1 + \tanh(\beta)}; \quad v_{opt}(s) = \max\left[0, \min\left(v_0, \frac{s - s_0}{T}\right)\right]$$

Table 10.1 Parameter of two variants of the Optimal Velocity Model (OVM)

Parameter	Typical value highway	Typical value city traffic
Adaptation time τ	0.65 s	0.65 s
Desired speed v_0	120 km/h	54 km/h
Transition width Δs [v_{opt} according to Eq. (10.21)]	15 m	8 m
Form factor β [v_{opt} according to Eq. (10.21)]	1.5	1.5
Time gap T [v_{opt} according to Eq. (10.22)]	1.4 s	1.2 s
Minimum distance gap s_0 [v_{opt} according to Eq. (10.22)]	3 m	2 m

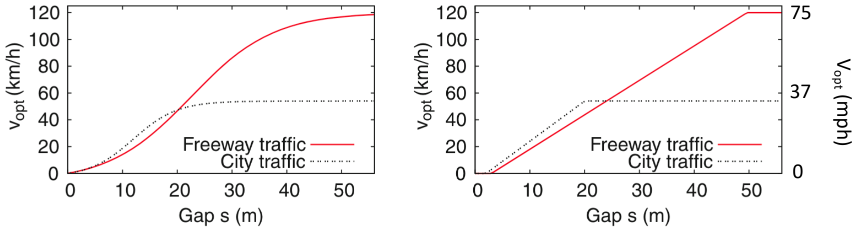


Fig. 10.4 Optimal velocity functions (10.21) (*left*) and (10.22) (*right*) for the parameter values of Table 10.1

M. Treiber and A. Kesting, “Chapter 10: Elementary Car-Following Models,” *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

Optimal velocity model (OVM) (Bando, 1995)

- Nice theoretical properties
  - String stability
  - Often used for theoretical analysis of microscopic traffic flow
- Quantitatively unrealistic
  - Generates accidents or unrealistic accelerations
  - Behavior is sensitive to model parameters
- Source of issues
  - Model doesn’t account for relative speed Δv
  - I.e., behavior is identical regardless of if leader is faster or slower than ego vehicle

M. Treiber and A. Kesting, “Chapter 10: Elementary Car-Following Models,” *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

Intelligent Driver Model (IDM)

- Simplest complete model producing generally realistic acceleration profiles

$$a(s, v, \Delta v) = a\left[1 - \left(\frac{v}{v_0}\right)^{\delta} - \left(\frac{s^*(v, \Delta v)}{s}\right)^2\right]$$

Acceleration goes to 0 as vehicle approaches desired speed v_0 (lower $\delta \in [1, \infty)$ indicates smoother relaxation)

Intelligent braking strategy

where

$$s^*(v, \Delta v) = s_0 + \max\left(0, vT + \frac{v\Delta v}{2\sqrt{ab}}\right)$$

desired spacing

- Remarks
  - Relax towards desired speed v_0
  - Equilibrium safe distance: $s_0 + vT$

Table 11.2 Model parameters of the Intelligent Driver Model (IDM) and typical values in different scenarios (vehicle length 5 m unless stated otherwise)

Parameter	Typical value Highway	Typical value City traffic
Desired speed v_0	120 km/h	54 km/h
Time gap T	1.0 s	1.0 s
Minimum gap s_0	2 m	2 m
Acceleration exponent δ	4	4
Acceleration a	1.0 m/s^2	1.0 m/s^2
Comfortable deceleration b	1.5 m/s^2	1.5 m/s^2

Treiber, M., Hennecke, A. & Helbing, D. (2000) Congested traffic states in empirical observations and microscopic simulations. Phys. Rev. E 62, 1805–1824. (doi:10.1103/PhysRevE.62.1805)
M. Treiber and A. Kesting, "Chapter 11: Car-following Models Based on Driving Strategies," *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

IDM's Intelligent braking strategy (intuition)

$$a(s, v, \Delta v) = a\left[1 - \left(\frac{v}{v_0}\right)^{\delta} - \left(\frac{s^*(v, \Delta v)}{s}\right)^2\right]$$

where

$$s^*(v, \Delta \mathrm{v}) = \mathrm{s}_0 + \max\left(0, \mathrm{vT} + \frac{\mathrm{v}\Delta v}{2\sqrt{ab}}\right)$$

Treiber, M., Hennecke, A. & Helbing, D. (2000) Congested traffic states in empirical observations and microscopic simulations. Phys. Rev. E 62, 1805–1824. (doi:10.1103/PhysRevE.62.1805)
M. Treiber and A. Kesting, "Chapter 11: Car-following Models Based on Driving Strategies," *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

Intelligent Driver Model (IDM) – validation

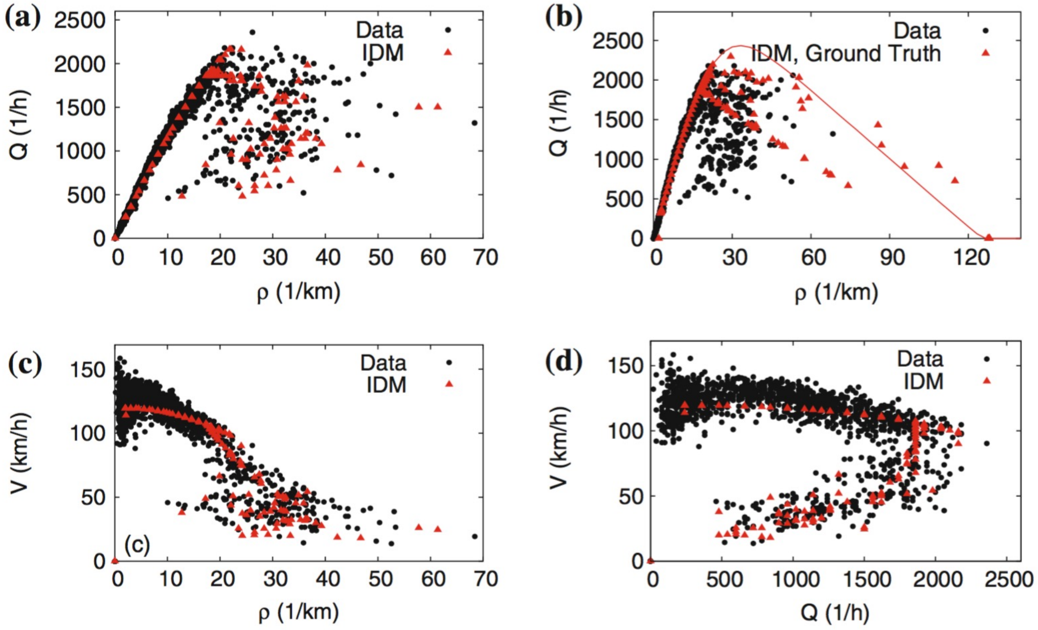
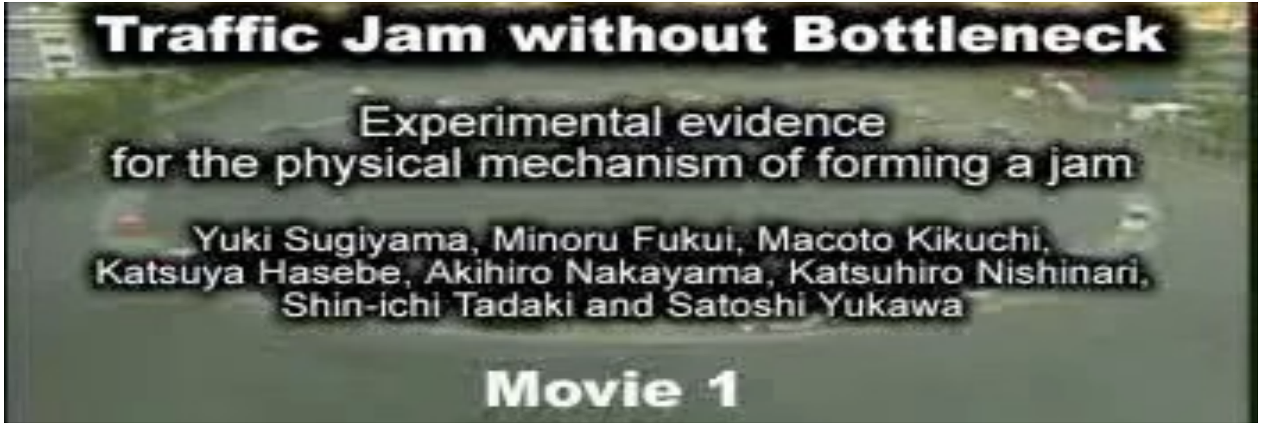


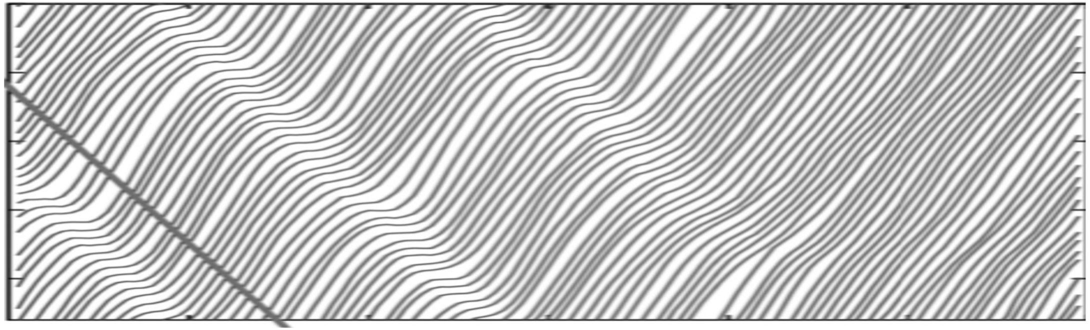
Fig. 11.5 **a** Fundamental diagram, **c** speed-density diagram, and **d** speed-flow diagram showing data from a virtual detector in the highway simulation shown in Fig. 11.4 (positioned 1 km upstream of the ramp). For comparison, empirical data from a real detector on the Autobahn A5 near Frankfurt, Germany, is shown. Velocities have been calculated using arithmetic means in both the real data and the simulation data. **b** Flow-density diagram with the same empirical data but using the real (local) density for the IDM simulation rather than the density derived from the virtual detectors

M. Treiber and A. Kesting, "Chapter 11: Car-following Models Based on Driving Strategies," *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

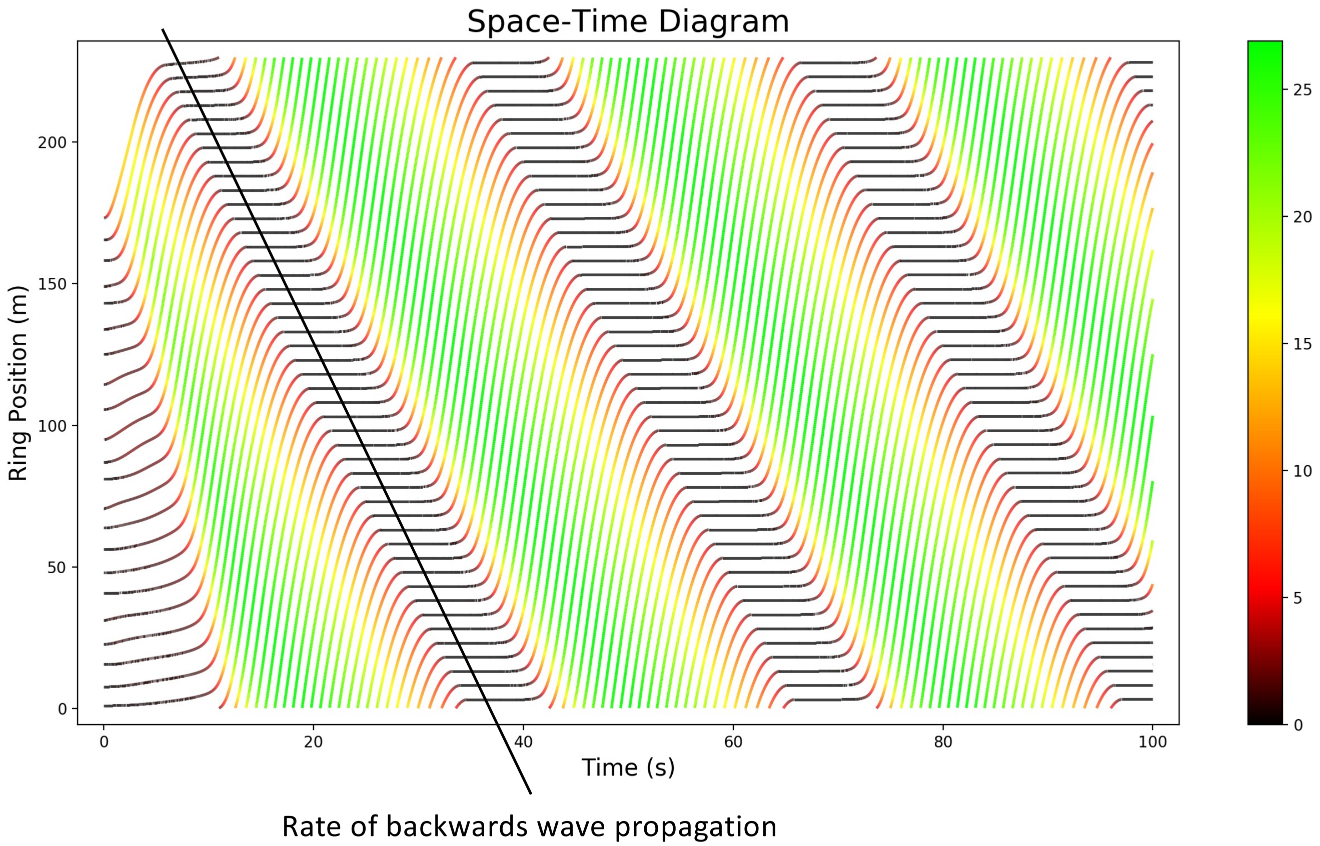
Recall: Traffic waves



Vehicle trajectories (Sugiyama et al. 2008)



Reproduced using IDM



C. Wu, A. R. Kreidieh, K. Parvate, E. Vinitsky, and A. M. Bayen, “Flow: A modular learning framework for mixed autonomy traffic,” *IEEE Transactions on Robotics (T-RO)*, vol. 38, no. 2, pp. 1270–1286, Jul. 2021, doi: [10.1109/TRO.2021.3087314](https://doi.org/10.1109/TRO.2021.3087314).

IDM – Demo

http://www.traffic-simulation.de/ring.html

Car following modeling principles

- Parsimonious: each model parameter describes one aspect of driving, is interpretable, and takes on plausible values.

- Varying realism depending on downstream application
  - Completeness of model
  - Accident-free
  - Incorporation of real driving behavior, such as keeping a safe distance or preferring comfortable accelerations

M. Treiber and A. Kesting, "Chapter 11: Car-following Models Based on Driving Strategies," *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

Discussion: Build your own car-following model

1. What aspects of longitudinal driving behavior might be missing from the car following models we have discussed?

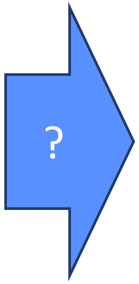
2. Using what surrounding information would you model such behavior?

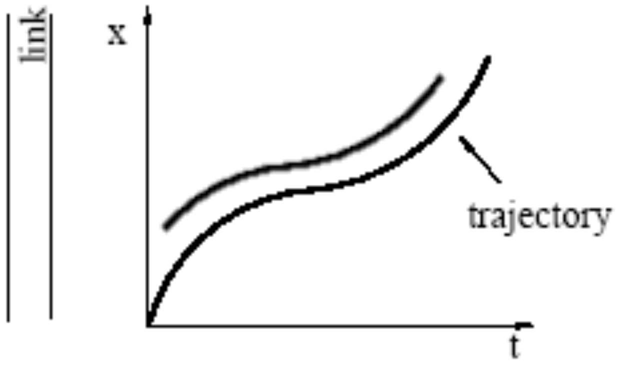
Outline

1. Vehicle motion
2. Driving behavior and car following models
3. **Numerical integration**
4. Advanced driving behavior models

From forces of motion to a trajectory

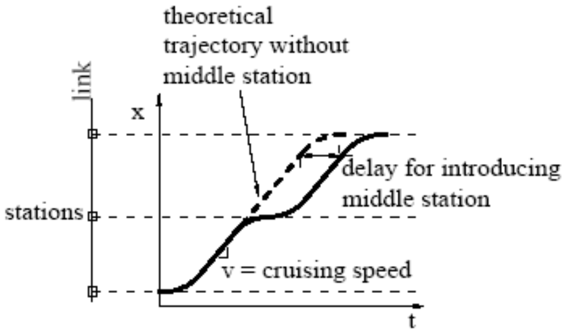
$$a(v, x, t, \ldots)$$



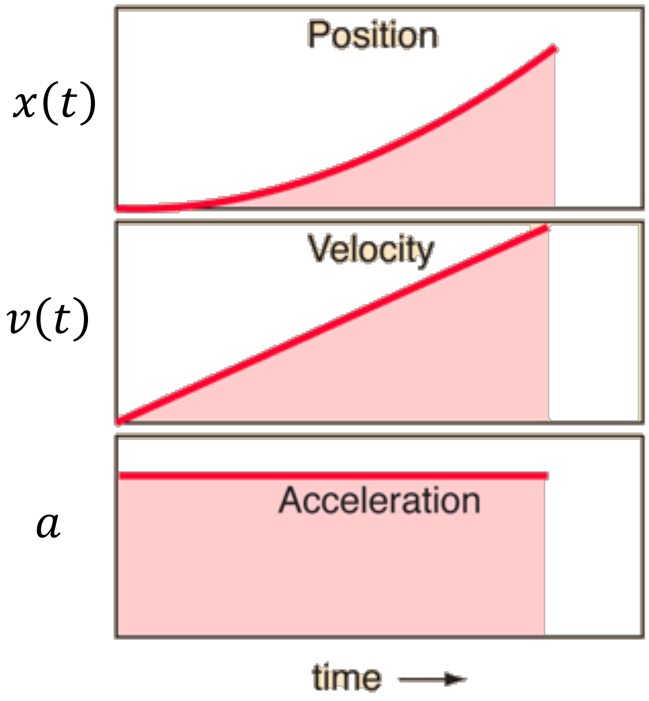


Special case: constant acceleration a

- Recall the basic equations of motion:
  - $v(t) = v_0 + at$
  - $x(t) = x_0 + vt + \frac{1}{2}at^2$



Transit station placement problem (Lecture 2)



Time-dependent acceleration

- Acceleration changes over time

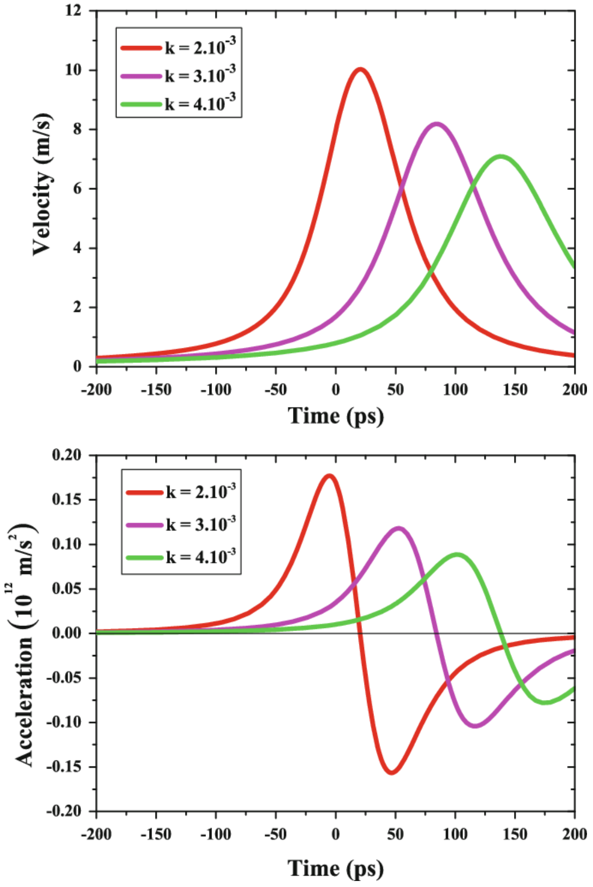
$$a(t)$$

- Velocity

$$v(t) = v_0 + \int_0^t a(t)dt$$

- Position

$$x(t) = x_0 + \int_0^t v(t)dt$$



General case

- Acceleration depends on vehicle state and surrounding information

$$a(v(x,t), x(t), t, \dots)$$

- Velocity

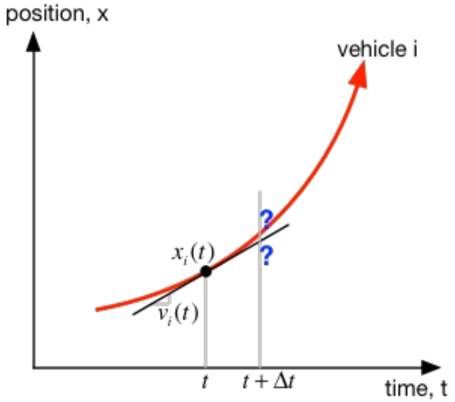
$$v(x,t) = v_0 + \int_0^t a(v(x,t), x(t), t, \dots)dt$$

- Position

$$x(t) = x_0 + \int_0^t v(x,t)dt$$

General solution: numerical integration

- Motivation: ODEs are in general extremely difficult to solve by hand
- Approach: Simulate and then analyze the ODE numerically.
  - *The catch*: Digital computers are discrete-time devices.
    Car-following models (ODEs) are continuous-time functions.
  - Need an **integration scheme** to approximate the numerical solution.



Given the situation at this moment, how to approximate what will happen in the next time increment (e.g., next second)?

Numerical integration: acceleration → velocity

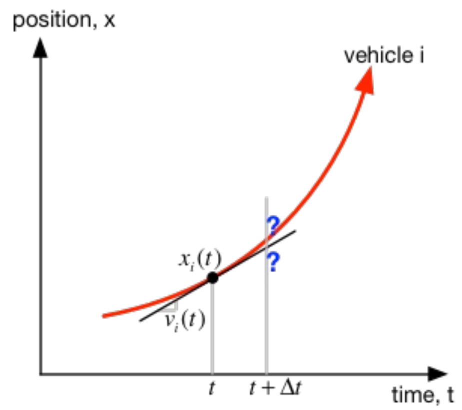
- Instead of allowing the acceleration to vary continuously in time, **assume** it is constant over very small (discrete) time intervals.
  - Let $\Delta t \ll 1$ be the time step.
- Given the velocity at the beginning of the timestep, we can compute the velocity at the end of the time step by approximating the acceleration:

$$v(t+\Delta t) = v(t) + \int_t^{t+\Delta_t} a(x,t,v)$$

$$v(t+\Delta t) \approx v(t) + \int_t^{t+\Delta_t} a_{approx}\,dt$$

$$v(t+\Delta t) \approx v(t) + a_{approx}\Delta_t$$

- Need to choose an appropriate a_{approx}.



Suppose that in a computer simulation, given the situation this time, what will happen in the next time increment (e.g., next second)?

Numerical integration

- To get the **position** of vehicle i at time $t + \Delta t$:

- For the continuous-time function (e.g., continuous CFM)

$$\dot{v}_i(t) = A(\cdot)$$
$$\overset{NI}{\rightarrow} \dot{x}_i(t) \approx v_i(t+\Delta t) = v_i(t) + \Delta t \dot{v}_i(t)$$
$$\overset{NI}{\rightarrow} x_i(t+\Delta t) \approx x_i(t) + \Delta t \dot{x}_i(t)$$

- For the discrete-time function (e.g., discrete CFM)

$$\dot{x}_i(t) = v_i(t+\Delta t) = V(\cdot)$$
$$\overset{NI}{\rightarrow} x_i(t+\Delta t)$$

Forward Euler: first-order method

Suppose we wish to approximate the solution of the initial value problem

$$\dot{y}(t) = f\big(t, y(t)\big), \qquad \text{and} \qquad y(t_0) = y_0$$

Let Δt be the length of time step, and $t_n = t_0 + n\,\Delta t$. One step of Euler's method from t_n to t_{n+1} gives

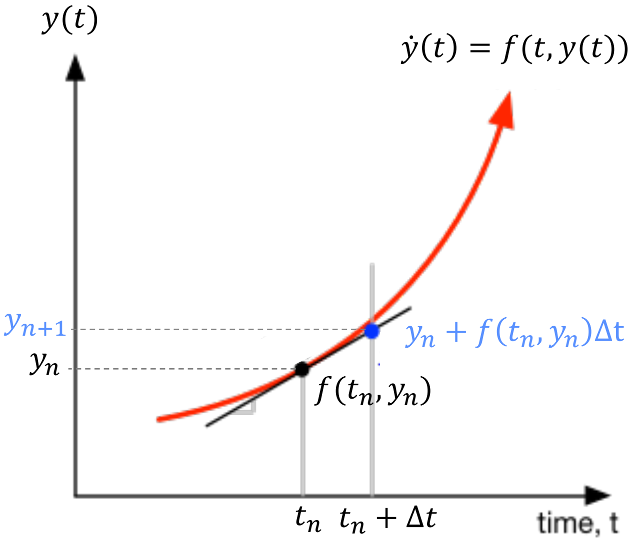
$$y_{n+1} = y_n + f(t_n, y_n)\Delta t$$

Based on a truncated Taylor series expansion, i.e., if we expand y in the neighborhood of $t = t_n$, we get

$$y(t_n + \Delta t) \equiv y_{n+1}$$

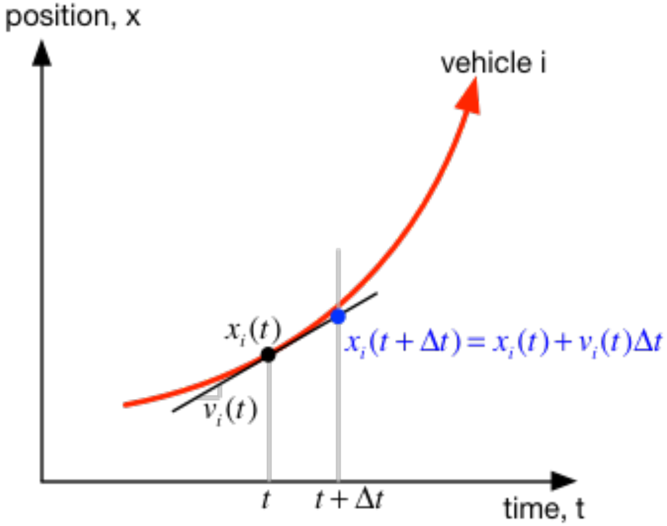
$$= y(t_n) + \Delta t \frac{dy}{dt}\Big|_{t_n} + O(\Delta t^2)$$

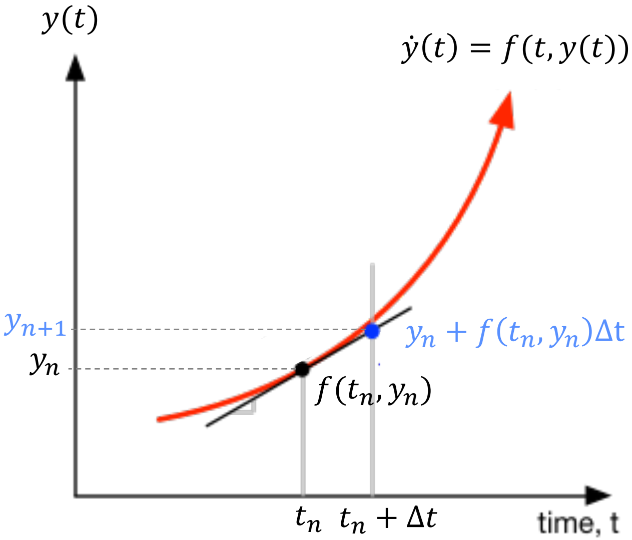
$$= y_n + f(t_n, y_n)\Delta t + O(\Delta t^2)$$



Forward Euler for vehicle motion

- In car-following models, given:
  - $x_i(t) \rightarrow y(t_n) = y_n$ and
  - function $v(t + \Delta t) \rightarrow \dot{y}(t)$
- Calculate $x_i(t + \Delta t) \rightarrow y(t)$
- Also see the **error** generated.





Heun's method (trapezoidal rule): second-order method

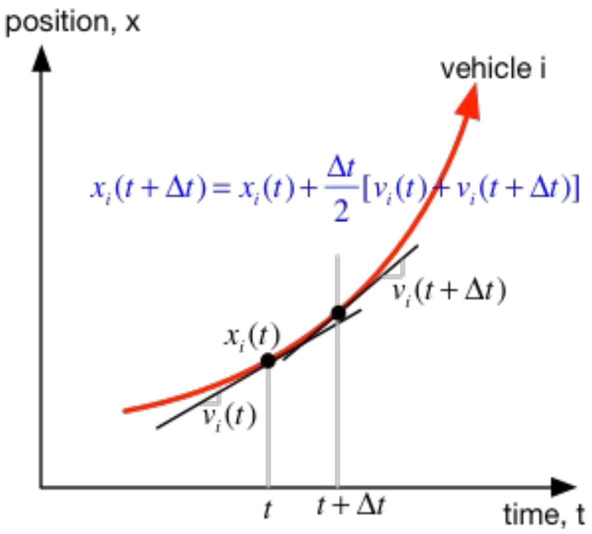
For the same initial value problem, we first calculate the intermediate value $\tilde{y}_{i+1}$ (i.e., slope at $t + \Delta t$)

$$\tilde{y}_{n+1} = y_n + f(t_n, y_n)\Delta \mathrm{t}$$

Then, the final approximation y_{n+1} is based on the average of the slopes at t and $t + \Delta t$

$$y_{n+1} = y_n + \frac{\Delta t}{2}[f(t_n, y_n) + f(t_{n+1}, \tilde{y}_{n+1})]$$

Example: vehicle motion



Runge Kutta methods

- Forward Euler and Heun's method belong to a family of numerical integration methods called Runge Kutta (RK) methods.
  - Shorthand: RK1, RK2 for first and second order methods, respectively.

RK4:

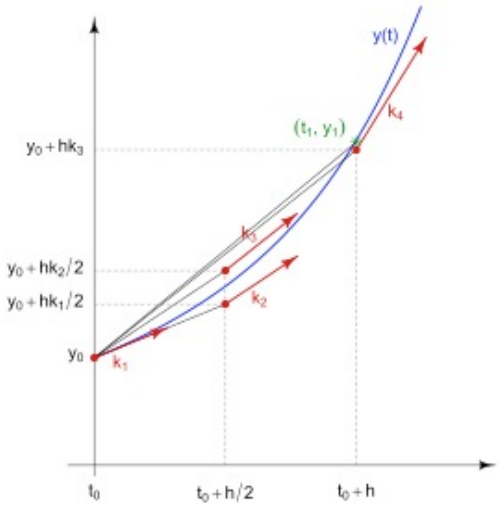


Image source: Wikipedia

- Most common: RK4.
- There is even a RK6.
- Trade-offs: computation cost, accuracy, numerical stability (growth in error over many steps)

$$y_{n+1} = y_n + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$
$$t_{n+1} = t_n + h$$

where

$$\begin{aligned} k_1 &= f(t_n, y_n), \\ k_2 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_1}{2}\right), \\ k_3 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_2}{2}\right), \\ k_4 &= f(t_n + h, y_n + hk_3). \end{aligned}$$

Simulation of car-following models

- Under similar assumptions about the acceleration, we can compute the velocity.
- Simulation of a trajectory: Follow these procedures over and over again to generate a trajectory (on a computer)

t	x	v	a
t_0	x_0	v_0	$a_0 = a(v_0, x_0)$
$t_1 = t_0 + \Delta_t$	$x_1 = x_0 + v_0\Delta_t$	$v_1 = v_0 + a_0\Delta_t$	$a_1 = a(v_1, x_1)$
$t_2 = t_1 + \Delta_t$	$x_2 = x_1 + v_1\Delta_t$	$v_2 = v_1 + a_1\Delta_t$	$a_2 = a(v_2, x_2)$
$t_3 = t_2 + \Delta_t$	$x_3 = x_2 + v_2\Delta_t$	$v_3 = v_2 + a_2\Delta_t$	$a_3 = a(v_3, x_3)$
...	...	...	...
...	...	...	...
...	...	...	...

- Simulation of traffic: Repeat these procedures for each vehicle.
- Now, you can build your own traffic jam! (Computational Lab #1)

Outline

1. Vehicle motion
2. Driving behavior and car following models
3. Numerical integration
4. **Advanced driving behavior models**

Lane changing behavior and other discrete choices

- General decision model: Utility maximization with discrete choice
- Discrete set K of alternatives
  - Ex. Lane change: left, right, or none.
  - Ex. Merging onto priority road: stopping or waiting for a sufficient gap between the main-road vehicles
- Acceleration as utility: $U^{(\beta,k)} \coloneqq a^{(\beta,k)}$, for driver β, alternative $k \in K$
- Ego-driver α selects the option of maximum utility

$$k_{selected} = \arg\max_{k'} U^{(\alpha,k')}$$

M. Treiber and A. Kesting, "Chapter 14: Lane-Changing and Other Discrete-Choice Situations," *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

Lane changing behavior

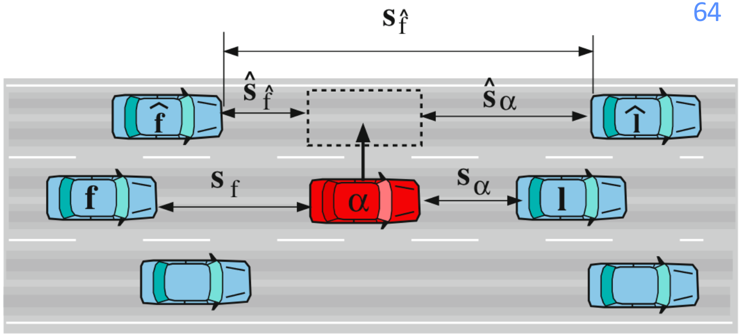


Fig. 14.1 Notation for a lane change of the *center* vehicle α to the *left*. All quantities with a hat pertain to the new situation after the (possibly hypothetical) lane change

- WLOG, consider 2 options (change to left lane or stay in current lane)
- Change lanes if utility of another lane is greater than current lane
$$\hat{a}_\alpha - a_\alpha > 0$$
- Add a threshold to prevent marginal advantage
$$\hat{a}_\alpha - a_\alpha > \Delta a > 0$$
- Add a bias term to model asymmetric behavior, $a_{bias} > \Delta a$ (e.g., right-overtaking ban on most European highways)
$$\hat{a}_\alpha - a_\alpha > \Delta a + a_{bias} > 0$$

M. Treiber and A. Kesting, "Chapter 14: Lane-Changing and Other Discrete-Choice Situations," *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

Lane changing behavior (MOBIL)

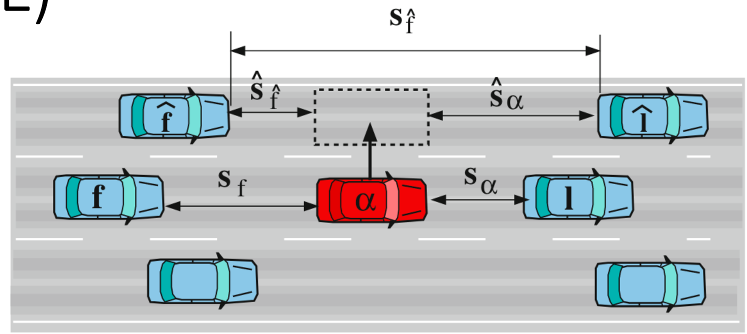


Fig. 14.1 Notation for a lane change of the *center* vehicle α to the *left*. All quantities with a hat pertain to the new situation after the (possibly hypothetical) lane change

- Add a politeness term ($p \in [0,1]$, typically ≈ 0.2)

  $$\hat{a}_\alpha - a_\alpha + p(\hat{a}_{\hat{f}} - a_{\hat{f}} + \hat{a}_f - a_f) > \Delta a + a_{bias} > 0$$

  - Special case: altruistic driver ($p = 1$), a lane change takes place only if sum of acceleration of all affected drivers increases with the maneuver
  - This model is called MOBIL (**M**inimizing **o**verall **b**raking deceleration **i**nduced by **l**ane changes)

M. Treiber and A. Kesting, "Chapter 14: Lane-Changing and Other Discrete-Choice Situations," *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

More complex driving models

- Multiple leaders
- Reaction times
- Perception thresholds and action point models
- Responding to brake lights
- Distracted driving
- Driving fatigue
- Imperfect driving
- Estimation errors
- Field of vision

- Trade-offs: Simplicity vs fit error. These models have more parameters, and thus may fit data better. However, similar resulting behavior can often be observed in simpler models.

M. Treiber and A. Kesting, "Chapter 12: Modeling Human Aspects of Driving Behavior," *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

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3. MIT 10.001: Numerical Solution of Ordinary Differential Equations http://web.mit.edu/10.001/Web/Course_Notes/Differential_Equations_Notes/lec24.html

4. Slides adapted from:
   - Prof. Dan Work (Vanderbilt CE 3501)
   - Prof. Victor L. Knoop (TU Delft)
   - Prof. Zhengbing He (Beijing Jiaotong University)