# 1.041/1.200 Spring 2024: Recitation 3 

Date: Feb 26, 2:00 PM

## 1 Problem 1: Fundamental Diagrams

Vehicle A traveling on a single-lane freeway joins a 0.5 mile long queue in front of a bottleneck at time $t=0 \mathrm{~min}$. At this point in time, the queue already contains 100 vehicles. The rate at which new vehicles join the queue is $80 \mathrm{veh} / \mathrm{min}$ and existing vehicles pass through the bottleneck at a rate of $50 \mathrm{veh} / \mathrm{min}$. The maximum capacity of the freeway is $100 \mathrm{veh} / \mathrm{min}$. When there is no queue, the free-flow speed of vehicles through the same bottleneck is $1 \mathrm{mile} / \mathrm{min}$. Based on these information, answer the following questions.

1. Determine the delay and the time in queue for vehicle A.
2. Determine the density of the queue.
3. Plot the triangular flow-density relation for the freeway based on the given information.

## 2 Problem 2 : Fundamental Diagrams

A road segment has the following speed-density relationship: traffic flows at an average free flow speed, $v_{f}$, for average densities between zero and $k_{c}$ (the critical density) and for average densities greater than $k_{c}$, the functional relationship between the speed and density forms a parabola, like in Figure 2. At the maximum density $k_{j}$, the gradient of the function $v(k)$ is zero, i.e., $v^{\prime}\left(k_{j}\right)=0$.


Figure 2

1. Find the expression for speed $v(k)$, which is considered as a function of the density $k$ for $0 \leq k$ $\leq k_{j}$.
2. What is the maximum flow of this road segment?

## 3 Code implementation of numerical integration

See Rec3Integration.ipynb.

