Macroscopic traffic models

Traffic Flow Theory

Cathy Wu

1.041/1.200 Transportation: Foundations and Methods

Readings

1. C. Daganzo, *Fundamentals of transportation and traffic operations*, vol. 30. Pergamon Oxford, 1997. Chapter 4: Traffic flow theory. URL.

Outline

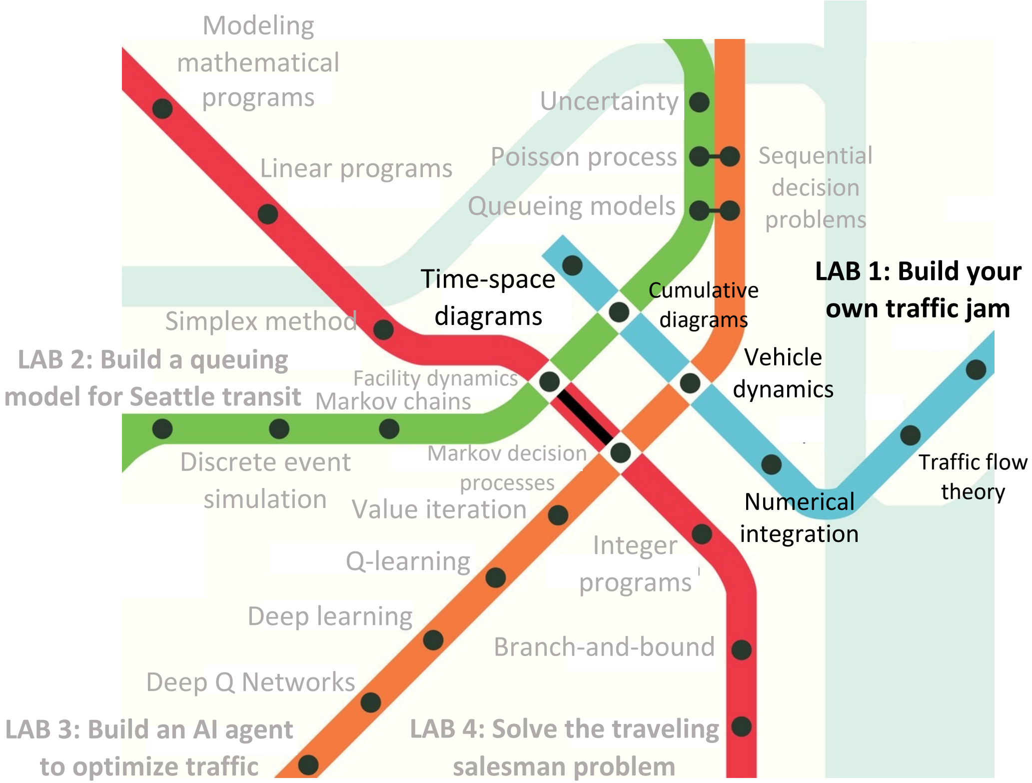
1. Basic assumptions of traffic flow theory
2. Fundamental diagrams (FDs)
3. Highway delay problem

Unit 1: Traffic flow fundamentals

Unit 1

Modeling

Deterministic



Outline

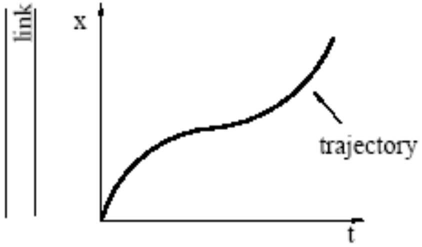
Traffic flow theory

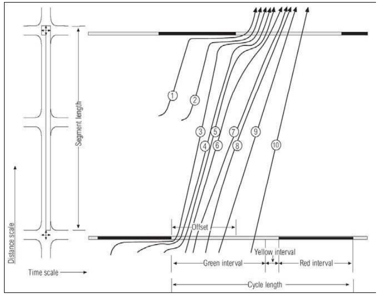
- Today: From traffic flow (traffic streams) to traffic flow theory

- **Traffic flow theory**:
  - Models and hypotheses for explaining traffic flow
  - I.e., what would happen to traffic streams if they were to flow on roads under different conditions, potentially not yet observed

- Models vs data

$$a_{\mathrm{IDM}}(s, v, \Delta v)$$





Basic assumptions

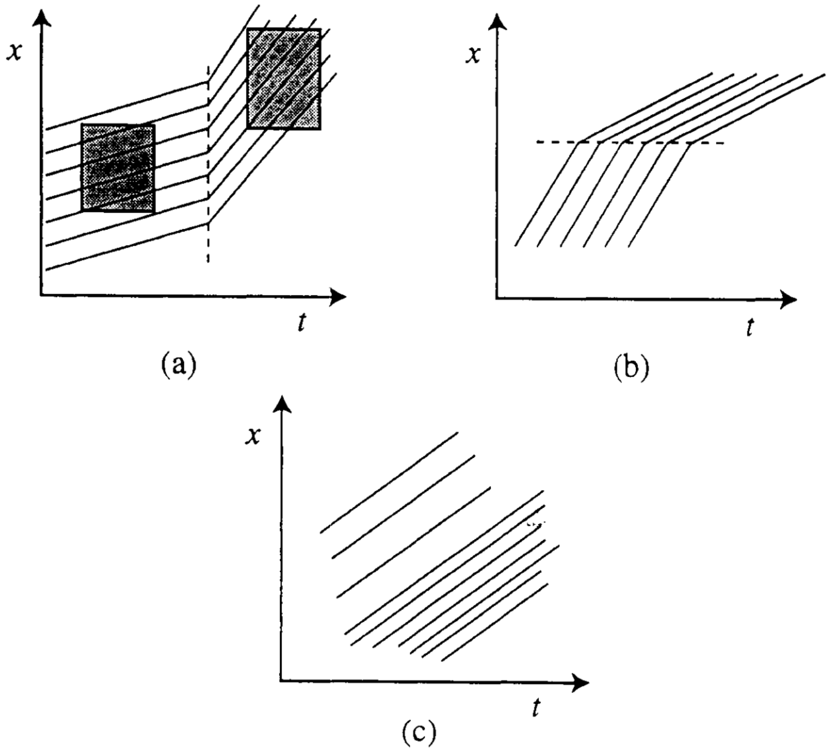
1. Study of a single traffic stream, flowing on a facility with a single entrance and a single exit
2. Uninterrupted traffic
   - Traffic regulated by interactions between vehicles, as opposed to being regulated by external means
   - E.g. on a highway or at unsignalized intersections, as opposed to traffic lights, stop signs.
3. Stationary traffic conditions (vs. time and space-varying dynamics)

Stationary vs non-stationary traffic

Stationary traffic conditions

(vs. time- or space-varying dynamics):

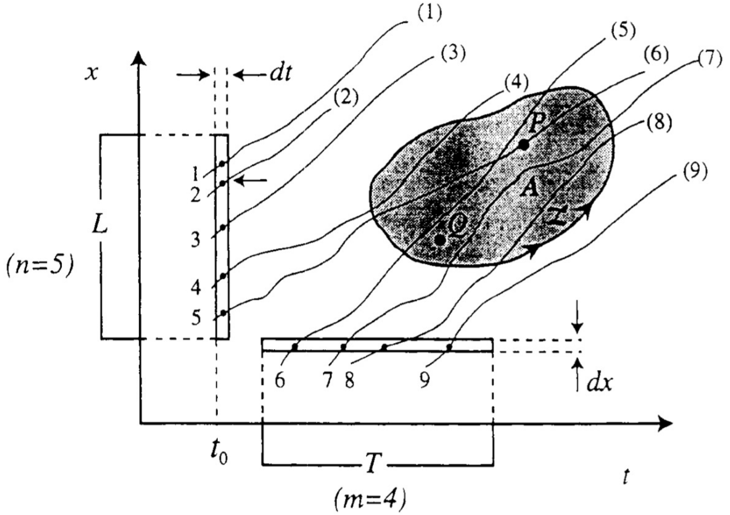
- Traffic is stationary if it is a superposition of families of trajectories that are each parallel and equidistant.



Examples of non-stationary traffic

Traffic stream variables

- Main variables
  - Flow
  - Time headway
  - Density
  - Spacing
  - Speed (space-mean, time-mean)

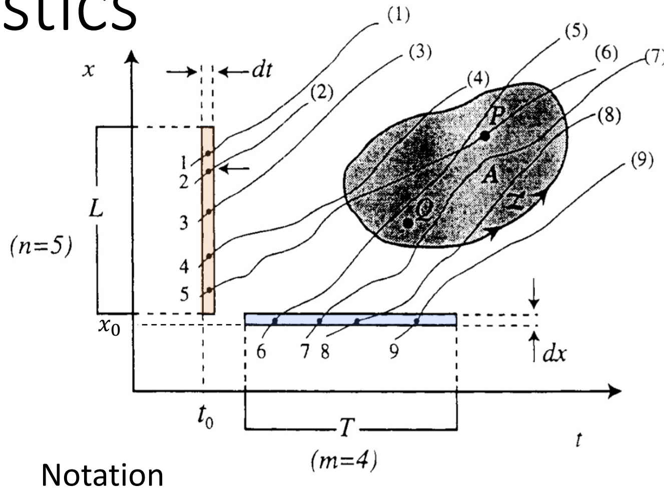


- Aim: Obtain relationships that hold "on average"; i.e. for large stationary time-space regions containing many vehicles

Formulas for traffic characteristics

Table 4.1.. Generalized formulas for various traffic characteristics using two observation methods. Boxed expressions correspond to the original definitions introduced in Chapter 1:

	Method of Observation	
	Instantaneous photograph at time t_0 **(section length, L)**	**Observation from a fixed location** x_0 **(duration, T)**
Density, k(**A**)	$\boxed{n/L}$	$\frac{1}{T}\sum_{j=1}^{m} p_j = \frac{1}{T}\sum_{j=1}^{m}\frac{1}{u_j}$
Flow, q(**A**)	$\frac{1}{L}\sum_{i=1}^{n} v_i$	$\boxed{m/T}$
Space-mean speed, v(**A**)	$\boxed{\frac{1}{n}\sum_{i=1}^{n} v_i}$	$[\frac{1}{m}\sum_{j=1}^{m} p_j]^{-1} = \left[\frac{1}{m}\sum_{j=1}^{m}\frac{1}{u_j}\right]^{-1}$
Average pace, p(**A**)	$[\frac{1}{n}\sum_{i=1}^{n} v_i]^{-1}$	$\frac{1}{m}\sum_{j=1}^{m} p_j$
t(**A**)	ndt	$dx\sum_{j=1}^{m} p_j$
d(**A**)	$dt\sum_{i=1}^{n} v_i$	mdx



- Notation
  - v_i = velocity (mi/hr) of vehicle i in section L at time t_0
  - p_j = pace (hr/mi) of vehicle j during duration T at location x_0
  - u_j = velocity (mi/hr) of vehicle j, $u_j = \frac{1}{p_j}$
  - $t(\boldsymbol{A})$ = total time spent in $\boldsymbol{A}$ by all vehicles
  - $d(\boldsymbol{A})$ = total distance traveled by vehicles in $\boldsymbol{A}$
- If traffic is stationary, then the two columns coincide, i.e. same values.
  - Proof: $v = v_i = \frac{1}{p_j}, \forall i, j$
- Then, for instance, density can be measured by counts at a fixed location.

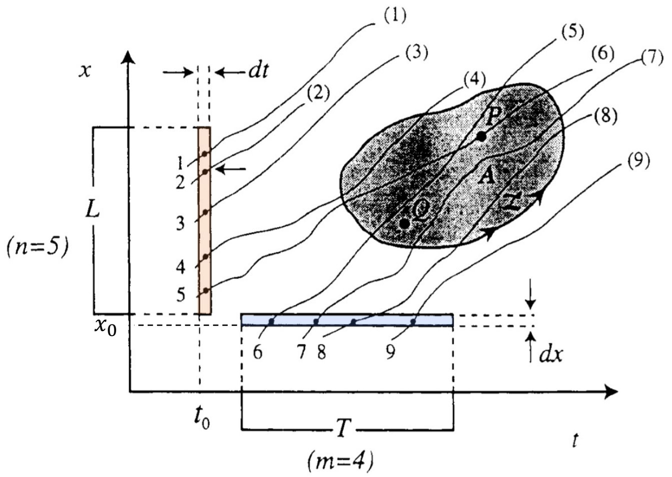
C. Daganzo, *Fundamentals of transportation and traffic operations*, vol. 30. Pergamon Oxford, 1997. Chapter 4: Traffic flow theory.

Outline

1. **Basic assumptions of traffic flow theory**
   a. Key variables
   b. **Time vs space means**
2. Fundamental diagrams (FDs)
3. Highway delay problem

Time and space means

- Space-mean: averages taken at an instant over a space interval
- Time-mean: averages taken at a specific locatio (with time-varying over an interval)
- Speed:
  - $\overline{v}_s$: space-mean speed
  - $\overline{v}_t$: time-mean speed



- Other *vehicle characteristics* can be averaged across space or time. E.g., occupancies (number of persons per vehicle), energy consumption, emissions, etc.
  - There is no a priori reason to expect averages taken across space or time to be the same.
  - Example: You own two cars, they are both driven an equal distance of 100 miles. One gets 20 miles per gallon (mpg), the other 50 mpg. Is the average mpg 35 (i.e. $\frac{50+20}{2}$)?

Time and space means

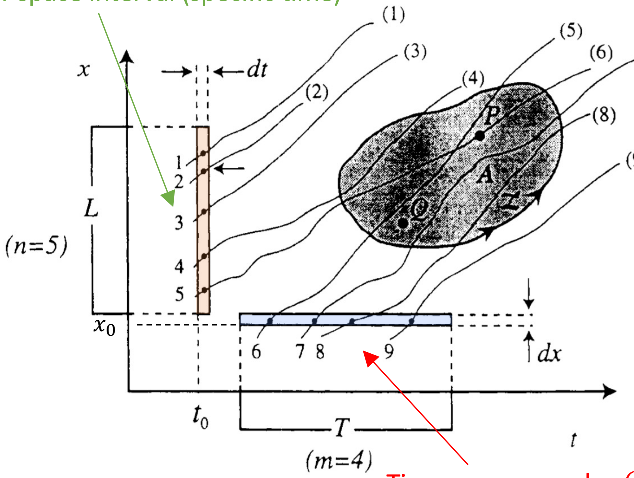
Table 4.1.. Generalized formulas for various traffic characteristics using two observation methods. Boxed expressions correspond to the original definitions introduced in Chapter 1:

	Method of Observation	
	Instantaneous photograph at time t_0 **(section length, L)**	**Observation from a fixed location** x_0 **(duration, T)**
Space-mean speed, v(A)	$\boxed{\frac{1}{n}\sum_{i=1}^{n} v_i}$	$[\frac{1}{m}\sum_{j=1}^{m} p_j]^{-1} = \left[\frac{1}{m}\sum_{j=1}^{m}\frac{1}{u_j}\right]^{-1}$
Time-mean speed $v_t(A)$	$\left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{v_i}\right]^{-1}$	$\frac{1}{m}\sum_{j=1}^{m}\frac{1}{p_j} = \frac{1}{m}\sum_{j=1}^{m} u_j$

Harmonic mean

Arithmetic mean

Space-mean speed $v(A) \equiv v_s(A)$
mean over space interval (specific time)



Time-mean speed $v_t(A)$
mean over time duration (specific location)

- Notation:
  - v_i = velocity (mi/hr) of vehicle i
  - p_j = pace (hr/mi) of vehicle j
  - u_j = velocity (mi/hr) of vehicle j, $u_j = \frac{1}{p_j}$
- If traffic is stationary, then time-mean speed = space-mean speed.
  - Proof: $v = v_i = \frac{1}{p_j}, \forall i, j$

C. Daganzo, *Fundamentals of transportation and traffic operations*, vol. 30. Pergamon Oxford, 1997. Chapter 4: Traffic flow theory.

Time and space means in practice

- Time-mean speeds: Often how dual inductance loop detectors in traffic management systems are configured
  - Ex. arithmetic average of vehicle speeds over 20-second intervals

- Space-mean speeds: In nearly all cases of traffic analysis, space-mean speeds should be used
  - Statistically more stable in short segments/durations
  - Weighs slower vehicles' speeds more heavily

Space-mean speed $v(\boldsymbol{A}) \equiv v_s(\boldsymbol{A})$
mean over space interval (specific time)

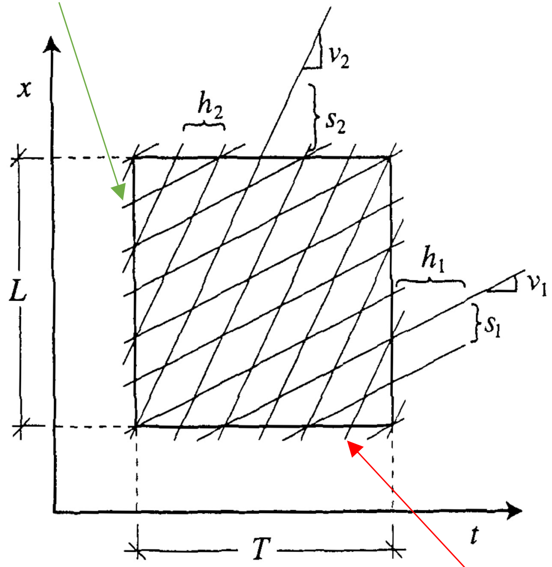


Figure 1.4 Time-space trajectories of two vehicle families.

Time-mean speed $v_t(\boldsymbol{A})$
mean over time duration (specific location)

Turner, Shawn M., William L. Eisele, Robert J. Benz, and Douglas J. Holdener. *Travel time data collection handbook*. No. FHWA-PL-98-035. United States. Federal Highway Administration, 1998. Chapter 1: Introduction. Link.
C. Daganzo, *Fundamentals of transportation and traffic operations*, vol. 30. Pergamon Oxford, 1997. Chapter 1: Time-space diagrams.

Time and space means in practice

- In practice, time-mean and space-mean speeds differ by 1-5%

- Differences are greater when there is more variability in speed (more congestion)

Table 1-1. Comparison of Time-Mean and Space-Mean Speeds

Data Items	Run 1	Run 2	Run 3	Run 4	Run 5	Sum	Average	Variance
Travel Time (sec)	153	103	166	137	127	686	137.2	
Running Time (sec)	142	103	141	137	127	650	130.0	
Stopped Delay Time (sec)	11	0	25	0	0	36	7.2	
Average Travel Speed (km/h)	44.7	66.4	41.2	49.9	53.9	256	51.2	95
Average Running Speed (km/h)	48.1	66.4	48.4	49.9	53.9	n.a.	52.6	

Section Length = 1.9 km

Difference between Time-Mean Speed and Space-Mean Speed
Time-Mean Speed = ∑(speeds) / no. of runs = 256 / 5 = 51.2 km/h
Space-Mean Speed = no. of runs × distance / ∑ (travel times) = 5 x 1.9 / 686 = 49.8 km/h
Therefore, difference = 1.4 km/h

Check Equation 1-5: Time-Mean Speed ≈ 49.8 + 95 / 49.8 ≈ 51.7 km/h ≈ 51.2 km/h

Turner, Shawn M., William L. Eisele, Robert J. Benz, and Douglas J. Holdener. *Travel time data collection handbook*. No. FHWA-PL-98-035. United States. Federal Highway Administration, 1998. Chapter 1: Introduction. Link.

Outline

1. Basic assumptions of traffic flow theory
2. **Fundamental diagrams (FDs)**
   a. **Basic relationship**
   b. FDs versus time-space diagrams
3. Highway delay problem

Basic relationship of traffic flow

$$q = v_s k$$

where:

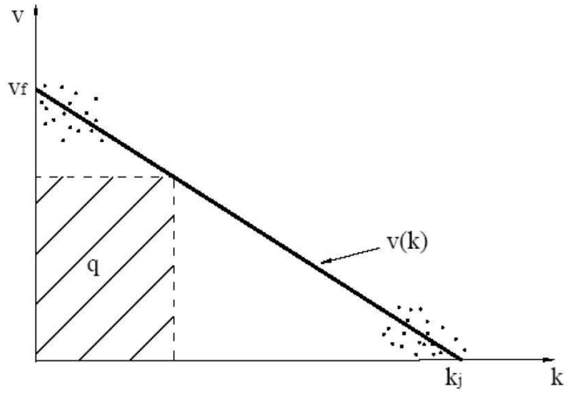
- q: flow [veh/h]
- v_s: speed (space-mean speed) [mi/h] or [km/h]
- k: density [veh/mi] or [veh/km]

- These are the three fundamental variables of traffic flow.

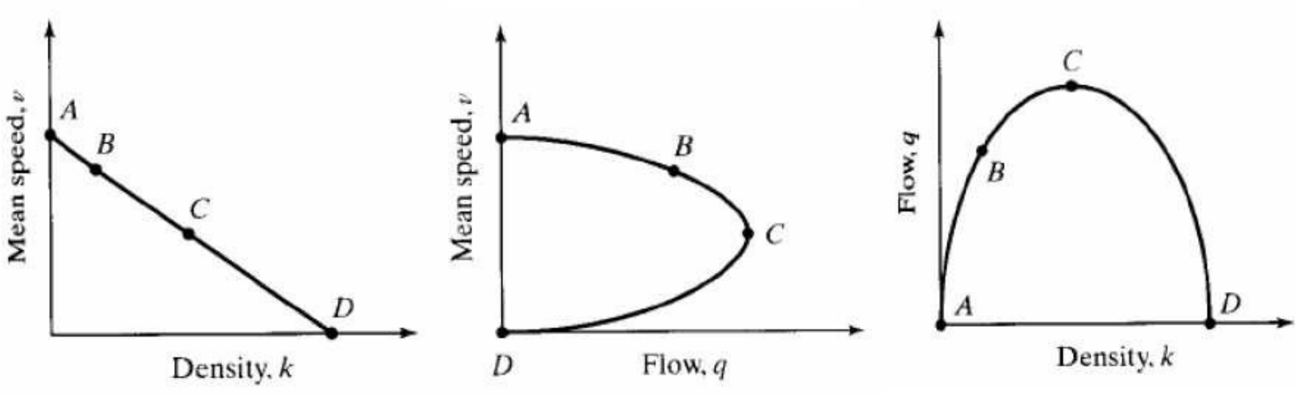
Traffic stream models

(1) Speed-density model

- Greenshields (1935), seminal work, assumes a linear relationship between speed and density
- From experimental data:
  - Light traffic → high speed
  - Heavy traffic → low speed (near zero).
- v_f : free flow speed
- k_{cap} = k_j : jam density



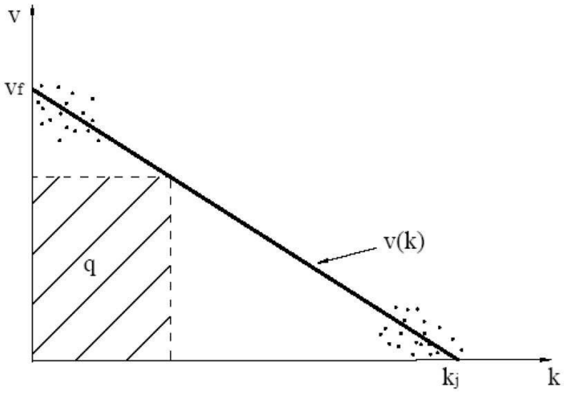
Traffic stream models



Traffic stream models are interrelated

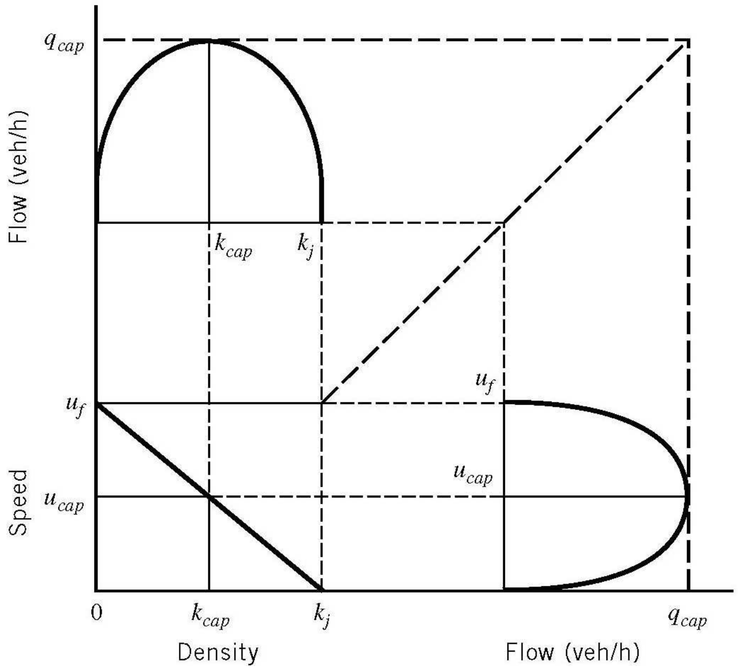
(1) Speed-density model

- Greenshields (1935), seminal work, assumes a linear relationship between speed and density
- v_f : free flow speed
- k_{cap} = k_j : jam density
- What is the corresponding relationship between flow and density?



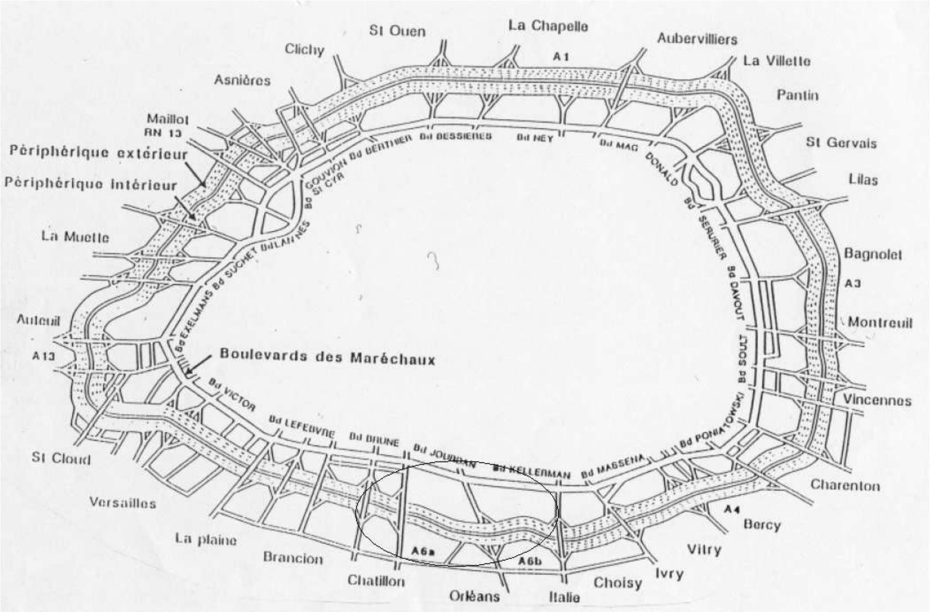
Interpretation of traffic stream models

- Each diagram relates the three fundamental variables, and are therefore called fundamental diagrams.
- For a given road, a fundamental diagram is fitted based on measurements
  - Points on the diagram describe possible traffic conditions
  - These relationships are postulated to be true "on average"



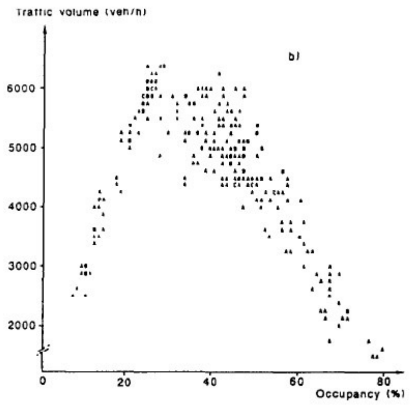
Fundamental Diagram in practice

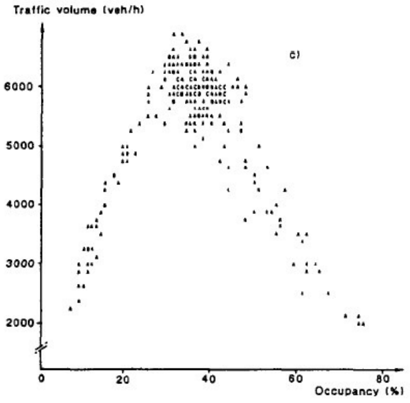
- Paris: boulevard periphérique
- Three locations (detectors)
- Measurements of car passages (traffic counts) and occupancies for each detector over a chosen time interval
- Source: Papageorgiou et al. (1990) Modelling and real-time control of traffic flow on the southern part of Boulevard Peripherique in Paris: Part I: Modelling, Transportation Research: Part A

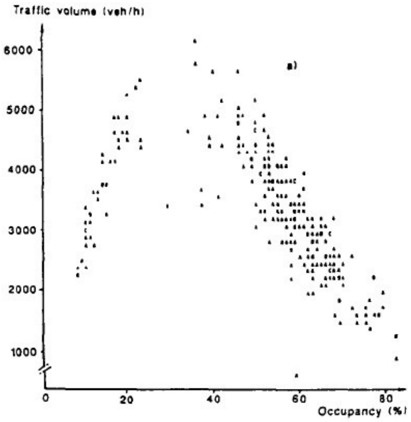


Fundamental Diagram in practice

- Flow-density model
- Measurements of car passages (traffic counts) and occupancies for each detector over time interval: 6am-10am of Nov. 27 1987
- Flow-density (volume-occupancy) diagrams taken over one-minute-intervals for a given location
- Traffic Occupancy (in %) (Def.: % of loop occupancy in a given time period)
- Conditions: heavily congested traffic after 7:40am, rainy

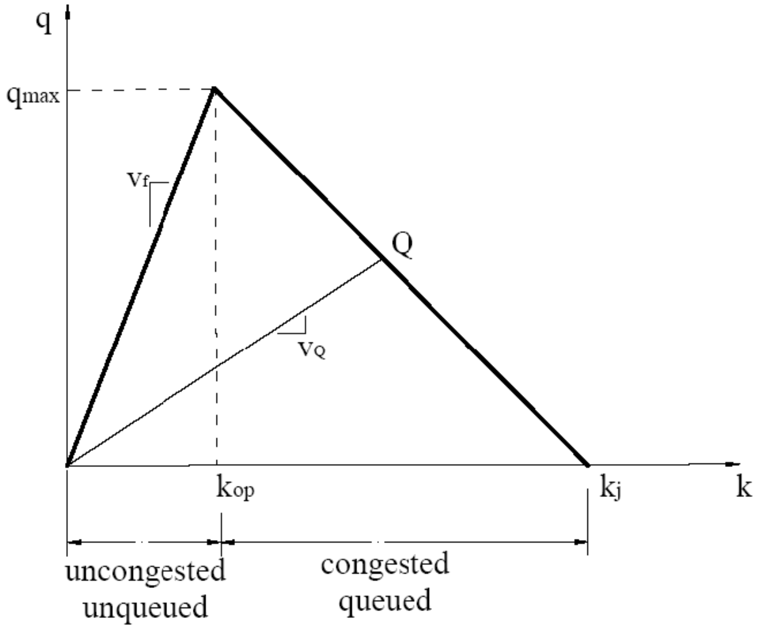






Triangle model

(3) Flow-density model



Outline

1. Basic assumptions of traffic flow theory
2. **Fundamental diagrams (FDs)**
   a. Basic relationship
   b. **FDs versus time-space diagrams**
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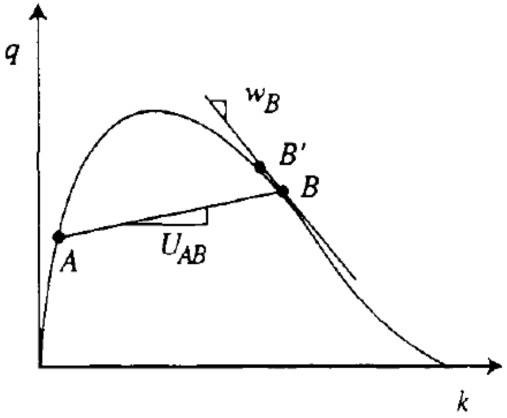
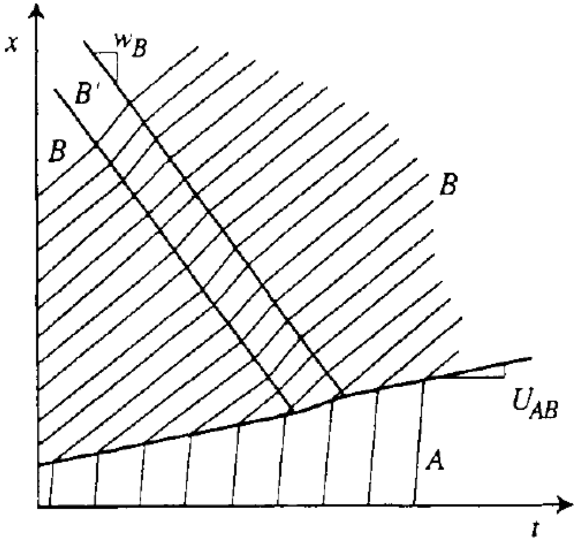
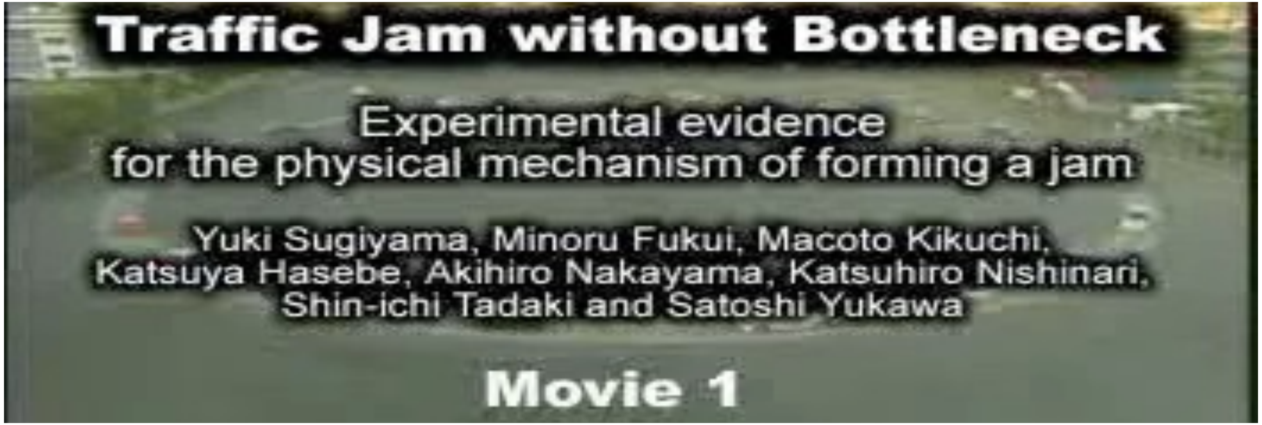
Fundamental diagram + (t,x)-diagram



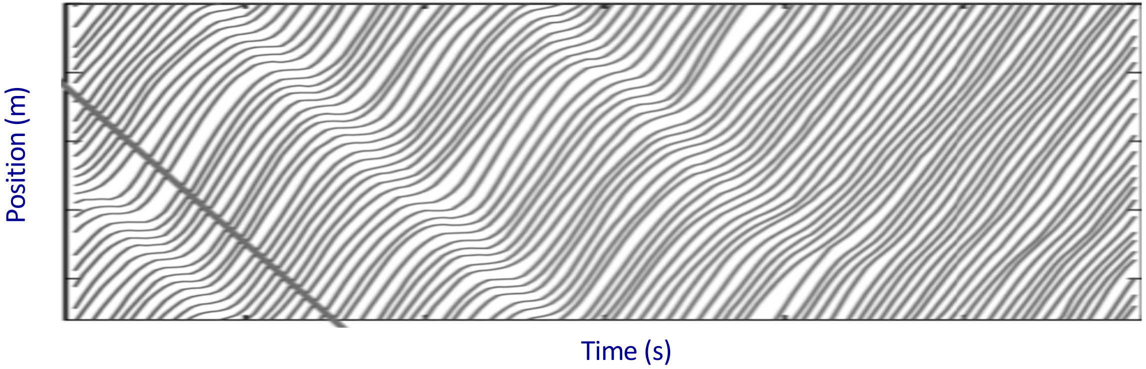
Source: C. Daganzo (1997)



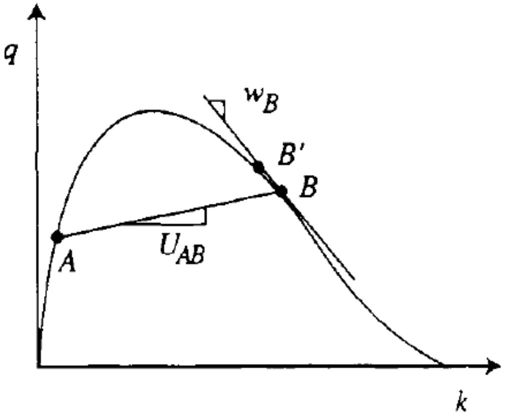
Recall: traffic waves



Vehicle trajectories (Sugiyama et al. 2008)



Traffic waves: fundamental diagram characterization



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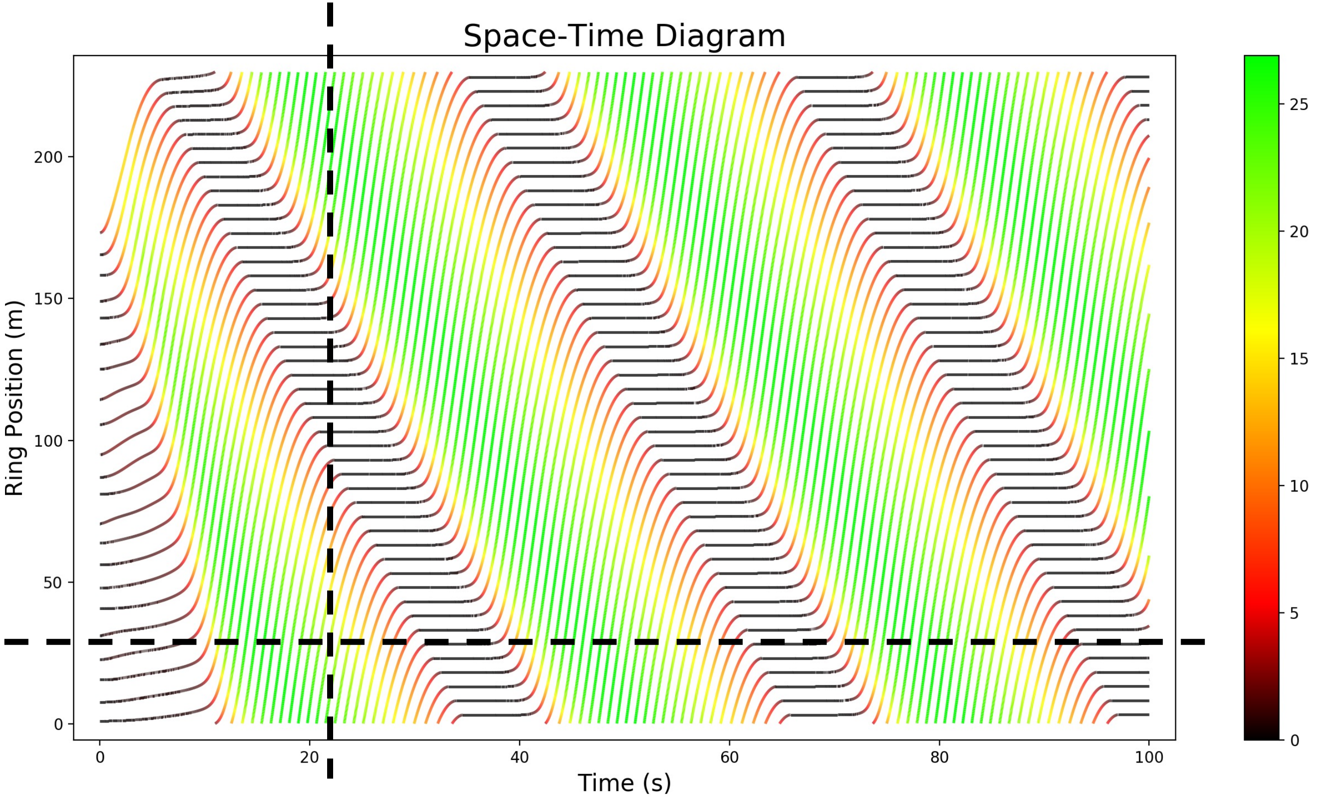

Wu, et al. IEEE T-RO, in review.

Outline

1. Basic assumptions of traffic flow theory
2. Fundamental diagrams (FDs)
3. **Highway delay problem**

Highway delay problem

A freeway exhibits a triangular flow-density relation with parameters: v_f (free flow speed), q_{max} (capacity) and k_j (jam density)

1. Plot the fundamental diagram and derive an expression for the function that gives the (space-mean) speed as a function of density inside a queue.
   - Don't forget to specify the range of k for which the equation holds.
2. If $k_j = 600$ veh/mile, $v_f = 1$ mile/min and $q_{max} = 100$ veh/min. Determine the delay experienced by a vehicle that joins a 2 mile queue caused by a bottleneck that flows at $q = 50$ veh/min.

Source: C. Daganzo

Highway delay problem

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References

1. C. Daganzo, *Fundamentals of transportation and traffic operations*, vol. 30. Pergamon Oxford, 1997. Chapter 4: Traffic flow theory
2. Prof. Nikolas Geroliminis' lecture Fundamentals of Traffic Operations and Control, Spring 2010 EPFL
3. Lecture notes: Principles of Highway Engineering and Traffic Analysis (2009) by Fred Mannering, Scott Washburn and Walter Kilareski
4. Lecture notes: 9th Dynamic Traffic Flow Modeling and Control (2010) by Prof. Markos Papageorgiou
5. Fred Hall (1997) Traffic stream characteristics. (available online)
6. Many slides adapted from Carolina Osorio