Spring 2026

Microscopic traffic models

Vehicle motion, behavior, and simulation

Cathy Wu

1.041/1.200 Transportation: Foundations and Methods

Readings

1. M. Treiber and A. Kesting, “Chapter 10: Elementary Car-Following Models,” *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

2. M. Treiber and A. Kesting, “Chapter 11: Car-following Models Based on Driving Strategies,” *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

3. (For fun) Phil Koopman, “L126 AV Trajectories: Newtonian Mechanics vs. the Real World,” YouTube, 2022. URL. (Start until 8:22)

Outline

1. Car following models
2. Numerical integration
3. Advanced driving behavior
4. Vehicle dynamics

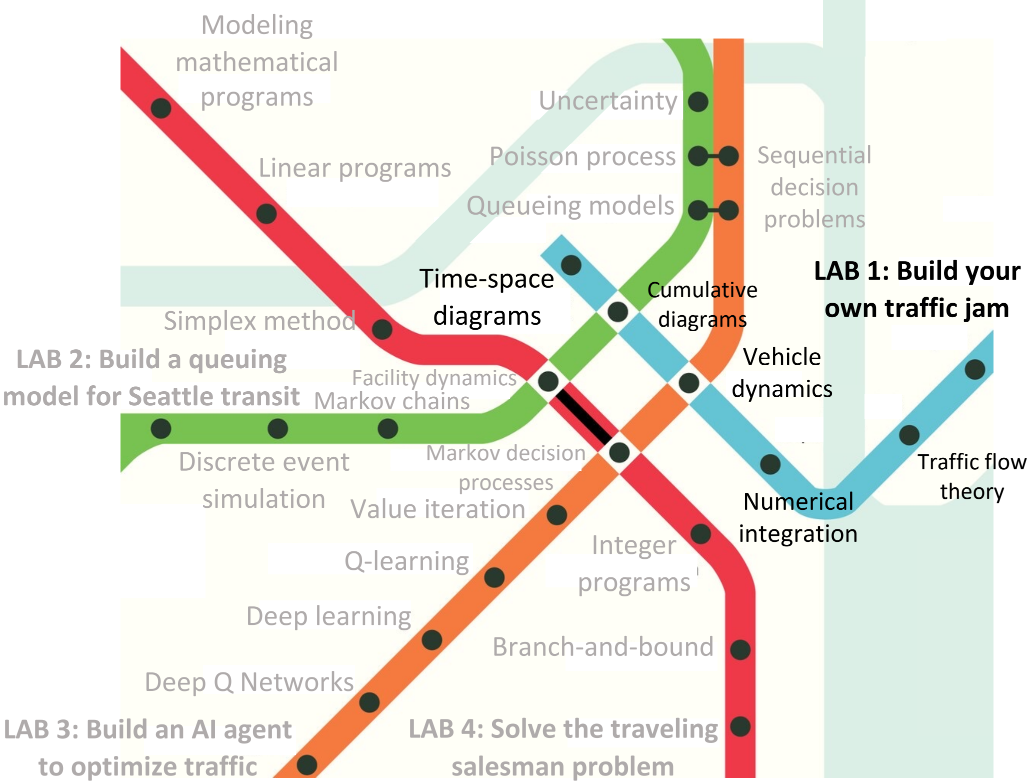
Unit 1: Traffic flow fundamentals

○

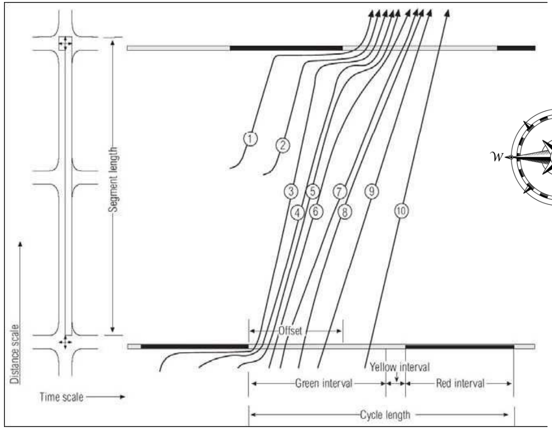
Unit 1

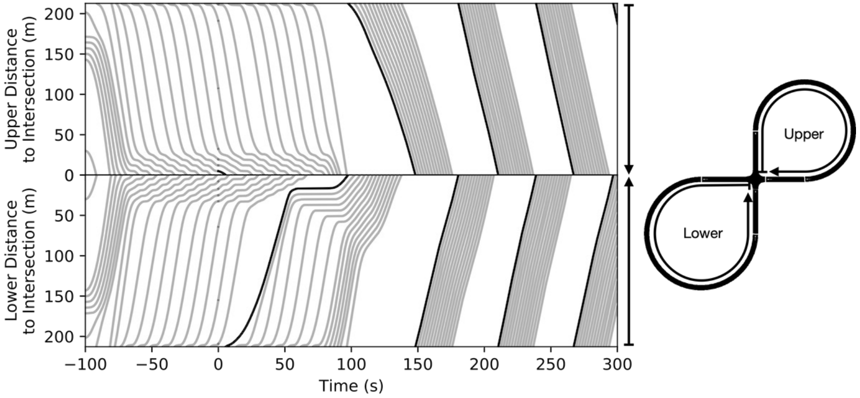
Modeling

Deterministic

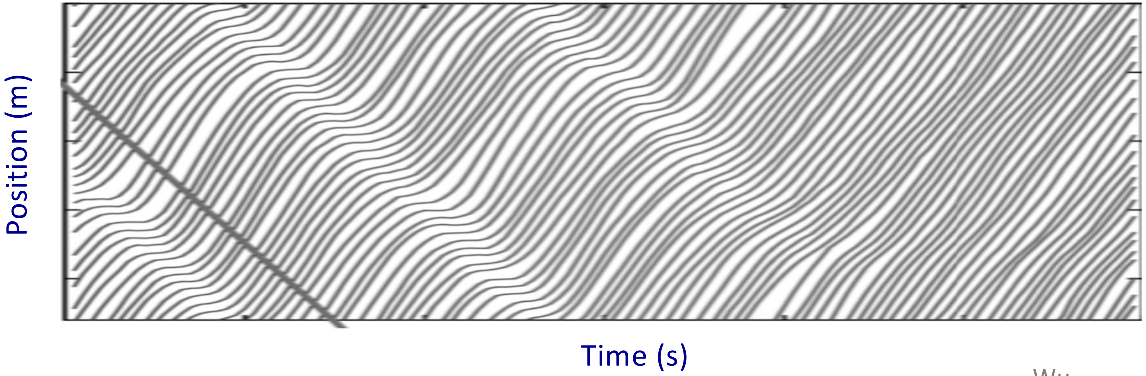


Today: Generating vehicle trajectories





Vehicle trajectories (Sugiyama et al. 2008)



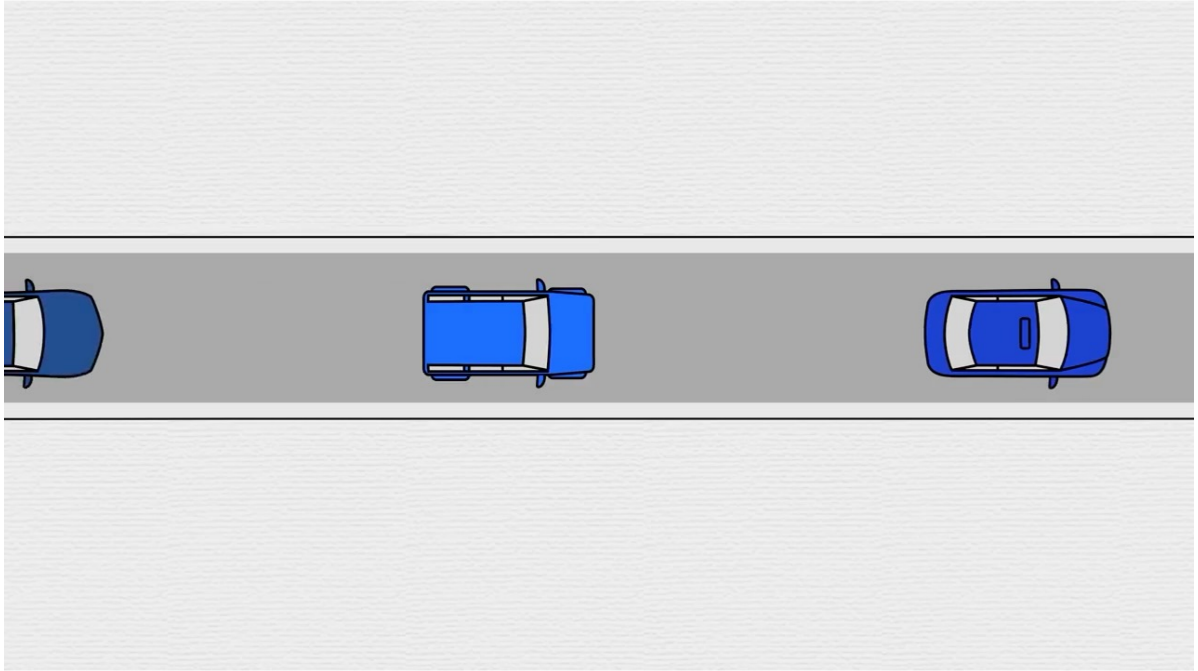
Daganzo (2007); Sugiyama (2008); Yan, et al. (2022)

Outline

1. **Car following models**
2. Numerical integration
3. Advanced driving behavior
4. Vehicle dynamics

A basic traffic system

- Consider a single lane road with multiple vehicles
  - Vehicles exhibit car following behavior
  - No traffic signals, no lane changes, no merging, no stop signs



Video source: CGP Grey, 2016

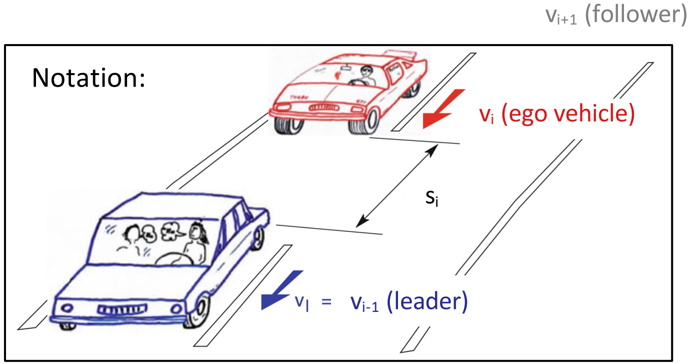
Continuous-time car-following model

Mathematical model that describe how a vehicle moves on a road longitudinally.

Defined by an acceleration function a:

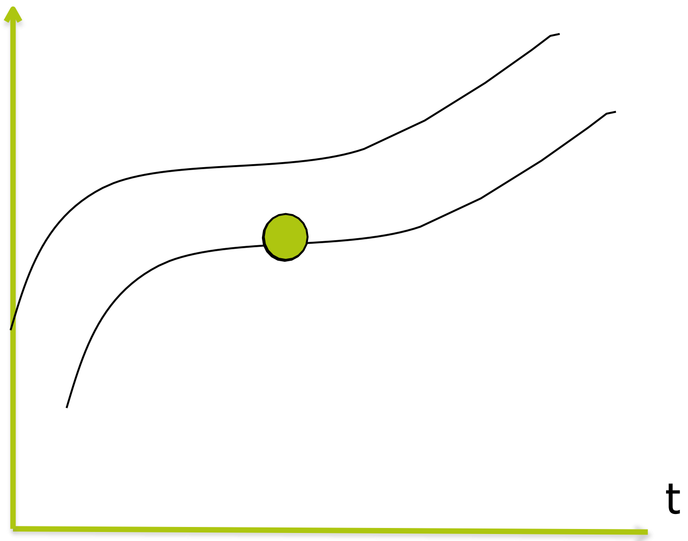
$$\dot{v}_i(t) = a\big(x_i(t), x_{i-1}(t), v_i(t), v_{i-1}(t)\big)$$

- $x_i(t)$ and $v_i(t)$ are the position and speed of vehicle i at time t, respectively
- Vehicle $i\ - 1$ is the leader of vehicle i
- Input: position and speed of vehicles i and $i\ - 1$ at time t
- Output: changes in speed



Newell simple car-following model (2002)

1. Translate the trajectory in time
2. Translate the trajectory in space



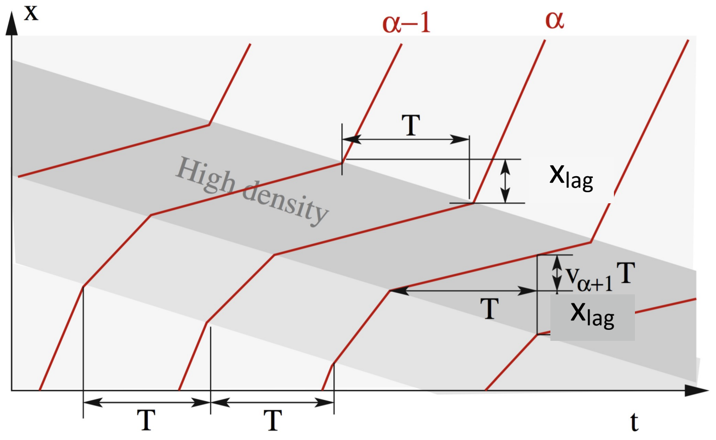
Newell G.F. (2002) A simplified car-following theory: a lower order model. ITS, UC Berkeley.
M. Treiber and A. Kesting, "Chapter 10: Elementary Car-Following Models," *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

Newell’s car following model (2002)

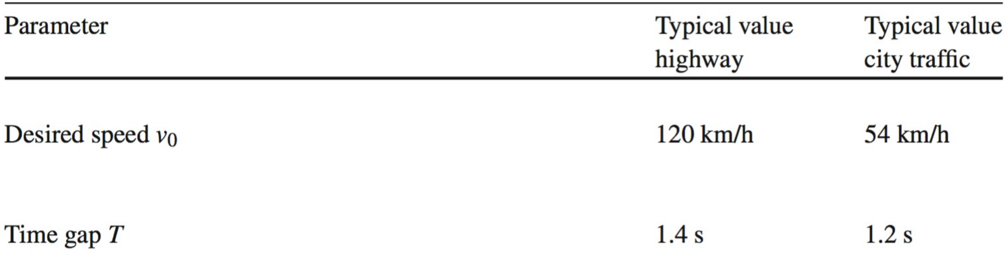
- The simplest car following model
  - $x_\alpha(t+T) = x_{\alpha-1}(t) - x_{lag}$
  - $v_\alpha(t+T) = v_{\alpha-1}(t)$

  Time lag

  Distance lag



- Not well-defined
  - E.g., needs a first trajectory
  - Cannot reproduce traffic phenomena by its
- Can produce any shifted pattern, even if not realistic



Parameter	Typical value highway	Typical value city traffic
Desired speed v_0	120 km/h	54 km/h
Time gap T	1.4 s	1.2 s

Newell G.F. (2002) A simplified car-following theory: a lower order model. ITS, UC Berkeley.
M. Treiber and A. Kesting, “Chapter 10: Elementary Car-Following Models,” *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

Optimal velocity model (OVM) (Bando, 1995)

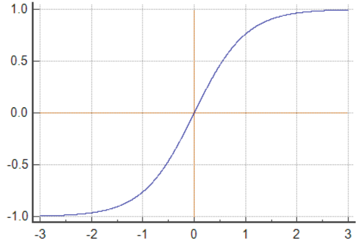
- Intuition: Relax speed towards an optimal velocity v_{opt}.

$$a(s,v) = \frac{v_{opt}(s) - v}{\tau}$$

- s: spacing between ego and leading vehicle (headway)
- v: velocity of ego vehicle
- $v_{opt}(s)$ is a target velocity function

$$v_{opt}(s) = v_0 \frac{\tanh\left(\frac{s}{\Delta s} - \beta\right) + \tanh(\beta)}{1 + \tanh(\beta)}; \quad v_{opt}(s) = \max\left[0, \min\left(v_0, \frac{s - s_0}{T}\right)\right]$$

tanh(s)



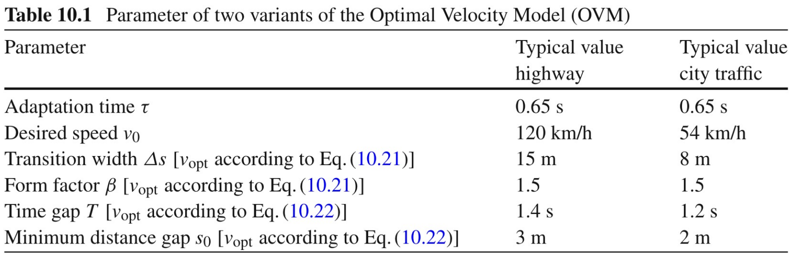


Table 10.1 Parameter of two variants of the Optimal Velocity Model (OVM)

Parameter	Typical value highway	Typical value city traffic
Adaptation time τ	0.65 s	0.65 s
Desired speed v_0	120 km/h	54 km/h
Transition width Δs [v_{opt} according to Eq. (10.21)]	15 m	8 m
Form factor β [v_{opt} according to Eq. (10.21)]	1.5	1.5
Time gap T [v_{opt} according to Eq. (10.22)]	1.4 s	1.2 s
Minimum distance gap s_0 [v_{opt} according to Eq. (10.22)]	3 m	2 m

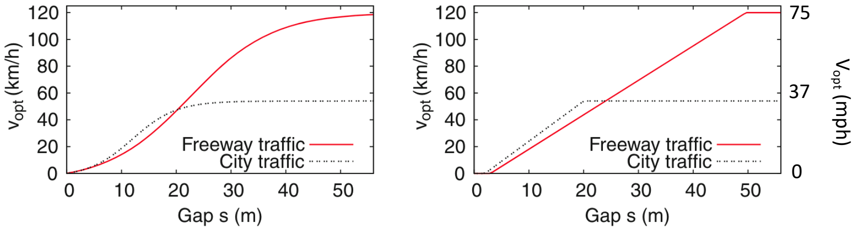


Fig. 10.4 Optimal velocity functions (10.21) (*left*) and (10.22) (*right*) for the parameter values of Table 10.1

M. Treiber and A. Kesting, "Chapter 10: Elementary Car-Following Models," *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

Optimal velocity model (OVM) (Bando, 1995)

- Nice theoretical properties
  - String stability
  - Often used for theoretical analysis of microscopic traffic flow

- Quantitatively unrealistic
  - Generates accidents or unrealistic accelerations
  - Behavior is sensitive to model parameters

- Source of issues
  - Model doesn’t account for relative speed Δv
  - I.e., behavior is identical regardless of if leader is faster or slower than ego vehicle

M. Treiber and A. Kesting, “Chapter 10: Elementary Car-Following Models,” *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

Intelligent Driver Model (IDM)

- Simplest complete model producing generally realistic acceleration profiles

$$a(s, v, \Delta v) = a\left[1 - \overbrace{\left(\frac{v}{v_0}\right)^{\delta}} - \left(\frac{s^*(v, \Delta v)}{s}\right)^2\right]$$

where

$$s^*(v, \Delta \mathrm{v}) = \mathrm{s}_0 + \max\left(0, \mathrm{vT} + \frac{\mathrm{v}\Delta v}{2\sqrt{ab}}\right)$$

Acceleration goes to 0 as vehicle approaches desired speed v_0

(lower $\delta \in [1, \infty)$ indicates smoother relaxation)

Intelligent braking strategy

desired spacing

- Remarks
  - Relax towards desired speed v_0
  - Equilibrium safe distance: $s_0 + vT$
  - $\Delta v := v - v_{leader}$

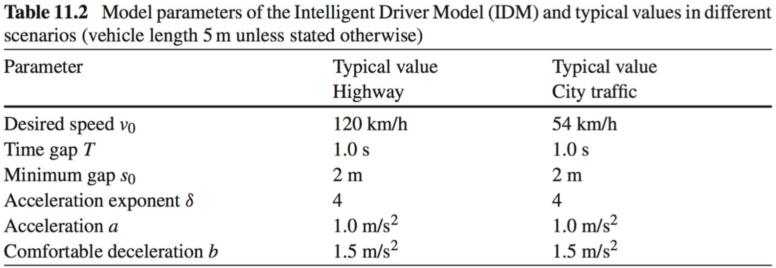


Table 11.2 Model parameters of the Intelligent Driver Model (IDM) and typical values in different scenarios (vehicle length 5 m unless stated otherwise)

Parameter	Typical value Highway	Typical value City traffic
Desired speed v_0	120 km/h	54 km/h
Time gap T	1.0 s	1.0 s
Minimum gap s_0	2 m	2 m
Acceleration exponent δ	4	4
Acceleration a	1.0 m/s^2	1.0 m/s^2
Comfortable deceleration b	1.5 m/s^2	1.5 m/s^2

Treiber, M., Hennecke, A. & Helbing, D. (2000) Congested traffic states in empirical observations and microscopic simulations. Phys. Rev. E 62, 1805–1824. (doi:10.1103/PhysRevE.62.1805)
M. Treiber and A. Kesting, "Chapter 11: Car-following Models Based on Driving Strategies," *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

IDM’s Intelligent braking strategy (intuition)

$$a(s,v,\Delta v) = a\left[1-\left(\frac{v}{v_0}\right)^{\delta}-\left(\frac{s^*(v,\Delta v)}{s}\right)^{2}\right]$$

where

$$s^*(v,\Delta \mathrm{v}) = \mathrm{s}_0 + \max\left(0, \mathrm{vT} + \frac{\mathrm{v}\Delta v}{2\sqrt{ab}}\right)$$

Treiber, M., Hennecke, A. & Helbing, D. (2000) Congested traffic states in empirical observations and microscopic simulations. Phys. Rev. E 62, 1805–1824. (doi:10.1103/PhysRevE.62.1805)
M. Treiber and A. Kesting, “Chapter 11: Car-following Models Based on Driving Strategies,” *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

Intelligent Driver Model (IDM) – validation

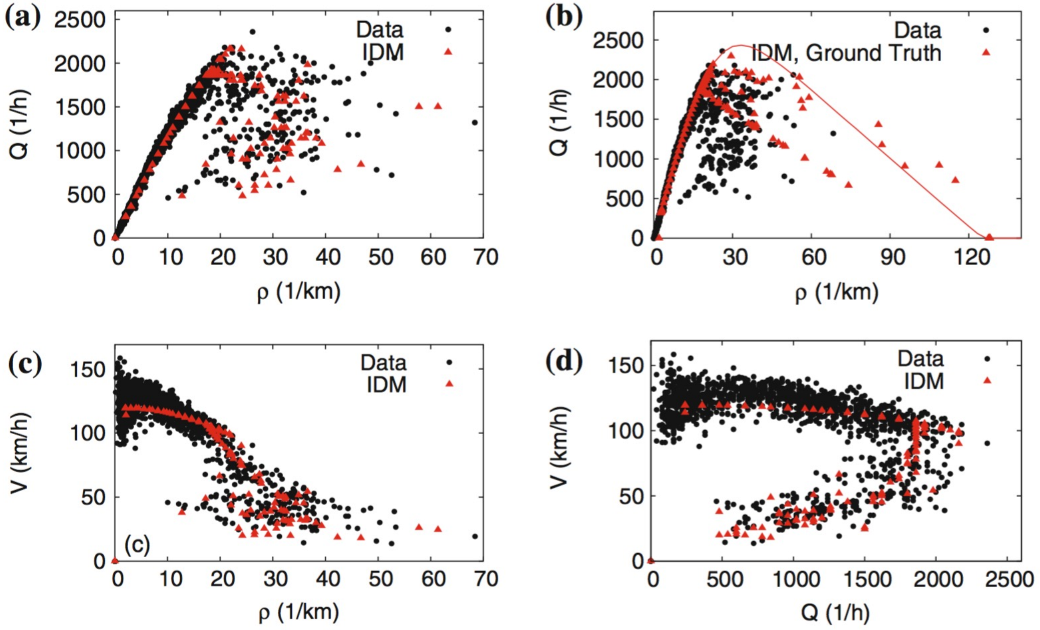
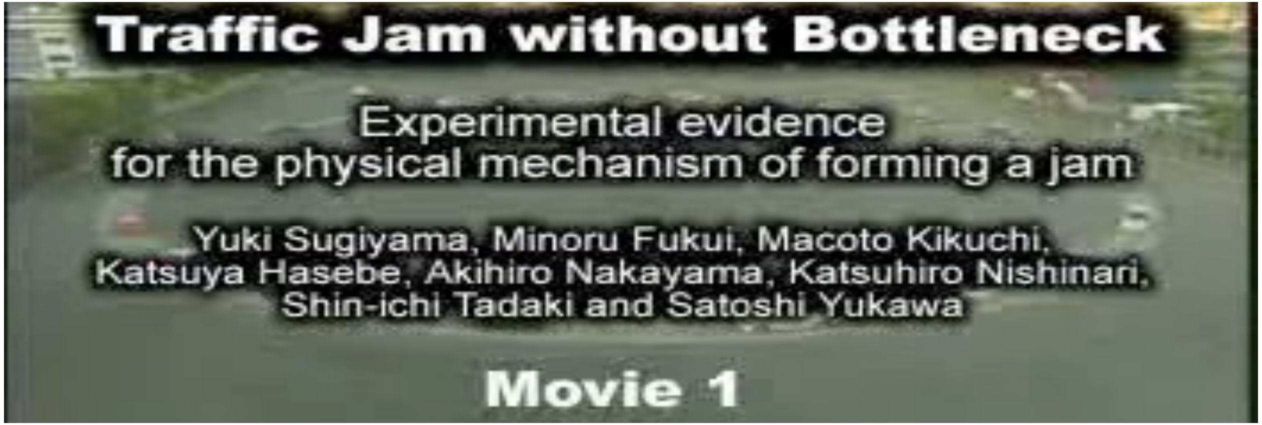


Fig. 11.5 **a** Fundamental diagram, **c** speed-density diagram, and **d** speed-flow diagram showing data from a virtual detector in the highway simulation shown in Fig. 11.4 (positioned 1 km upstream of the ramp). For comparison, empirical data from a real detector on the Autobahn A5 near Frankfurt, Germany, is shown. Velocities have been calculated using arithmetic means in both the real data and the simulation data. **b** Flow-density diagram with the same empirical data but using the real (local) density for the IDM simulation rather than the density derived from the virtual detectors

M. Treiber and A. Kesting, "Chapter 11: Car-following Models Based on Driving Strategies," *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

Recall: Traffic waves



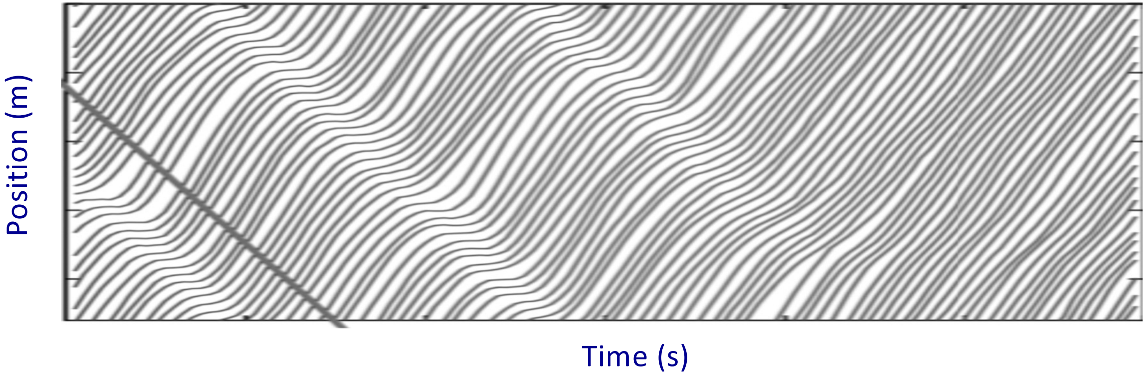
Traffic Jam without Bottleneck

Experimental evidence
for the physical mechanism of forming a jam

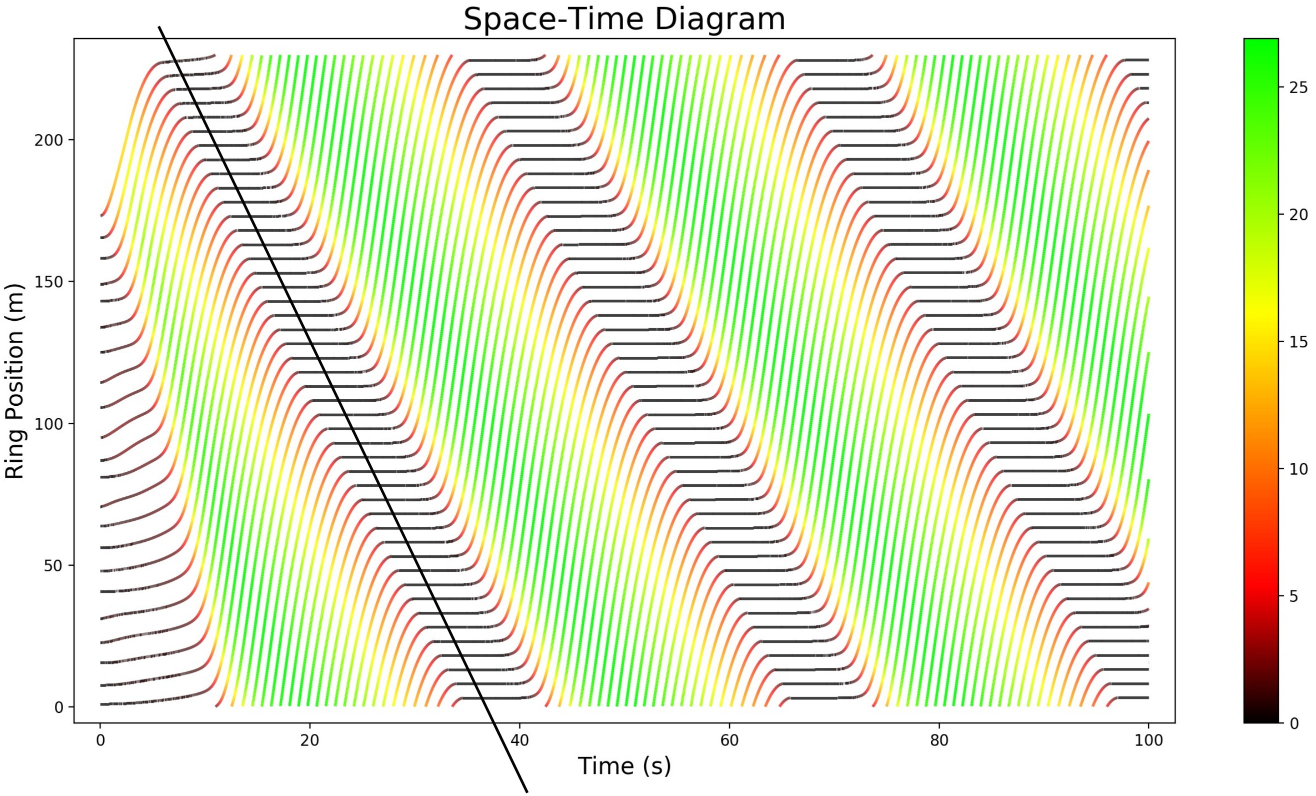
Yuki Sugiyama, Minoru Fukui, Macoto Kikuchi,
Katsuya Hasebe, Akihiro Nakayama, Katsuhiro Nishinari,
Shin-ichi Tadaki and Satoshi Yukawa

Movie 1

Vehicle trajectories (Sugiyama et al. 2008)



Reproduced using IDM



Rate of backwards wave propagation

C. Wu, A. R. Kreidieh, K. Parvate, E. Vinitsky, and A. M. Bayen, “Flow: A modular learning framework for mixed autonomy traffic,” *IEEE Transactions on Robotics (T-RO)*, vol. 38, no. 2, pp. 1270–1286, Jul. 2021, doi: 10.1109/TRO.2021.3087314.

IDM – Demo

http://www.traffic-simulation.de/ring.xhtml

Car following modeling principles

- Parsimonious: each model parameter describes one aspect of driving, is interpretable, and takes on plausible values.

- Varying realism depending on downstream application
  - Completeness of model
  - Accident-free
  - Incorporation of real driving behavior, such as keeping a safe distance or preferring comfortable accelerations

M. Treiber and A. Kesting, “Chapter 11: Car-following Models Based on Driving Strategies,” *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

Discussion: Build your own car-following model

1. What aspects of longitudinal driving behavior might be missing from the car following models we have discussed?

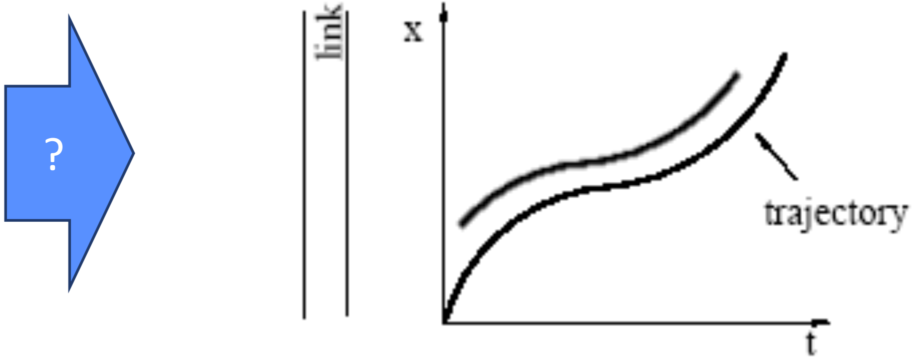
2. Using what surrounding information would you model such behavior?

Outline

1. Car following models
2. **Numerical integration**
3. Advanced driving behavior
4. Vehicle dynamics

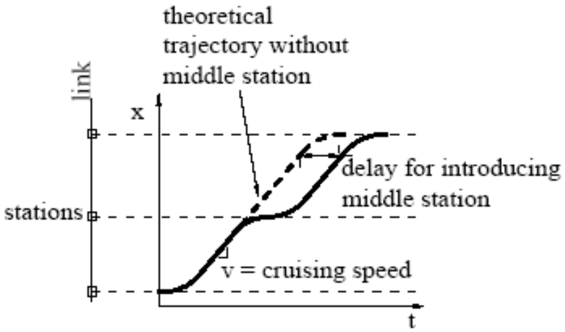
From longitudinal behavior model to a trajectory

$$a(v, x, t, \dots)$$

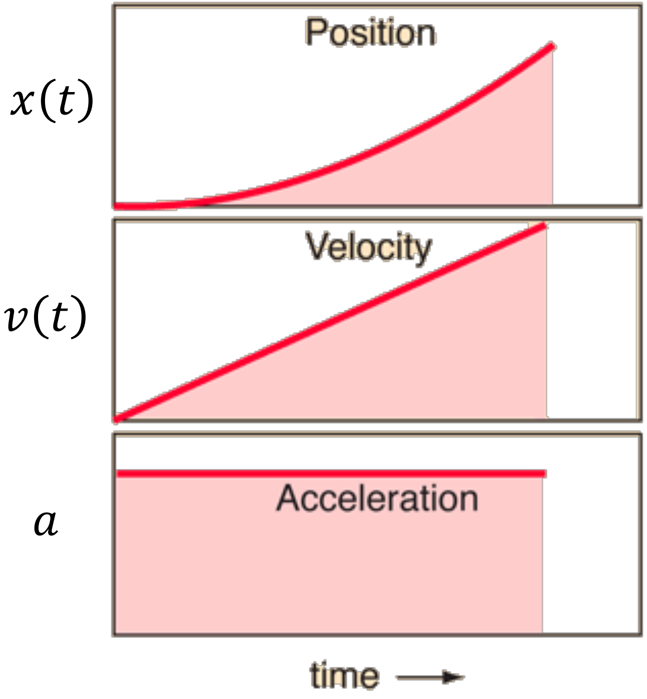


Special case: constant acceleration a

- Recall the basic equations of motion:
  - $v(t) = v_0 + at$
  - $x(t) = x_0 + vt + \frac{1}{2}at^2$



Transit station placement problem (Lecture 2)



Time-dependent acceleration

- Acceleration changes over time

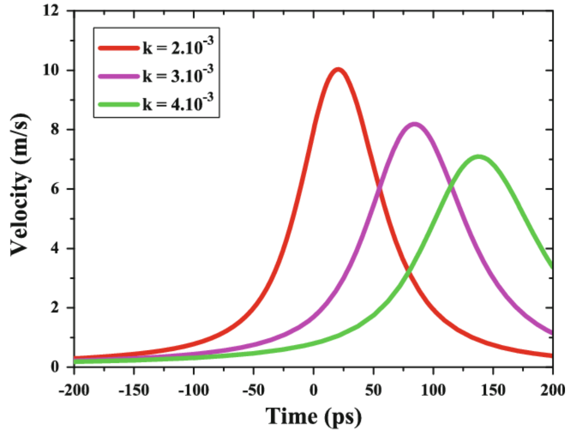
$$a(t)$$

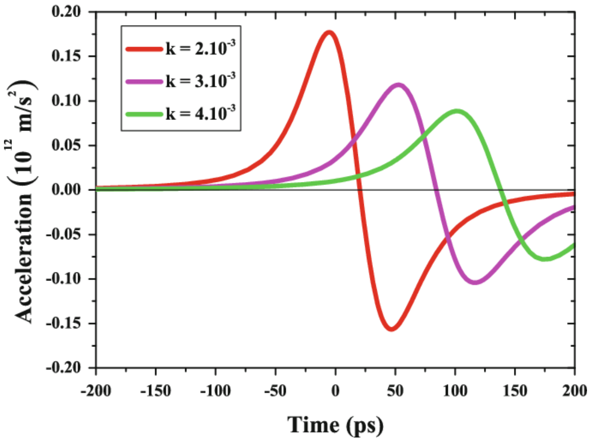
- Velocity

$$v(t) = v_0 + \int_0^t a(t)dt$$

- Position

$$x(t) = x_0 + \int_0^t v(t)dt$$





Wu

General case

- Acceleration depends on vehicle state and surrounding information

$$a(v(x,t),x(t),t,\dots)$$

- Velocity

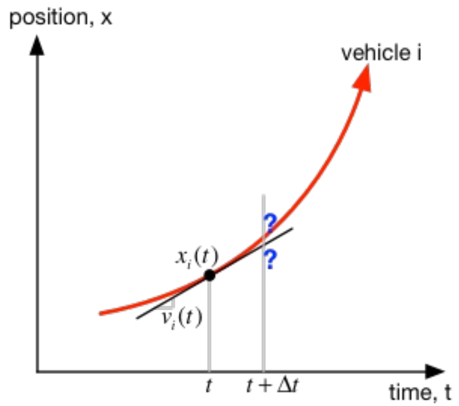
$$v(x,t) = v_0 + \int_0^t a(v(x,t),x(t),t,\dots)dt$$

- Position

$$x(t) = x_0 + \int_0^t v(x,t)dt$$

General solution: numerical integration

- Motivation: ODEs are in general extremely difficult to solve by hand
- Approach: Simulate and then analyze the ODE numerically.
  - *The catch*: Digital computers are discrete-time devices.
    Car-following models (ODEs) are continuous-time functions.
  - Need an **integration scheme** to approximate the numerical solution.



Given the situation at this moment, how to approximate what will happen in the next time increment (e.g., next second)?

Numerical integration: acceleration → velocity

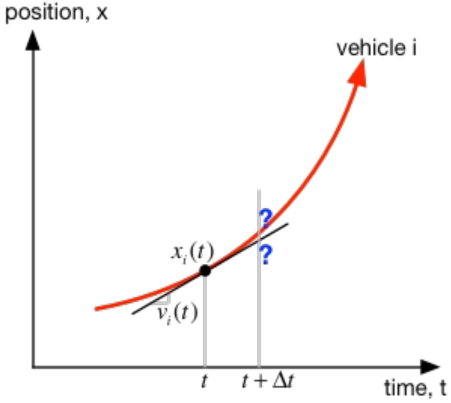
- Instead of allowing the acceleration to vary continuously in time, **assume** it is constant over very small (discrete) time intervals.
  - Let Δt ≪ 1 be the time step.
- Given the velocity at the beginning of the timestep, we can compute the velocity at the end of the time step by approximating the acceleration:

$$v(t+\Delta t) = v(t) + \int_{t}^{t+\Delta_t} a(x,t,v)$$

$$v(t+\Delta t) \approx v(t) + \int_{t}^{t+\Delta_t} a_{approx}\,dt$$

$$v(t+\Delta t) \approx v(t) + a_{approx}\Delta_{\mathrm{t}}$$

- Need to choose an appropriate a_{approx}.



Suppose that in a computer simulation, given the situation this time, what will happen in the next time increment (e.g., next second)?

Numerical integration

- To get the **position** of vehicle i at time $t + \Delta t$:

- For the continuous-time function (e.g., continuous CFM)

$$
\begin{aligned}
&\dot{v}_i(t) = A(\cdot) \\
&\xrightarrow{NI} \dot{x}_i(t) \approx v_i(t + \Delta t) = v_i(t) + \Delta t \dot{v}_i(t) \\
&\xrightarrow{NI} x_i(t + \Delta t) \approx x_i(t) + \Delta t \dot{x}_i(t)
\end{aligned}
$$

- For the discrete-time function (e.g., discrete CFM)

$$
\begin{aligned}
&\dot{x}_i(t) = v_i(t + \Delta t) = V(\cdot) \\
&\xrightarrow{NI} x_i(t + \Delta t)
\end{aligned}
$$

Forward Euler: first-order method

Suppose we wish to approximate the solution of the initial value problem

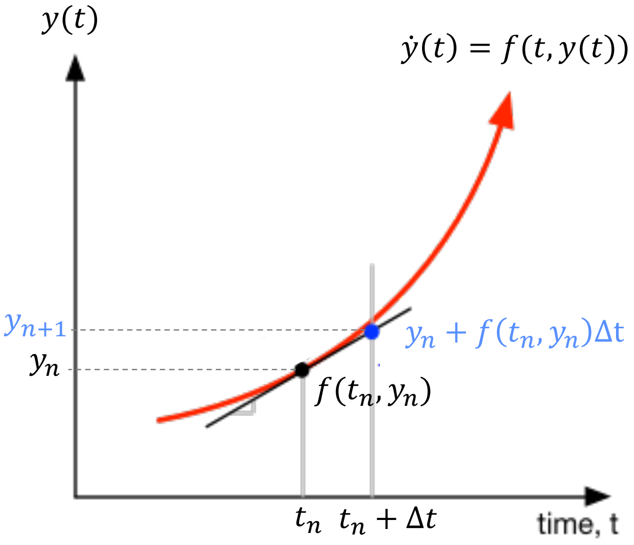
$$\dot{y}(t) = f\big(t, y(t)\big), \qquad \text{and} \qquad y(t_0) = y_0$$

Let Δt be the length of time step, and $t_n = t_0 + n\,\Delta t$. One step of Euler's method from t_n to t_{n+1} gives

$$y_{n+1} = y_n + f(t_n, y_n)\Delta t$$

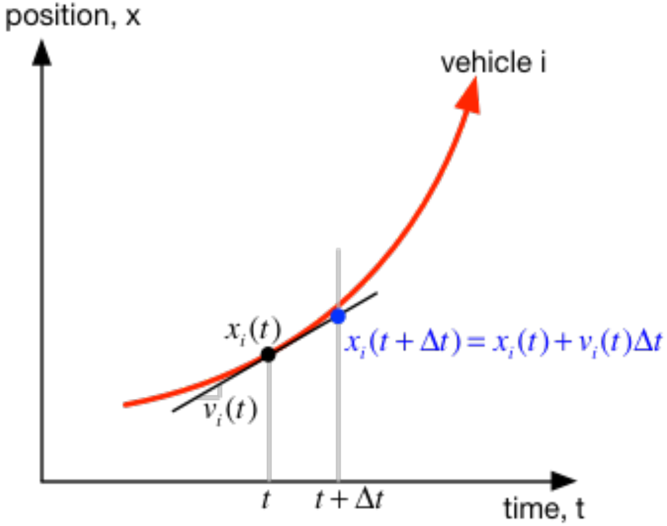
Based on a truncated Taylor series expansion, i.e., if we expand y in the neighborhood of $t = t_n$, we get

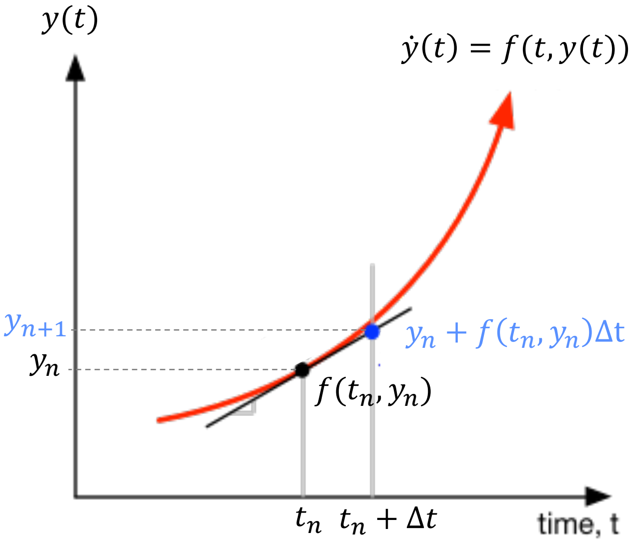
$$y(t_n + \Delta t) \equiv y_{n+1}$$
$$= y(t_n) + \Delta t \frac{dy}{dt}|_{t_n} + O(\Delta t^2)$$
$$= y_n + f(t_n, y_n)\Delta t + O(\Delta t^2)$$



Forward Euler for vehicle motion

- In car-following models, given:
  - $x_i(t) \rightarrow y(t_n) = y_n$ and
  - function $v(t + \Delta t) \rightarrow \dot{y}(t)$
- Calculate $x_i(t + \Delta t) \rightarrow y(t)$
- Also see the **error** generated.





Heun’s method (trapezoidal rule): second-order method

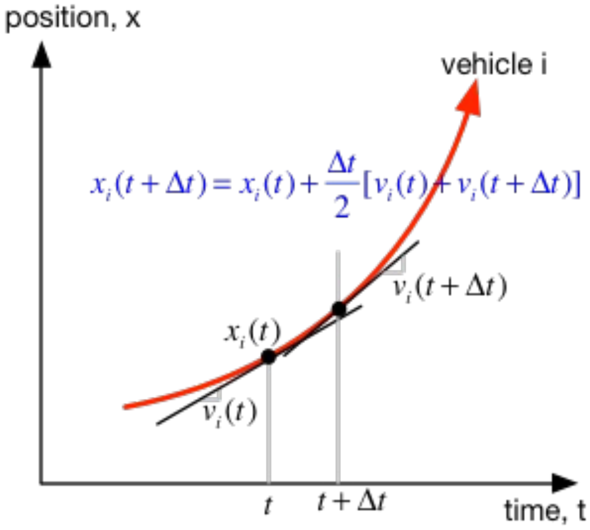
For the same initial value problem, we first calculate the intermediate value $\tilde{y}_{i+1}$ (i.e., slope at $t + \Delta t$)

$$\tilde{y}_{n+1} = y_n + f(t_n, y_n)\Delta\text{t}$$

Then, the final approximation y_{n+1} is based on the average of the slopes at t and $t + \Delta t$

$$y_{n+1} = y_n + \frac{\Delta t}{2}[f(t_n, y_n) + f(t_{n+1}, \tilde{y}_{n+1})]$$

Example: vehicle motion



Runge Kutta methods

- Forward Euler and Heun’s method belong to a family of numerical integration methods called Runge Kutta (RK) methods.
  - Shorthand: RK1, RK2 for first and second order methods, respectively.

RK4:

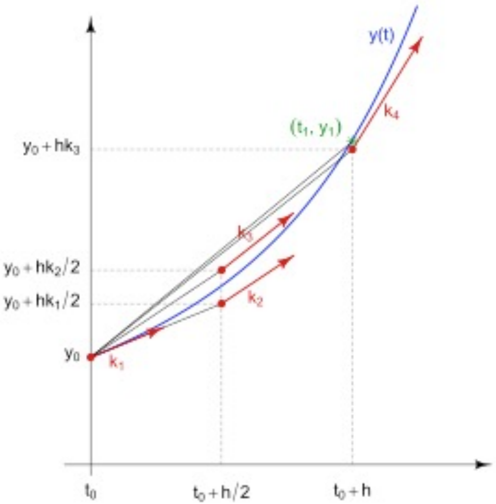


Image source: Wikipedia

- Most common: RK4.
- There is even a RK6.
- Trade-offs: computation cost, accuracy, numerical stability (growth in error over many steps)

$$y_{n+1} = y_n + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

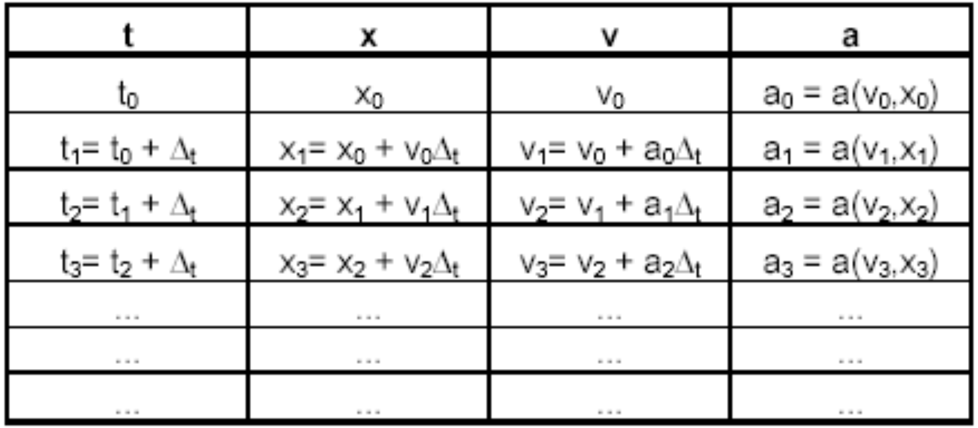
$$t_{n+1} = t_n + h$$

where

$$\begin{aligned} k_1 &= f(t_n, y_n), \\ k_2 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_1}{2}\right), \\ k_3 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_2}{2}\right), \\ k_4 &= f(t_n + h, y_n + hk_3)\,. \end{aligned}$$

Simulation of car-following models

- Under similar assumptions about the acceleration, we can compute the velocity.
- Simulation of a trajectory: Follow these procedures over and over again to generate a trajectory (on a computer)



t	x	v	a
t_0	x_0	v_0	$a_0 = a(v_0,x_0)$
$t_1 = t_0 + \Delta_t$	$x_1 = x_0 + v_0\Delta_t$	$v_1 = v_0 + a_0\Delta_t$	$a_1 = a(v_1,x_1)$
$t_2 = t_1 + \Delta_t$	$x_2 = x_1 + v_1\Delta_t$	$v_2 = v_1 + a_1\Delta_t$	$a_2 = a(v_2,x_2)$
$t_3 = t_2 + \Delta_t$	$x_3 = x_2 + v_2\Delta_t$	$v_3 = v_2 + a_2\Delta_t$	$a_3 = a(v_3,x_3)$
...	...	...	...
...	...	...	...
...	...	...	...

- Simulation of traffic: Repeat these procedures for each vehicle.
- Now, you can build your own traffic jam! (Computational Lab #1)

Outline

1. Car following models
2. Numerical integration
3. **Advanced driving behavior**
4. Vehicle dynamics

What influences driving behavior?

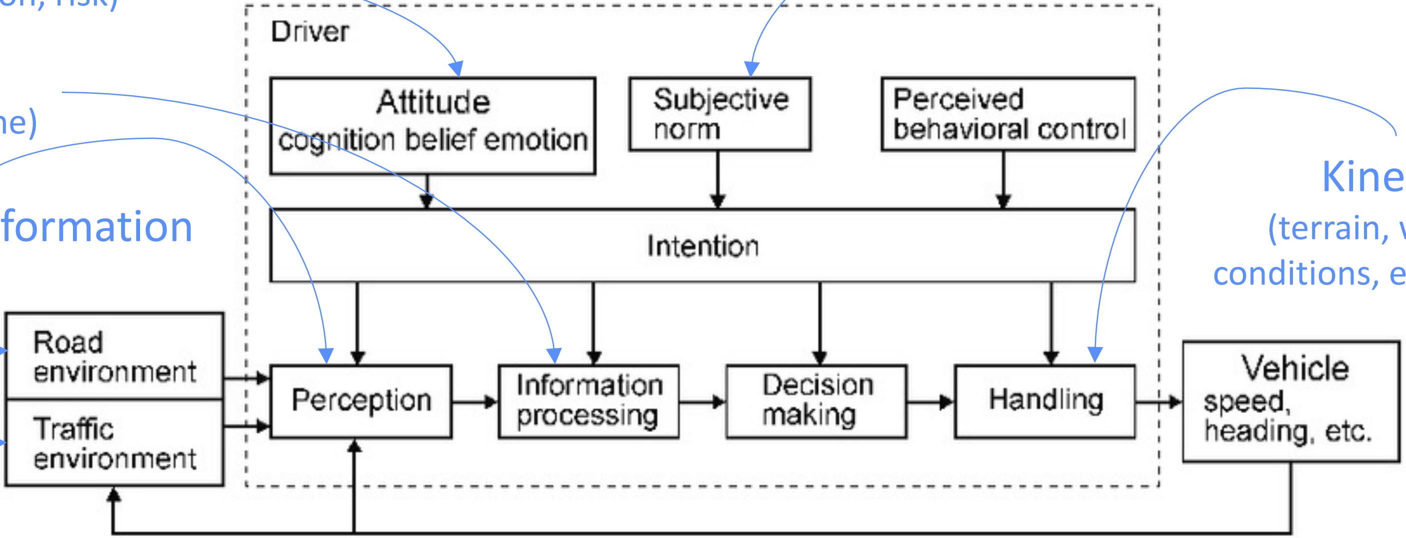
Driving style
(aggression, risk)

Delays
(reaction time)

Imperfect information

Driving culture
(Pittsburgh left, drink-driving, phone use while driving)

Kinetics
(terrain, weather conditions, engine type)



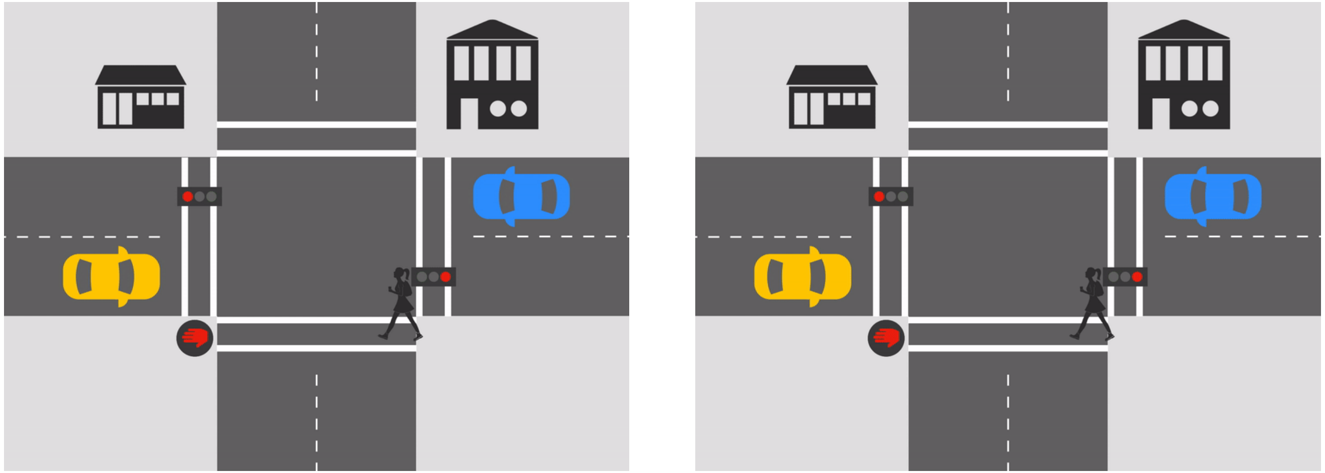
Other road users
(vehicles, pedestrians)

Rules & regulations
(traffic signals, speed limits)

Combined behavioural model to indicate factors that influence driver behaviour (after Van der Horst, 1998)

N. Schaap, “Driving behaviour in unexpected situations. A study into drivers’ compensation behaviour to safety-ciritical situations and the effects of mental workload, event urgency and task prioritization,” 2012.

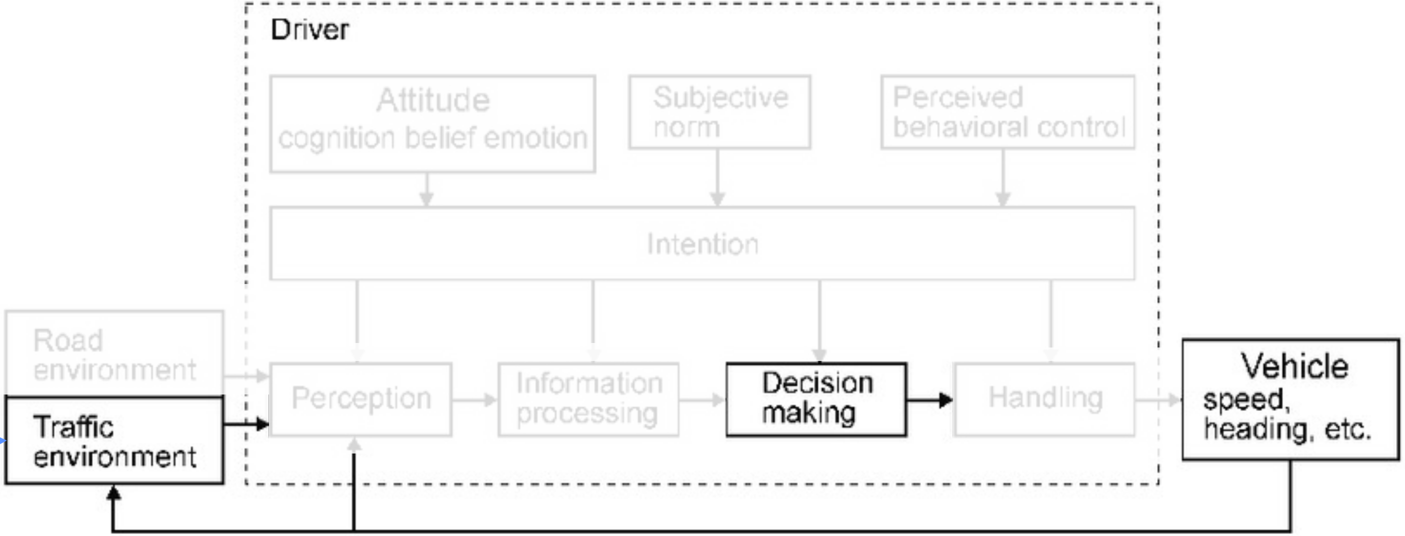
Pittsburgh left



The Pittsburgh left is imposing potential danger to the pedestrians crossing in the same direction.

Source: Wikipedia

What influences driving behavior? (Simplified)



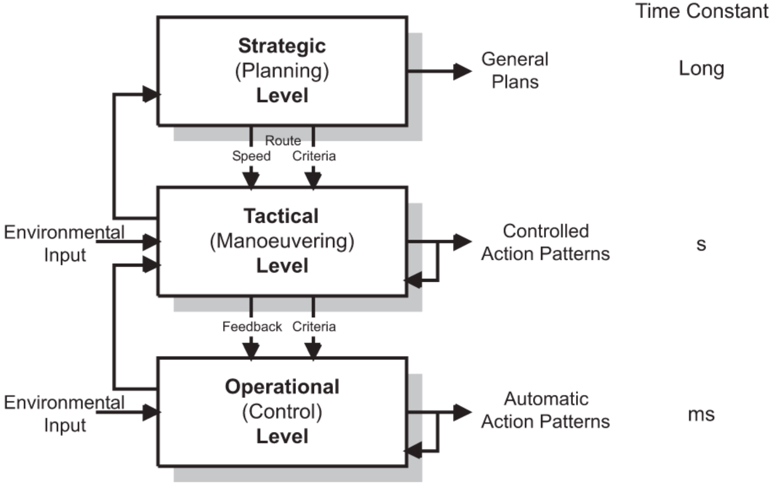
Other road users
(vehicles)

N. Schaap, “Driving behaviour in unexpected situations. A study into drivers’ compensation behaviour to safety-ciritical situations and the effects of mental workload, event urgency and task prioritization,” 2012.

[Michon, 1985]

Levels of the driving task

- Strategic
  - E.g., choice of turn / route
- Tactical
  - E.g., car following, overtaking, lane changing
- Operational / Control
  - Car operations (i.e., steering, operating throttle)
  - Lane keeping



Driver task hierarchy (after Michon 1985).

C. Lim, T. Sayed, and F. Navin, “A driver visual attention model. Part 1. Conceptual framework,” *Canadian Journal of Civil Engineering*, vol. 31, pp. 463–472, Feb. 2011, doi: 10.1139/l04-020.

[Michon, 1985]

Levels of the driving task

- Strategic
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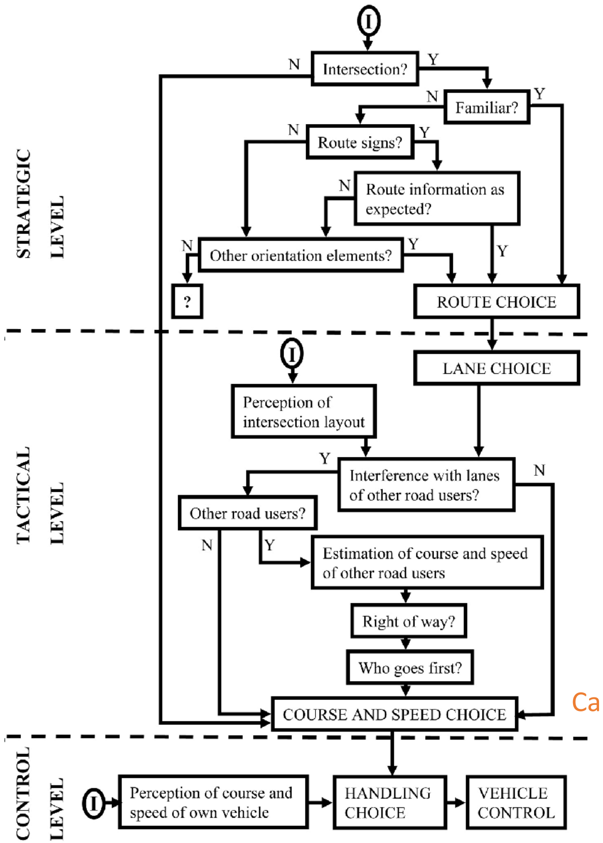

STRATEGIC LEVEL
I
Intersection?
N
Y
Familiar?
N
Y
Route signs?
N
Y
Route information as expected?
N
Y
Other orientation elements?
N
Y
?
ROUTE CHOICE
TACTICAL LEVEL
I
LANE CHOICE
Perception of intersection layout
Interference with lanes of other road users?
Y
N
Other road users?
N
Y
Estimation of course and speed of other road users
Right of way?
Who goes first?
COURSE AND SPEED CHOICE
CONTROL LEVEL
I
Perception of course and speed of own vehicle
HANDLING CHOICE
VEHICLE CONTROL

Car following

N. Schaap, “Driving behaviour in unexpected situations. A study into drivers’ compensation behaviour to safety-ciritical situations and the effects of mental workload, event urgency and task prioritization,” 2012.

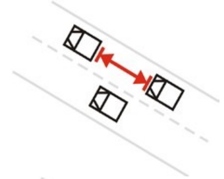
Basic inter-vehicle interactions

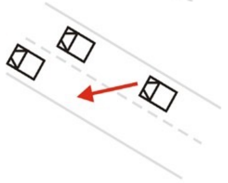
How do drivers interact with other (nearby) vehicles?

Summary of basic driving subtasks

	Roadway interaction subtask	Vehicle interaction subtasks
Longitudinal	Speed choices (free speed)	Car-following
Lateral	Lane choice	Lane changing, merging, overtaking

How do drivers accelerate to their desired speed?





Lane changing behavior and other discrete choices

- General decision model: Utility maximization with discrete choice
- Discrete set K of alternatives
  - Ex. Lane change: left, right, or none.
  - Ex. Merging onto priority road: stopping or waiting for a sufficient gap between the main-road vehicles
- Acceleration as utility: $U^{(\beta,k)} := a^{(\beta,k)}$, for driver β, alternative $k \in K$
- Ego-driver α selects the option of maximum utility

$$k_{selected} = \arg\max_{k'} U^{(\alpha,k')}$$

M. Treiber and A. Kesting, "Chapter 14: Lane-Changing and Other Discrete-Choice Situations," *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

Lane changing behavior

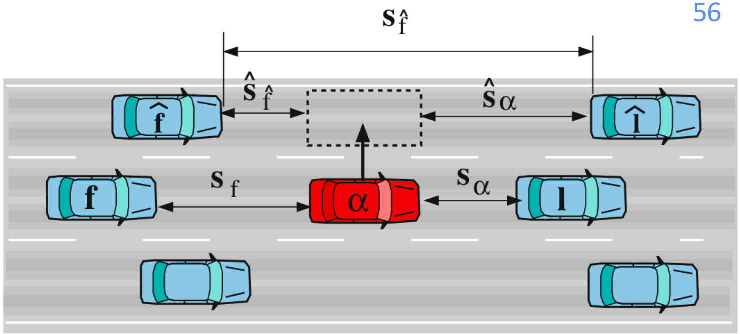


Fig. 14.1 Notation for a lane change of the *center* vehicle α to the *left*. All quantities with a hat pertain to the new situation after the (possibly hypothetical) lane change

- WLOG, consider 2 options (change to left lane or stay in current lane)
- Change lanes if utility of another lane is greater than current lane

$$\hat{a}_\alpha - a_\alpha > 0$$

- Add a threshold to prevent marginal advantage

$$\hat{a}_\alpha - a_\alpha > \Delta a > 0$$

- Add a bias term to model asymmetric behavior, $a_{bias} > \Delta a$ (e.g., right-overtaking ban on most European highways)

$$\hat{a}_\alpha - a_\alpha > \Delta a + a_{bias} > 0$$

M. Treiber and A. Kesting, “Chapter 14: Lane-Changing and Other Discrete-Choice Situations,” *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

Lane changing behavior (MOBIL)

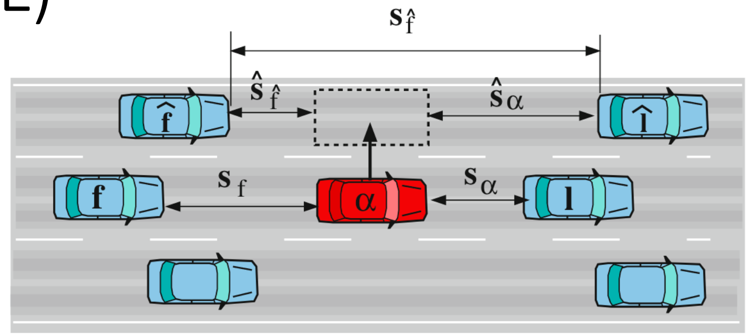


Fig. 14.1 Notation for a lane change of the *center* vehicle α to the *left*. All quantities with a hat pertain to the new situation after the (possibly hypothetical) lane change

- Add a politeness term ($p \in [0,1]$, typically ≈ 0.2)

$$\hat{a}_\alpha - a_\alpha + p(\hat{a}_{\hat{f}} - a_{\hat{f}} + \hat{a}_f - a_f) > \Delta a + a_{bias} > 0$$

  - Special case: altruistic driver ($p = 1$), a lane change takes place only if sum of acceleration of all affected drivers increases with the maneuver
  - This model is called MOBIL (**M**inimizing **o**verall **b**raking deceleration **i**nduced by **l**ane changes)

M. Treiber and A. Kesting, "Chapter 14: Lane-Changing and Other Discrete-Choice Situations," *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: 10.1007/978-3-642-32460-4.

More complex driving models

- Multiple leaders
- Reaction times
- Perception thresholds and action point models
- Responding to brake lights

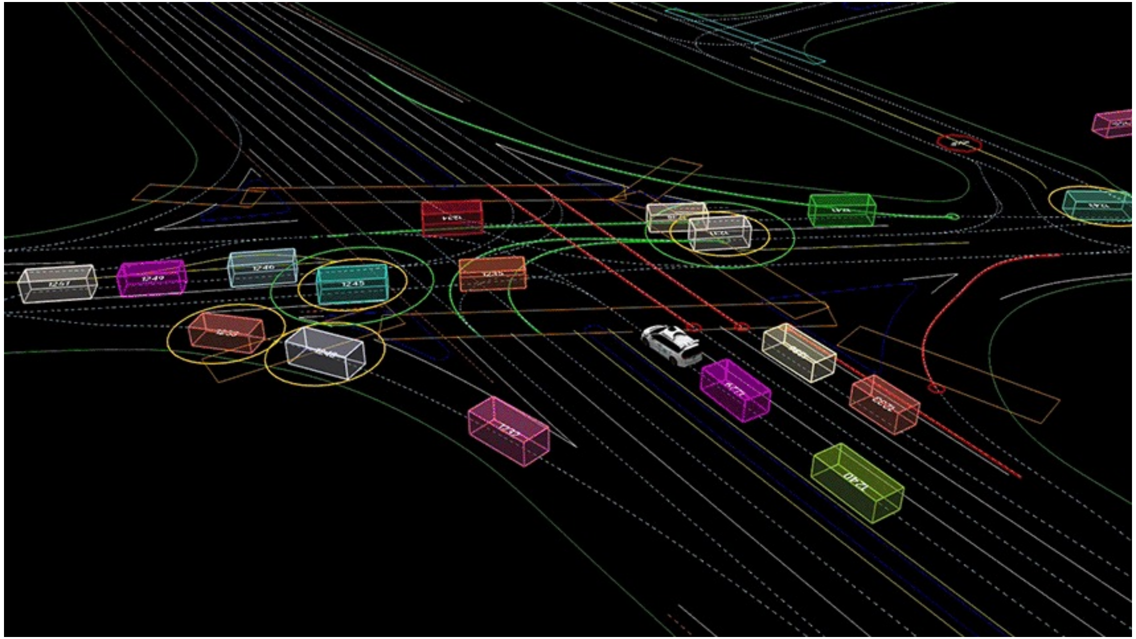
- Distracted driving
- Driving fatigue
- Imperfect driving
- Estimation errors
- Field of vision

- Trade-offs: Simplicity vs fit error. These models have more parameters, and thus may fit data better. However, similar resulting behavior can often be observed in simpler models.

M. Treiber and A. Kesting, “Chapter 12: Modeling Human Aspects of Driving Behavior,” *Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg*, 2013, doi: [10.1007/978-3-642-32460-4](https://doi.org/10.1007/978-3-642-32460-4).

Driving behavior foundation models

- Waymo Sim Agents Challenge (2023) / Waymo Open Motion Dataset



Captures road topology, road signs, and nearby vehicles in all lanes and driving history

[1] Montali et al., The Waymo Open Sim Agents Challenge, NeurIPS 2023

AV Traffic Simulations

Benchmarks

WAYMO

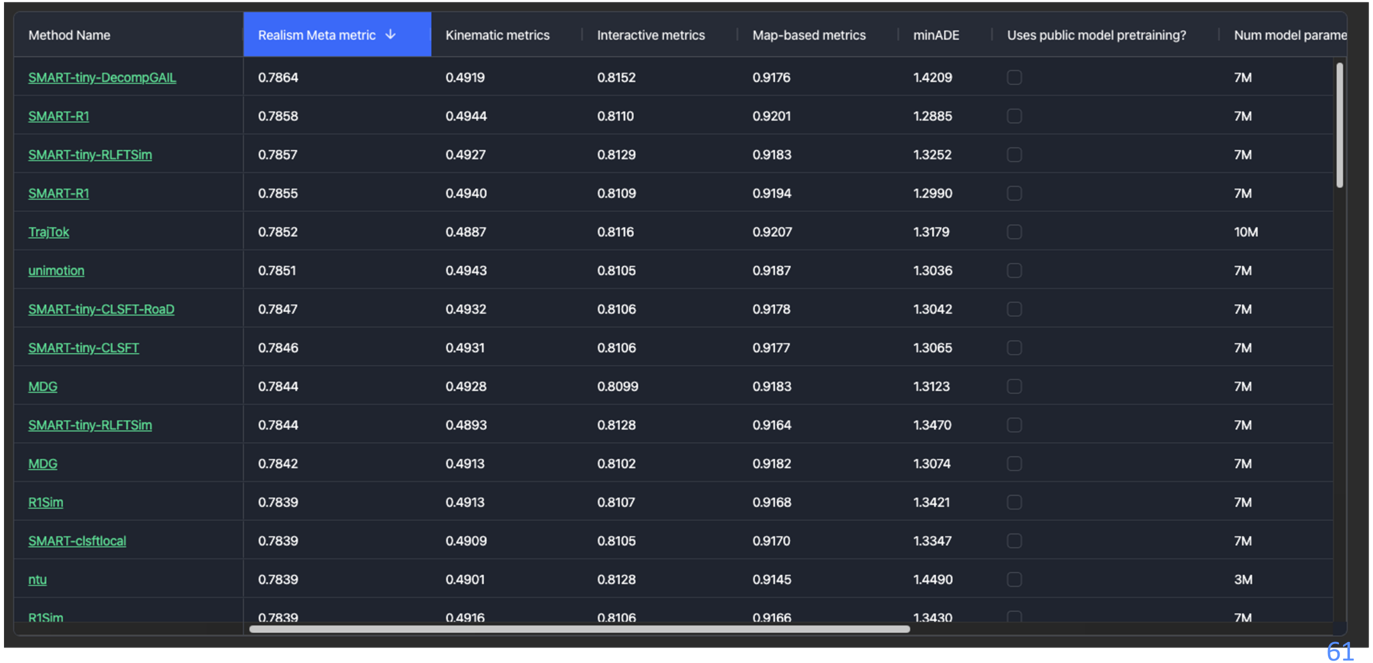
Datasets

WAYMO Open Dataset

NUSCENES by Motional

Argoverse

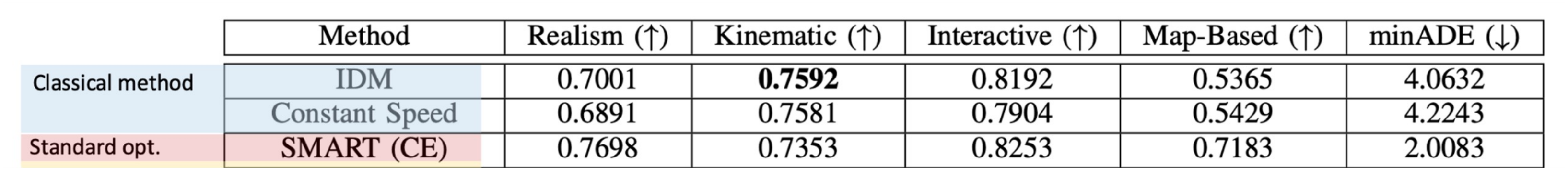
Methods



Method Name	Realism Meta metric ↓	Kinematic metrics	Interactive metrics	Map-based metrics	minADE	Uses public model pretraining?	Num model parame
SMART-tiny-DecompGAIL	0.7864	0.4919	0.8152	0.9176	1.4209	☐	7M
SMART-R1	0.7858	0.4944	0.8110	0.9201	1.2885	☐	7M
SMART-tiny-RLFTSim	0.7857	0.4927	0.8129	0.9183	1.3252	☐	7M
SMART-R1	0.7855	0.4940	0.8109	0.9194	1.2990	☐	7M
TrajTok	0.7852	0.4887	0.8116	0.9207	1.3179	☐	10M
unimotion	0.7851	0.4943	0.8105	0.9187	1.3036	☐	7M
SMART-tiny-CLSFT-RoaD	0.7847	0.4932	0.8106	0.9178	1.3042	☐	7M
SMART-tiny-CLSFT	0.7846	0.4931	0.8106	0.9177	1.3065	☐	7M
MDG	0.7844	0.4928	0.8099	0.9183	1.3123	☐	7M
SMART-tiny-RLFTSim	0.7844	0.4893	0.8128	0.9164	1.3470	☐	7M
MDG	0.7842	0.4913	0.8102	0.9182	1.3074	☐	7M
R1Sim	0.7839	0.4913	0.8107	0.9168	1.3421	☐	7M
SMART-clsftlocal	0.7839	0.4909	0.8105	0.9170	1.3347	☐	7M
ntu	0.7839	0.4901	0.8128	0.9145	1.4490	☐	3M
R1Sim	0.7839	0.4916	0.8106	0.9166	1.3430	☐	7M

Wu

Driving behavior foundation models



	Method	Realism (↑)	Kinematic (↑)	Interactive (↑)	Map-Based (↑)	minADE (↓)
Classical method	IDM	0.7001	**0.7592**	0.8192	0.5365	4.0632
	Constant Speed	0.6891	0.7581	0.7904	0.5429	4.2243
Standard opt.	SMART (CE)	0.7698	0.7353	0.8253	0.7183	2.0083

Competitive driving behavior foundation model (7M parameters) [2]

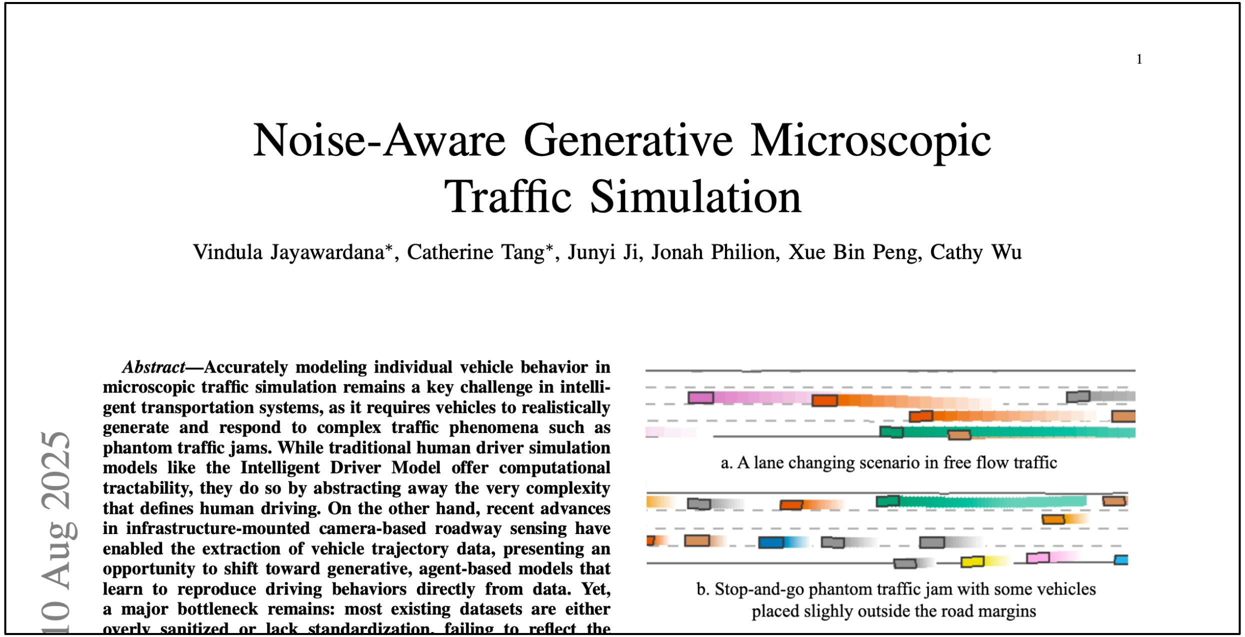
- **Kinematic-based features:** Linear speed, linear acceleration magnitude, angular speed, and angular acceleration magnitude
- **Interaction-based features:** TTC and distance to nearest object
- **Map-based features:** Road departures and distance to road edge
- **minADE:** How close the generated trajectory is to the ground-truth trajectory on average over all time steps
- **Realism:** A meta metric

[1] Montali et al., The Waymo Open Sim Agents Challenge, NeurIPS 2023

[2] Wu, et al., SMART: Scalable Multi-agent Real-time Motion Generation via Next-token Prediction, NeurIPS 2024

Active research topic

- Caveat: Developed by and for autonomous driving
- What are the utility / implications for traffic operations & planning?



1

Noise-Aware Generative Microscopic Traffic Simulation

Vindula Jayawardana*, Catherine Tang*, Junyi Ji, Jonah Philion, Xue Bin Peng, Cathy Wu

10 Aug 2025

Abstract—Accurately modeling individual vehicle behavior in microscopic traffic simulation remains a key challenge in intelligent transportation systems, as it requires vehicles to realistically generate and respond to complex traffic phenomena such as phantom traffic jams. While traditional human driver simulation models like the Intelligent Driver Model offer computational tractability, they do so by abstracting away the very complexity that defines human driving. On the other hand, recent advances in infrastructure-mounted camera-based roadway sensing have enabled the extraction of vehicle trajectory data, presenting an opportunity to shift toward generative, agent-based models that learn to reproduce driving behaviors directly from data. Yet, a major bottleneck remains: most existing datasets are either overly sanitized or lack standardization, failing to reflect the

a. A lane changing scenario in free flow traffic

b. Stop-and-go phantom traffic jam with some vehicles placed slighly outside the road margins

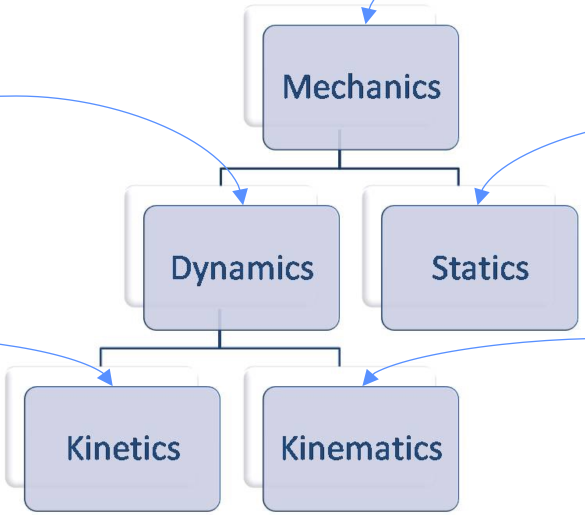
V. Jayawardana, C. Tang, J. Ji, J. Philion, X. B. Peng, and **C. Wu**, “Noise-Aware Generative Microscopic Traffic Simulation,” arXiv, 2025. doi: 10.48550/arXiv.2508.07453.

Outline

1. Car following models
2. Numerical integration
3. Advanced driving behavior
4. **Vehicle dynamics**

Vehicle dynamics

Objects that move

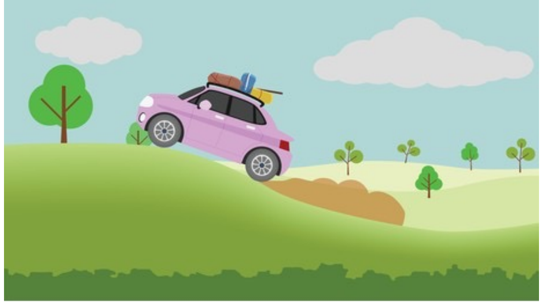


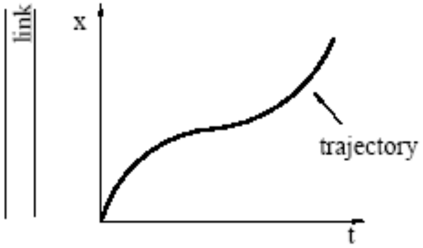
Body in acceleration
(forces on the motion)

Body in equilibrium
(at rest or constant velocity)

Newton’s laws of motion
(without consideration of physics)

Pure motion, e.g. behavior
(without consideration of physics)





Vehicle Kinematics

- The branch of mechanics that studies the motion of a body or a system of bodies without consideration given to its mass or the forces acting on it.

- $a(v, x, t)$ describes driving behavior.
  - Longitudinal motion
  - Lateral motion

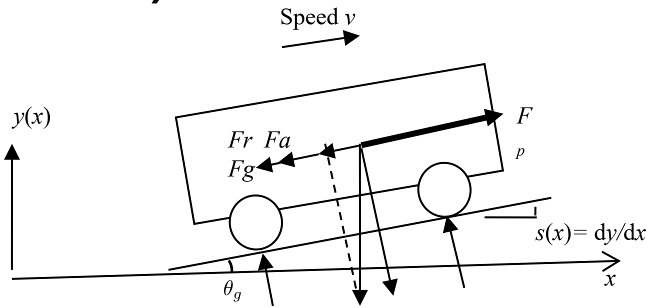
- Note that from basic kinematics:

$$a(v, x, t) = \dot{v} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

  - These are *ordinary differential equations* (ODEs) of $x(t)$ and $v(t)$. We can solve them to obtain the full motion of a vehicle.

Basic resistance forces (kinetics)

$$F_p = ma + F_a + F_g + F_r$$



- Aerodynamic resistance; $\mathrm{F_a}$
- $\mathrm{F_a} = -\frac{\rho}{2}\mathrm{DAv}^2$
  - ρ = air density
  - D = drag coefficient
  - A = frontal cross-sectional area of the vehicle
  - v = vehicle speed

- Grade resistance (from gravity); $\mathrm{F_g}$
- $F_g = -mg\sin\theta_g$
  - m = mass
  - θ_g = angle of the slope/grade
  - g = gravity acceleration

- Rolling resistance; $\mathrm{F_r}$
- $F_r = -\beta mv \quad (\ll F_a)$
  - $\beta \approx$ tire rigidity and road surface

Prof. Philip Koopman

Carnegie
Mellon
University

AV Trajectories: Newtonian Mechanics vs. The Real World

@PhilKoopman

References

1. Chap 10 (Elementary Car-Following Models) of Traffic Flow Dynamics: Data, Models and Simulation (2013)

2. Yang, Shaopu, Yongjie Lu, and Shaohua Li. "An overview on vehicle dynamics." International Journal of Dynamics and Control 1, no. 4 (2013): 385-395.

3. MIT 10.001: Numerical Solution of Ordinary Differential Equations http://web.mit.edu/10.001/Web/Course_Notes/Differential_Equations_Notes/lec24.html

4. Slides adapted from:
   - Prof. Dan Work (Vanderbilt CE 3501)
   - Prof. Victor L. Knoop (TU Delft)
   - Prof. Zhengbing He (Beijing Jiaotong University)