1.041/1.200 Spring 2024: Recitation 10

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1 Problem 1 : Linear Programs : Assignment Problem

Supply we have n customers and m warehouse. Customer $j(i = 1, 2, \ldots, n)$ wants d_j units of goods. Warehouse i has a stock of s_i unit of goods. Assume $\sum_{i=0}^{m} s_i \geq \sum_{j=0}^{n} d_j$. The cost to transport one unit of goods from warehouse i to customer j is c_{ij}. Assume that all warehouses can send goods to all customers.

a) Write an LP model to find a transportation plan that can meet the needs of all customers and has minimum cost.

2 Problem 2 : Linear Programs : Network Flow Problem

Let $\mathcal{N}$ be the set of all villages (nodes). Denote (i, j) as the one-way road (arc) connecting village i and $j(i, j \in \mathcal{N})$. For some village pair, there may not be an arc connecting them directly. Let $\mathcal{A}$ be the set of all existing arcs. A merchant wants to transport K units of goods from village $s \in \mathcal{N}$ to village $d \in \mathcal{N}$. The cost of transport one unit of goods on arc $(i, j) \in \mathcal{A}$ is c_{ij}. Also, on arc (i, j) the merchant can transport at most u_{ij} units of goods, i.e., the capacity of arc (i, j) is u_{ij}. Note that the merchant can also sale goods in the intermediate villages.

a) Write an LP model that can find a transport plan with minimum cost.

3 Problem 3 : Graphical Method

Consider the following IP:

$$\begin{aligned} \min \quad & x_1 - 2x_2 \\ \text{s.t.} \quad & -4x_1 + 6x_2 \leq 9 \\ & x_1 + x_2 \leq 4 \\ & x_1, x_2 \geq 0 \\ & x_1, x_2 \in \mathbb{Z} \end{aligned}$$

a) Solve the LP relaxation graphically and determine the optimal solution.