

1.041/1.200 Spring 2024: Recitation 4

Date: Mar 4, 2:00 PM

1 Problem 1: Poisson Process

You have been recruited as a summer intern at the MBTA's Green Line operations center. On your first day, you are given the task of observing passenger traffic at Northeastern University station during the afternoon off-peak in the inbound direction. Passenger arrivals on the platform can be assumed to arise according to a Poisson process with rate $\lambda_A = 120$ passengers per hour. The headway between two successive trains is exponentially distributed with an expected value of 6 minutes. You arrive at the station at 14:00 hours.

1. What is the probability that the time between two successive passenger arrivals is less than 5 minutes?
2. The first passenger you observe at the station arrives at 14:05 hours. It is now 14:10 hours and you still haven't observed any more passengers. What is the probability that the second passenger arrives after 14:15 hours?
3. What is the probability that you will observe 100 passengers arrive between 14:00 hours and 15:30 hours? How does this probability change if you know that another intern counted 90 passengers yesterday during the same time period?

For the following questions, we assume that the Green Line runs parallel to Bus Route #39 and the headway between successive buses is exponentially distributed with an expected value of 10 minutes. A passenger will board either the train or the bus, whichever arrives first at Northeastern after his own arrival.

4. What is the expected waiting time of a passenger under this scenario?
5. What is the probability that more than 5 buses arrive within the next hour?
6. What is the probability that the next two arrivals are both from green line trains?