

# 1.041/1.200 Spring 2024: Recitation 1

Date: Feb 12, 2:00 PM

## 1 Problem 1

We'll use time-space diagrams to look at how a temporary obstruction affects traffic flow on a single lane road. Denote an arbitrary location on the road as reference location  $l = 0$ , and assume that all vehicles travel down the road as follows:

- Vehicles attempt to travel 20 meters per second and will do so if none of the following spacing rules would be violated; otherwise a vehicle may travel slower.
- A vehicle always leaves at least 20 meters of space between itself and a stopped vehicle in front.
- A vehicle always leaves at least 60 meters of space between itself and a moving vehicle in front.

To simplify sketching plots, assume all vehicles can accelerate and decelerate instantly and drivers have instant reaction times - both unrealistic assumptions of course, but adequate for this problem.

1. Suppose there are four vehicles denoted V1, V2, V3, V4. At time  $t = 0$ , the first vehicle begins traveling down the road from location  $l = 0$ , and every 10 seconds thereafter another vehicle follows. Draw the time-space diagram representing these four vehicles trajectories. The time axis should include  $t = 0$  seconds to  $t = 40$  seconds.
2. Now suppose that when V1 reaches location  $l = 100\text{m}$ , a minor fender-bender occurs in front of V1 and the driver slams on the brakes. The driver of V1 takes one minute to ascertain that nobody is hurt and the road is clear, and then proceeds. Draw a time-space diagram with the trajectories of all four vehicles. The time axis should run from  $t = 0$  seconds to  $t = 80$  seconds.
3. For this sub-question, define the location of a traffic jam as the location of the last stopped car in the jam. Between times  $t = 15$  seconds and time  $t = 68$  seconds, how far does the (small) traffic jam move, and in what direction?

4. Define  $\delta_i$  as the time vehicle  $V_i$  reaches location  $l = 200\text{m}$ . Let  $\delta = \delta_1 + \delta_2 + \delta_3 + \delta_4$ . What is the value of  $\delta$ ?
5. Suppose that one of the four vehicles has slightly different behavior: instead of traveling at  $20\text{m/s}$  subject to spacing rules, this driver travels at  $25\text{m/s}$  subject to the same spacing rules. In order to minimize  $\delta$  (and thus delay due to the obstruction), which of the four vehicles should be extra-speedy? What is the improved value of  $\delta$ ?

## 2 Code the Time-Space Diagram

1. **Define your Axis:** Horizontal Axis (X-axis): Represents time, typically with time increasing as you move up the axis, measured in seconds, minutes, or hours. Vertical Axis (Y-axis): Represents space, such as distance along a road or path, often measured in kilometers or miles.
2. **Identify the entity and scenario:** Decide what you are tracking on the diagram. This could be individual vehicles, groups of pedestrians, or any entities moving through space over time. Also, it is important to notice on what is the time and space you are observing, e.g. a single lane or some more complexed traffic scenarios.
3. **Collect trajectory data:** Gather data on the positions of the entities at different times. This might involve recording the time when vehicles pass certain milestones or using sensors or video analysis to track movements.
4. **Plot the data points:** For each entity, plot points on the graph corresponding to its position at different times. For example, if a car starts at the  $0\text{ km}$  mark and reaches the  $1\text{ km}$  mark in  $5\text{ minutes}$ , you would plot a point at the intersection of  $5\text{ minutes}$  on the X-axis and  $1\text{ km}$  on the Y-axis.

Hands-on practice: download the `Rec1_trajectory.csv` and `Rec1_plot_time_space.ipynb` in Piazza. Save them in the same folder in your local machine.